Theoretisch-Physikalisches Institut
Friedrich-Schiller Universität Jena

Particles and Fields

Winter term 2018/19

Examples sheet 4

Exercise 8

We consider three real scalar fields $\phi^i(x)$ with $i \in \{1, 2, 3\}$ whose dynamics is described by

$$S[\phi^i] = \int d^4x \mathcal{L}[\phi^i, \partial_\mu \phi^i] = \int d^4x \left(-\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi^i(x) \partial_\nu \phi^i(x) - V(\phi^i \phi^i) \right) , \qquad (1)$$

where V is an arbitrary polynomial in $\phi^i \phi^i = (\phi^1(x))^2 + (\phi^2(x))^2 + (\phi^3(x))^2$.

(i) Derive the classical equations of motion for $\phi^i(x)$.

(ii) Show that the action (1) is invariant under the infinitesimal transformation

$$\delta_a \phi^i(x) = \epsilon^{ijk} a^j \phi^k(x) \, , \qquad (2)$$

where a^i is an infinitesimal parameter and ϵ^{ijk} is the antisymmetric tensor.

(iii) Show that the three Noether-currents J^i ($i \in \{1, 2, 3\}$), with components $J^{\mu i}$, are given by

$$J^{\mu i} = \epsilon^{ijk} \, \Pi^{\mu j} \, \phi_k \, , \qquad \Pi^{\mu j} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^j)} \, . \qquad (3)$$

Verify that the currents are conserved provided that the equations of motion are used.

In the second part of the examples class we will review the concept of Lie groups and Lie algebras. In particular, the exercise above can be reformulated in terms of the Lie algebra $\mathfrak{so}(3)$ whose structure constants are given by the antisymmetric tensor ϵ^{ijk}.