1. exercise sheet: Particles and Fields

Exercise 1:

Use the relation $\hbar c \simeq 197$ MeV fm valid in SI units to compute your body height in inverse eV for those units where $\hbar = 1 = c$.

Exercise 2:

Show that the particular Lorentz transformation Λ discussed in the lecture, corresponding to a boost along the x axis, can be written as $e^{-\zeta K_1}$, where

$$K_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \zeta = \mathrm{Artanh}\beta \qquad \Longrightarrow \quad \Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Convince yourself that a boost in a general direction given by the relative velocity vector $\boldsymbol{\beta}$ can be written as $e^{-\boldsymbol{\zeta}\cdot\boldsymbol{K}}$. Work out the relation between $\boldsymbol{\beta}$ and $\boldsymbol{\zeta}$ as well as the form of the matrices K_2 and K_3. (We have $\gamma = 1/\sqrt{1-\beta^2}$.)

Exercise 3:

Verify that the matrix Λ given above satisfies the relation

$$g_{\mu\nu} = g_{\kappa\lambda}\Lambda^{\kappa}{}_{\mu}\Lambda^{\lambda}{}_{\nu},$$

where the metric is $g = \mathrm{diag}(1, -1, -1, -1)$.