13. exercise sheet: Particles and Fields

Exercise 37:

Show that the defining property of the Lie algebra $[T^a, T^b] = if^{abc}T^c$ is also satisfied by the structure constants f^{abc} themselves by identifying $(T^a)^{bc} = -if^{abc}$. This characterizes the adjoint representation of the Lie algebra. Hint: use the Jacobi identity of the commutator.

Exercise 38:

Start from the covariant derivative $D_\mu = \partial_\mu - igA_\mu$ acting on fields in the fundamental representation, i.e. the gauge field is matrix valued, $A_\mu = A_\mu^a \tau^a$, where τ^a are the hermitean generators of an $\mathrm{SU}(N_\mathrm{c})$ Lie group. Show that the definition of the field strength $F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$ with $F_{\mu\nu} = F_{\mu\nu}^a \tau^a$ yields,

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c.$$

Exercise 39:

Given the classical Lagrangian for (Quantum) Chromodynamics,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi}i\not{D}\psi - m\bar{\psi}\psi,$$

derive the classical equation of motion for the gluon field A_μ^a.