2. exercise sheet: Particles and Fields

Exercise 3:

Use the Euler-Lagrange equations to derive the equations of motion for

(a) Maxwell's electrodynamics,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - J^\mu A_\mu.$$

(b) The theory of a complex Klein-Gordon field,

$$\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) - m^2 \phi^* \phi,$$

where $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$, $\phi_{1,2} \in \mathbb{R}$. Show that the equations of motion can also (more conveniently be obtained if ϕ and ϕ^* are considered as independent fields.

(c) Schrödinger theory,

$$\mathcal{L} = \psi^* i\partial_t \psi - \frac{1}{2m}(\nabla\psi^*) \cdot (\nabla\psi) - V(\mathbf{x})\psi^*\psi.$$

Use the same trick as in (b) and consider ψ and ψ^* as independent.

Exercise 4:

Consider the following Lagrange density (Proca theory)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\mu^2 A_\mu A^\mu - J^\mu A_\mu.$$

(a) Derive the equations of motion.

(b) Which condition has to be imposed on A_μ in order to maintain current conservation? How does this simplify the equations of motion?

(c) Consider the static limit, i.e., A_μ becomes independent of time. Let the current be given by a point charge $J_0 = q\delta^{(3)}(\mathbf{x})$, $J_i = 0$. How does the static potential A_0 look like? Interpret the quantity μ in the light of this result.

Aufgabe 5:

Determine the mass dimension of a Klein-Gordon field in D-dimensional spacetime.