Theoretisch-Physikalisches Institut
Friedrich-Schiller Universität Jena

Particles and Fields

Winter term 2018/19

Examples sheet 3

Exercise 5

Let us consider a scalar field theory which is invariant under translations $x^\mu \mapsto \tilde{x}^\mu = x^\mu - a^\mu$. In the lecture we derived that $\theta^\mu{}_\nu$ is conserved, i.e. $\partial_\mu \theta^\mu{}_\nu = 0$. Hence P_ν defined by

$$P_\nu = \int d^{d-1}\boldsymbol{y}\, \theta^0{}_\nu = \int d^{d-1}\boldsymbol{y} \left(-\Pi(t,\boldsymbol{y})\partial_\nu \phi(t,\boldsymbol{y}) + \mathcal{L}\delta^0_\nu \right)$$

is the associated Noether charge. Show explicitly by using the Poisson brackets that the field transformation $\delta\phi(t,\boldsymbol{x})$ is given by

$$\delta\phi(t,\boldsymbol{x}) = a^\mu \left\{ \phi(t,\boldsymbol{x}), P_\mu \right\}_{P.B.} .$$

In other words, the Noether charge P_μ is the generator of spacetime translations.

Exercise 6

Let us consider electrodynamics with Lagrangian $\mathcal{L} = -1/4 F_{\alpha\beta}F^{\alpha\beta}$. Show that the energy momentum tensor $\theta^\mu{}_\nu$ defined by

$$\theta^\mu{}_\nu = -\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\rho)}\, \partial_\nu A_\rho + \mathcal{L}\delta^\mu{}_\nu$$

reads

$$\theta^{\mu\nu} = (\partial^\nu A_\rho)F^{\mu\rho} - \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} .$$

Show that by adding $\partial_\lambda f^{\lambda\mu\nu}$ to $\theta^{\mu\nu}$ where $f^{\lambda\mu\nu} = A^\nu F^{\lambda\mu}$, we obtain the canonical energy momentum tensor

$$T^{\mu\nu} = \theta^{\mu\nu} + \partial_\lambda f^{\lambda\mu\nu} = F^\nu{}_\rho F^{\mu\rho} - \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} .$$

Exercise 7

Consider a free real scalar field with vanishing mass. Show that the action

$$S = -\frac{1}{2}\int d^4x \eta^{\mu\nu}\partial_\mu\phi(x)\,\partial_\nu\phi(x)$$

is invariant under the dilatation transformation

$$x^\mu \mapsto \tilde{x}^\mu = e^\alpha x^\mu , \qquad \phi(x) \mapsto \tilde{\phi}(\tilde{x}) = \phi(x)e^{-d_\phi \alpha}$$

for an appropriate choice of d_ϕ. Determine the infinitesimal version of the transformation and compute the associated Noether current. Verify that the Noether current is conserved.

Is the dilatation transformation still a symmetry if we add an interaction term of the form $\lambda\phi^4(x)$? What happens if we add a mass term for the scalar field? Generalize the results above to arbitrary spacetime dimensions d.