Theoretisch-Physikalisches Institut
Friedrich-Schiller Universität Jena

Particles and Fields

Winter term 2018/19

Examples sheet 1

to be solved in the examples classes

Exercise 1: Equation of motion for scalar field theory

We consider a self-interacting scalar field $\phi(x)$ of mass m with interaction potential $\mathcal{V}(\phi(x)) = g\,\phi(x)^n$. The Lagrange density reads

$$\mathcal{L}(\phi(x), \partial_\mu \phi(x)) = -\frac{1}{2}\eta^{\mu\nu}\partial_\mu \phi(x)\partial_\nu \phi(x) - \frac{1}{2}m^2\phi(x)^2 - \mathcal{V}(\phi(x))\,.$$

(i) Determine the corresponding equation of motion!

(ii) What are the possible boundary conditions if we consider

- Minkowski spacetime $\mathbb{R}^{d-1,1}$,
- Spacetimes of the form $\mathbb{R}^{d-2,1} \times S^1$ where one of the spatial coordinates is compactified to a circle of radius R,
- Spacetimes of the form $\mathbb{R}^{d-2,1} \times I$ where one of the spatial coordinates takes values in the interval $I = [y_0, y_1]$.

(iii) Solve the equation of motion for Minkowski spacetime $\mathbb{R}^{d-1,1}$ in the case of a non-interacting scalar field theory, i.e. for $g = 0$.

Exercise 2: Equations of motion for electrodynamics

Show that the equations of motion corresponding to the Lagrange density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}\,F^{\mu\nu} - A_\mu J^\mu$$

reduce to the Maxwell equations. The field strength tensor $F_{\mu\nu}$ is given in terms of the vector potential A_μ by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.