

7. EXERCISE SHEET: QUANTUM FIELD THEORY

Aufgabe 15:

Wick's theorem shares some structures with a Gaussian integral: Consider the following "expectation values"

$$\langle x^{2n} \rangle := \int_{-\infty}^{\infty} dx x^{2n} e^{-\frac{1}{2}ax^2}, \quad a > 0, \quad n \in \mathbb{N}.$$

- (a) First determine the integral, e.g. by iteratively taking derivatives of a Gaussian integral $\int dx e^{\frac{1}{2}ax^2}$ with respect to a .
- (b) Now consider – analogously to Wick's theorem – the generating function

$$Z(J) := \langle e^{Jx} \rangle$$

with a source $J \in \mathbb{R}$. Solve the corresponding integral by completing the square and compute the expectation values by taking derivatives with respect to the source.

Aufgabe 16:

Convince yourself diagrammatically that the number of closed loops L of a connected diagram is given by

$$L = I - V + 1$$

where I is the number of inner lines and V is the number of vertices. (Hint: start with $L = 0$).

Aufgabe 17:

Consider a ϕ^3 theory with $\mathcal{H}_I = \frac{g}{3!}\phi^3$ and determine the scattering amplitudes for

- (a) 2-to-2 scattering to order g^2 and
- (b) 2-to-3 scattering to order g^3 .

For the (b) part, it suffices to work out the analytic representation of only one of the diagrams explicitly.