1. EXERCISE SHEET: QUANTUM FIELD THEORY

Aufgabe 1:

Determine your body height (or your age) in units of inverse GeV.

Aufgabe 2:

Two grounded conductors attract each other solely due to their interactions with the vacuum fluctuations of the radiation field. Determine the scaling of the distance dependence of this *Casimir force* for the case of two parallel infinitely extended perfectly conducting plates as a function of their distance a. For this, you only need to argue on the basis of dimensional considerations using the fact that the origin of the effect is a relativistic quantum (field) theory. Consider the force per unit surface element.

Aufgabe 3:

Consider the Green's function equation for the D-dimensional Euclidean Laplace operator with mass m,

$$(-\partial^2 + m^2)G(x-x') = \delta^{(D)}(x-x'), \quad \partial^2 = \partial_\mu\partial_\mu, \quad \mu = 1, \dots, D, \quad D > 2.$$

Determine the Green's function $G(x-x')$. How does $G(x-x')$ look like in the limit $m \to 0$? What is the behavior of G for large distances $(x-x') \gg (1/m)$?

Hints:

(a) First perform a Fourier transformation and determine $\widetilde{G}(p)$ in momentum space.

(b) For the Fourier transformation back to coordinate space, use the "propertime" representation

$$\frac{1}{p^2+m^2} = \int_0^\infty dT\, e^{-(p^2+m^2)T}.$$

(c) The following integral representation of the modified Bessel function (MacDonald function) may be useful:

$$K_\nu(x) = \frac{1}{2}\left(\frac{2}{x}\right)^\nu \int_0^\infty dT\, T^{\nu-1}\, e^{-T-\frac{x^2}{4T}}, \quad K_\nu(x) = K_{-\nu}(x).$$

Its asymptotic behavior is:

$$K_\nu(x \gg 1) \simeq \sqrt{\frac{\pi}{2x}}\, e^{-x}, \quad K_\nu(x \ll 1) \simeq \frac{\Gamma(\nu)}{2}\left(\frac{2}{x}\right)^\nu.$$