

Asymptotically safe scenarios in particle physics

Holger Gies

Institute for Theoretical Physics
Heidelberg University



Outline

1 Asymptotic safety



2 Scalar O(N) Theories



3 Fermionic Theories



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1 Asymptotic safety



2 Scalar O(N) Theories



3 Fermionic Theories



Why physics beyond the standard model.

- abundance of parameters
- origin of flavor physics
- origin of neutrino physics

Conceptual issues

- hierarchy problem (naturalness problem, fine-tuning problem)

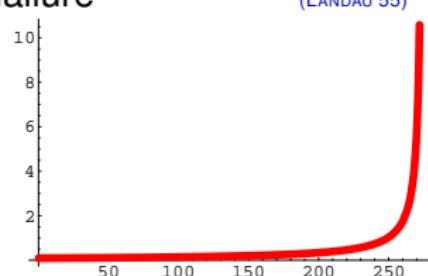
$$\Lambda_{\text{UV}} \gg \Lambda_{\text{EW}} \quad (\gg \Lambda_{\text{QCD}})$$

- triviality problem of U(1) and scalar Higgs sector
("Landau pole singularities")

Triviality problem.

- ▷ QED: perturbation theory predicts its own failure

$$\frac{1}{e_R^2} - \frac{1}{e_\Lambda^2} = \beta_0 \ln \frac{\Lambda}{m_R}, \quad \beta_0 = \frac{N_f}{6\pi^2}$$



- ▷ e_R^2 and m_R fixed:

$$\implies \Lambda_L \simeq m_R \exp\left(\frac{1}{\beta_0} e_R\right) \simeq 10^{272} \text{GeV (2 loop)}$$

Landau pole singularity

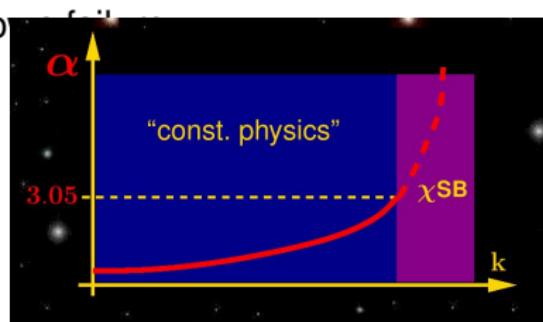
- ▷ $\lim (\Lambda/m_R) \rightarrow \infty:$ $\implies e_R \rightarrow 0$

Triviality

Triviality problem.

- ▷ QED: perturbation theory predicts its own

$$\frac{1}{e_R^2} - \frac{1}{e_\Lambda^2} = \beta_0 \ln \frac{\Lambda}{m_R}, \quad \beta_0 = \frac{N_f}{6\pi^2}$$



(HG, JAECKEL'04)

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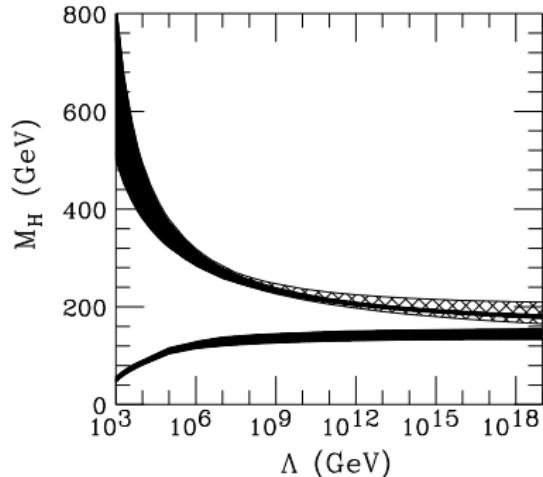
- ▷ $\lim (\Lambda/m_R) \rightarrow \infty:$ $\implies e_R \rightarrow 0$

Triviality

Triviality problem . . .

⇒ . . . scale of maximal UV extension

- ▷ triviality of the scalar Higgs sector:



(HAMBYE,RIESSELMANN'97)

⇒ SM Higgs mass bounds from Landau pole position

Hierarchy problem $\Lambda_{\text{UV}} \ggg \Lambda_{\text{EW}}$.

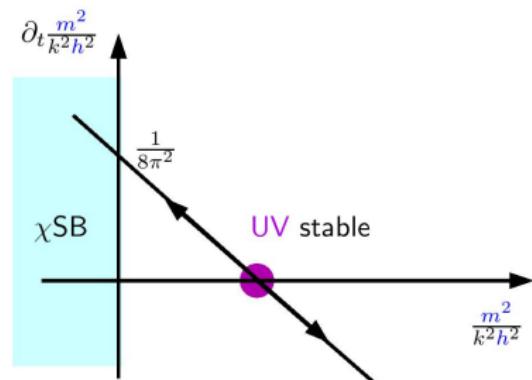
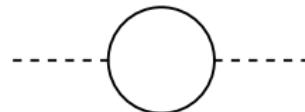
- renormalization of the scalar mass (e.g., $\Lambda_{\text{UV}} = 10^{16} \text{ GeV}$)

$$\underbrace{m_R^2}_{\sim 10^4 \text{ GeV}^2} \sim \underbrace{m_\Lambda^2}_{\sim 10^{32} \left(1 + \dots 10^{-28}\right) \text{ GeV}^2} - \underbrace{\delta m^2}_{\sim 10^{32} \text{ GeV}^2}$$

- RG viewpoint ($\partial_t = k \frac{d}{dk}$)

e.g., Yukawa theory:

$$\partial_t \frac{m^2}{k^2 h^2} = -2 \frac{m^2}{k^2 h^2} + \frac{1}{8\pi^2}$$

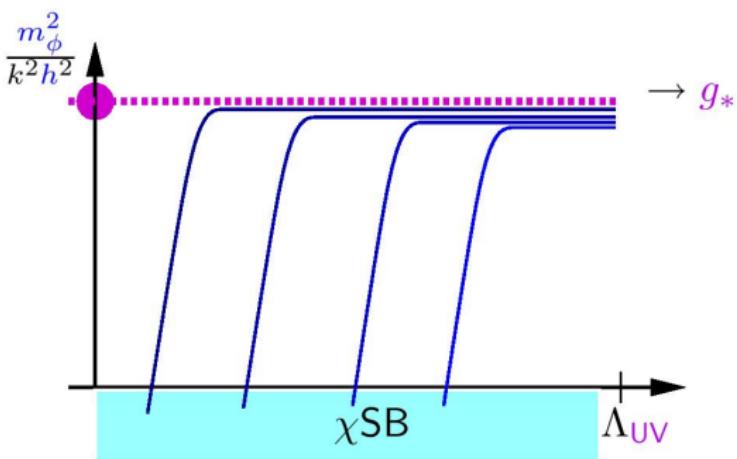


$\dots \cong$ Finetuning Problem.

- “coupling”: $\mathbf{g} := \frac{m^2}{\hbar^2 k^2}$, “ β function”: $\partial_t \mathbf{g} = \beta(\mathbf{g})$

▷ critical exponent Θ

$$\Theta = - \frac{\partial \beta(g_*)}{\partial g} = 2$$



$\Rightarrow \Theta$ ~ measure for the required finetuning

Renormalizability.

- IR physics well separated from UV physics
(... cutoff Λ independence)
- # of physical parameters $\Delta < \infty$
(... predictive power)

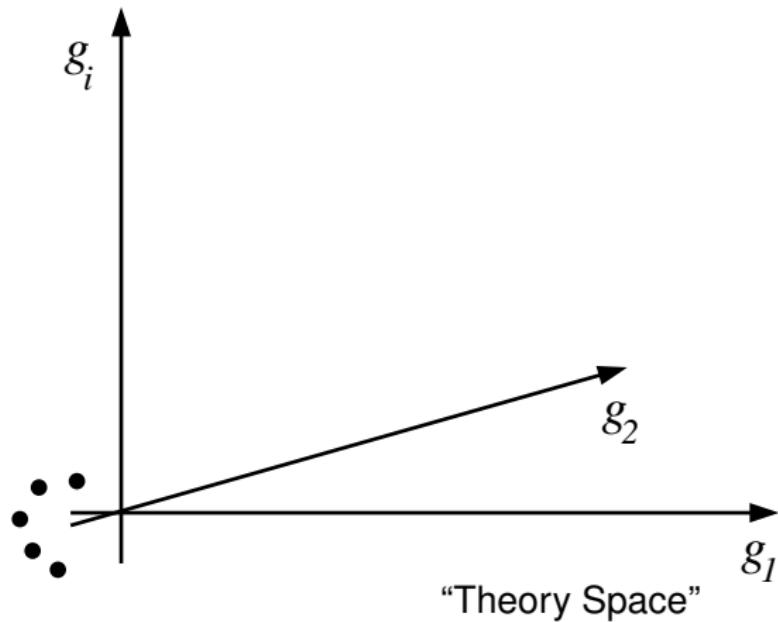
\implies realized by perturbative RG ...

\implies ... and by “Asymptotic Safety”

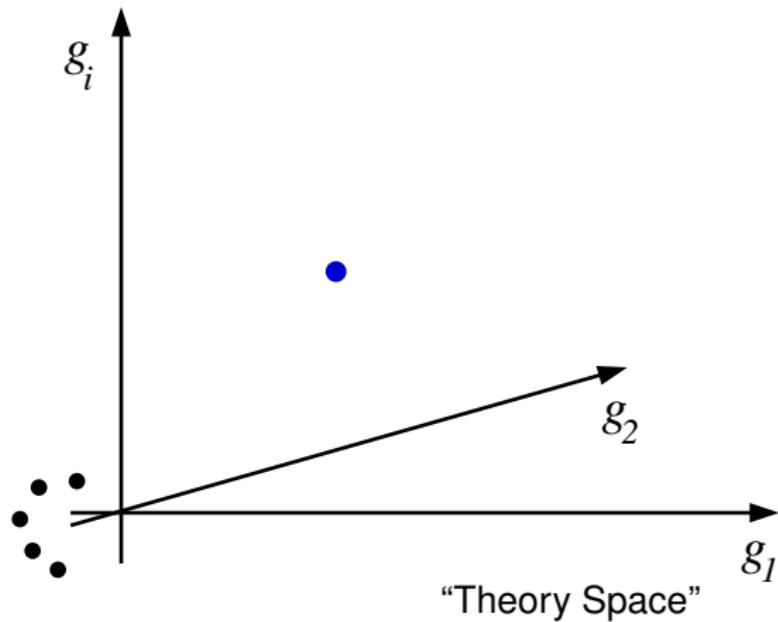
(WEINBERG'76)

(GELL-MANN, LOW'54)

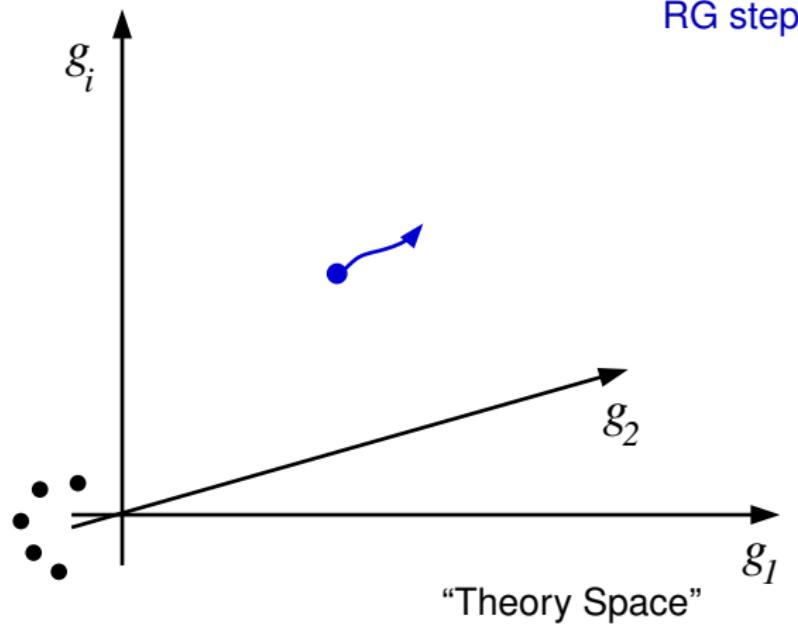
Asymptotic Safety.



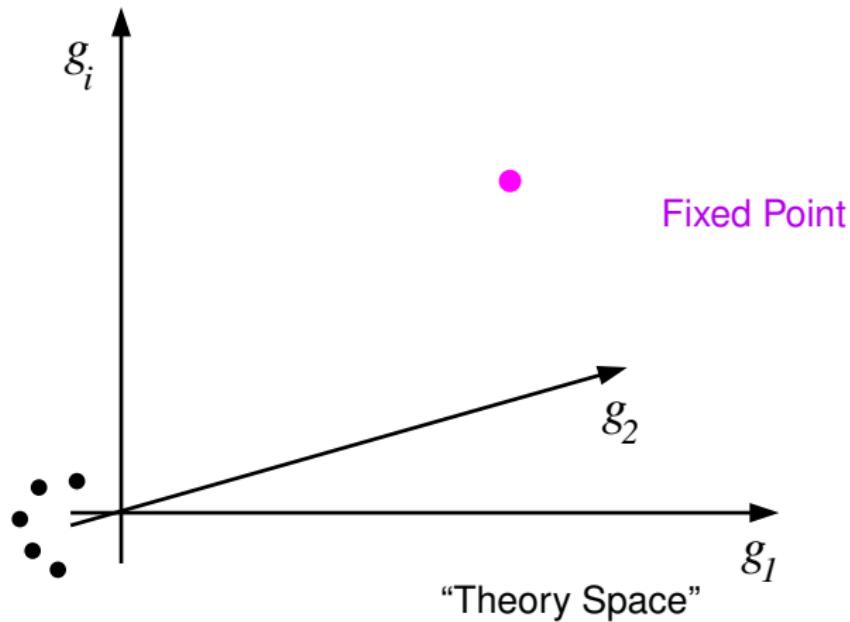
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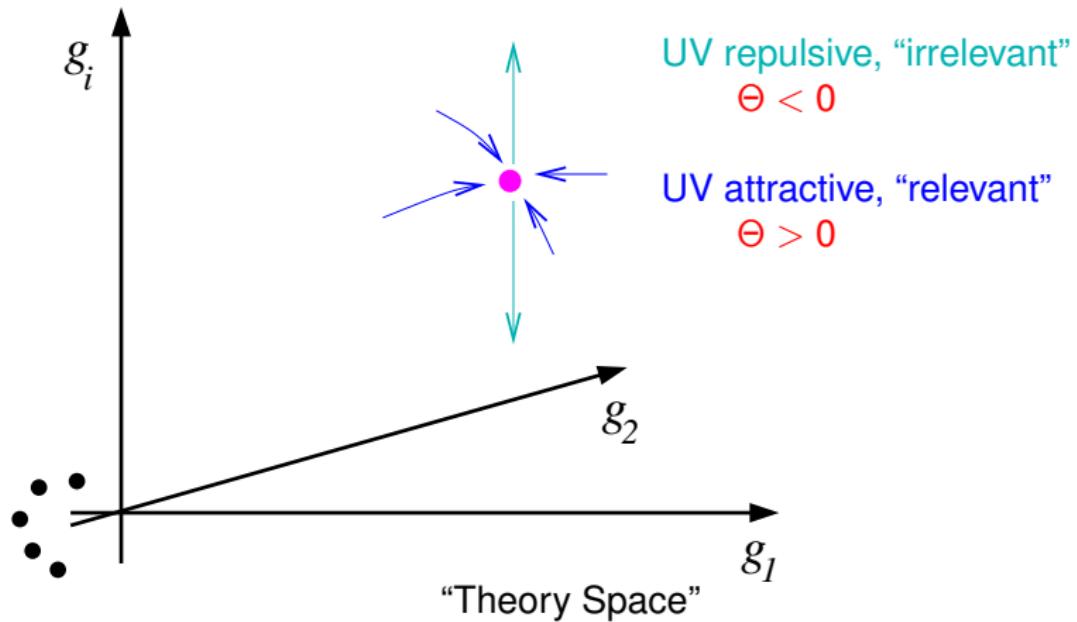
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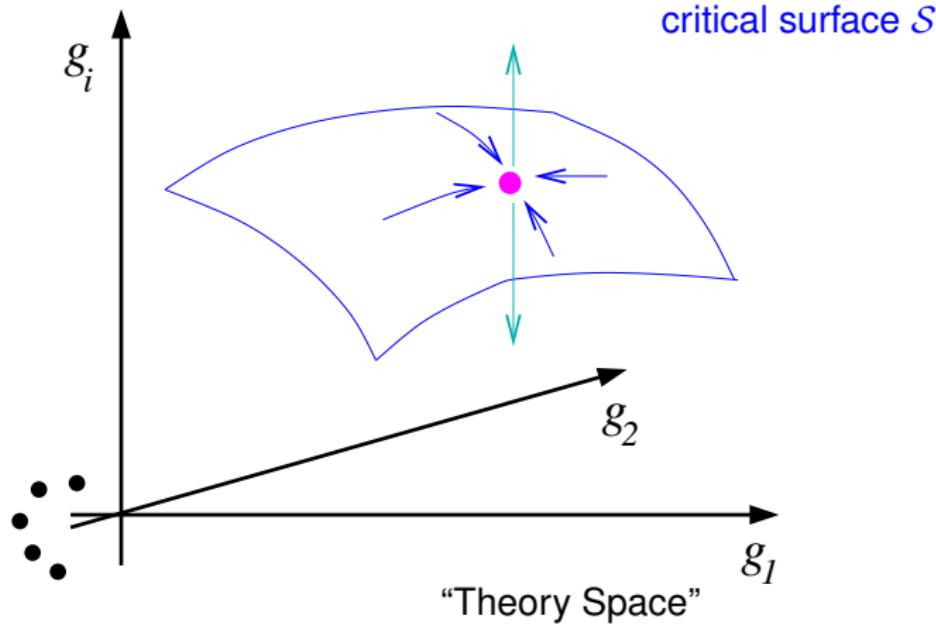
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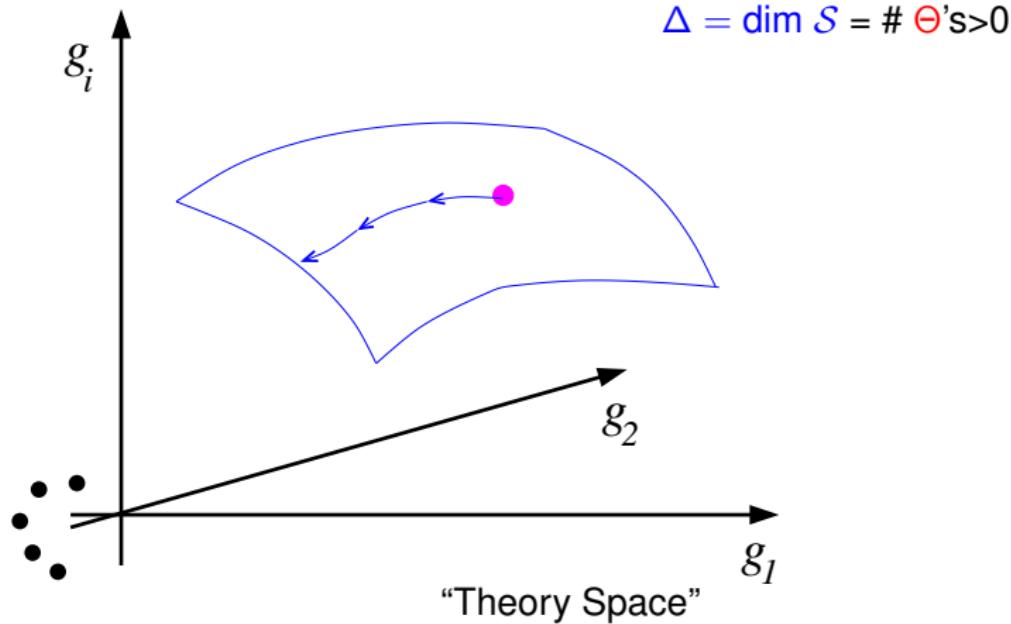
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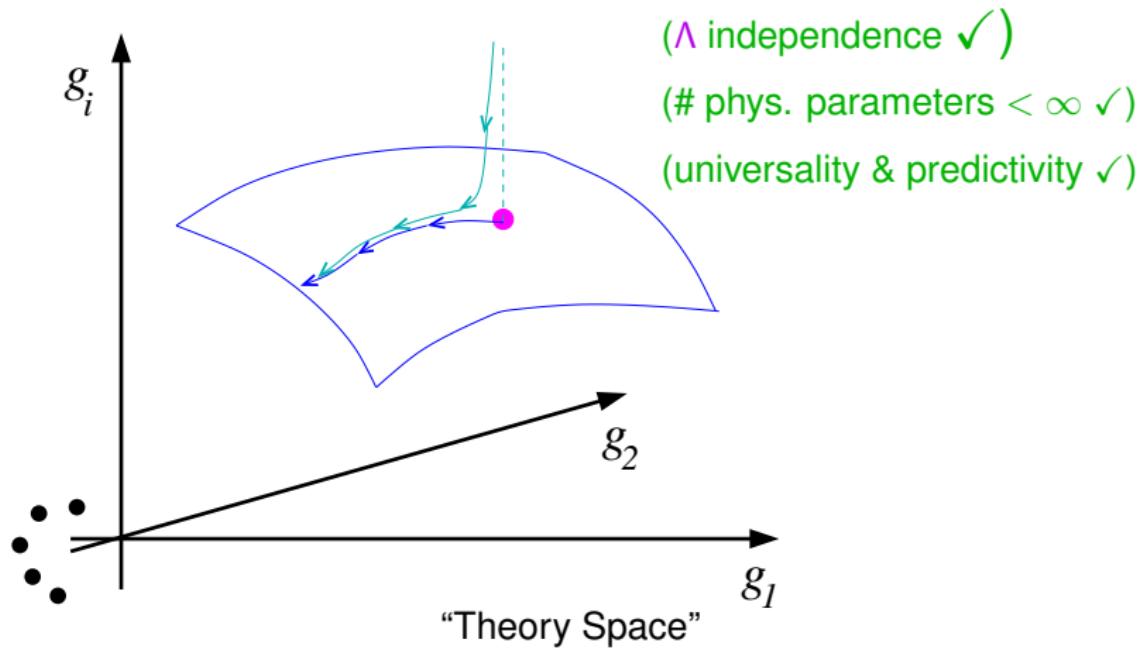
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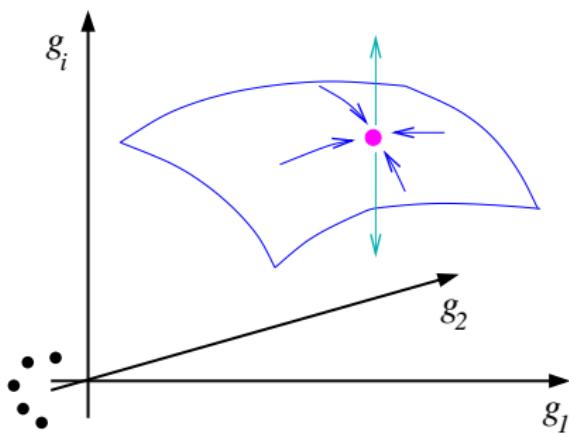
Asymptotic Safety.



Asymptotic Safety.



Asymptotic Safety.



▷ FP regime:

$$\partial_t g_i = B_i^j (g_j - g_{*j}) + \dots$$

▷ stability matrix

$$B_i^j = \frac{\partial \beta_i(g_*)}{\partial g_j}$$

▷ critical exponents:

$$\{\Theta\} = \text{spect}(-B_i^j)$$

“Theory Space”

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2 Scalar $O(N)$ Theories



3 Fermionic Theories



Asymptotically Safe Scalars ?

(SYMANZIK '70)

- ▶ no non-Gaußian FP is known

(LUSCHER ET AL.'88)

- ▶ Gaußian FP: asymptotically free ?

- ▶ polynomial potentials ?

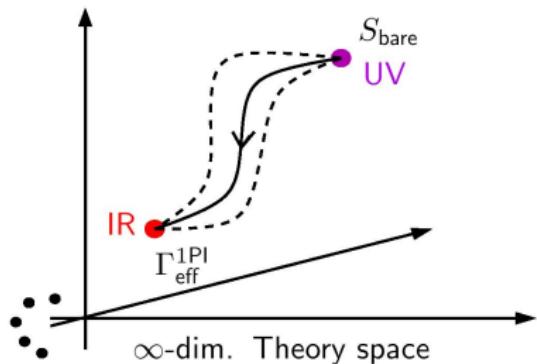
$$U(\phi^2) = \pm \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 + \frac{\lambda_6}{6!} \phi^6 + \frac{\lambda_8}{8!} \phi^8 \dots$$

Asymptotically Safe Scalars ?

- ▶ functional RG:

(WEGNER&HOUGHTON'73, WETTERICH'93)

$$\partial_t \Gamma_k = \frac{1}{2} \operatorname{Tr} \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$



- ▶ gradient expansion

$$\Gamma_k[\phi] = \int_x \left(U_k(\phi^2) + \frac{1}{2} (\partial_\mu \phi)^2 \dots \right)$$

- ▶ scaling variables:

$$\rho = \frac{1}{2} k^{-2} \phi^2, \quad u(\rho) = k^{-4} U_k(\phi^2)$$

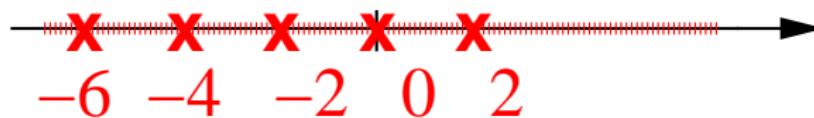
Flow in the FP regime.

- ▶ linearized flow, Gaußian FP $u_* = 0$:

(HALPERN&HUANG'95)

$$\partial_t u(\rho) = \textcolor{red}{B} u(\rho), \quad \textcolor{red}{B} = -4 + 2\rho\partial_\rho - \frac{\kappa[\textcolor{red}{R}_k]}{2} (2\rho\partial_\rho^2 + N\partial_\rho)$$

- ▶ critical exponents Θ :



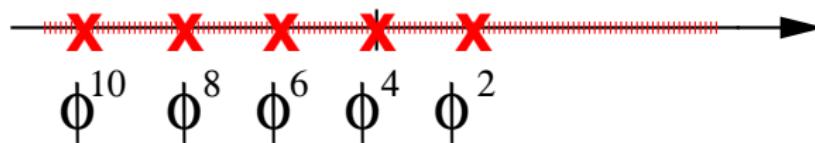
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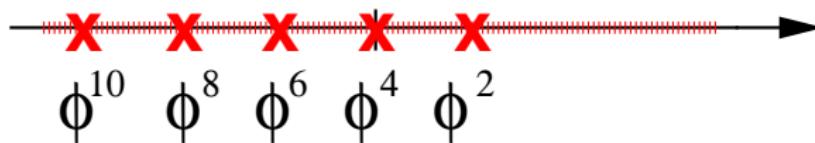
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- ▶ critical exponents Θ :



- ▶ Halpern-Huang potentials with “FP-distance parameter” r

$$u_k(\rho) = -e^{-\Theta t} \frac{2\kappa r}{4-\Theta} \left[M\left(\frac{\Theta-4}{2}, \frac{N}{2}; \frac{2}{\kappa}\rho\right) - 1 \right]$$

Halpern-Huang Potentials.

▷ Halpern-Huang potentials

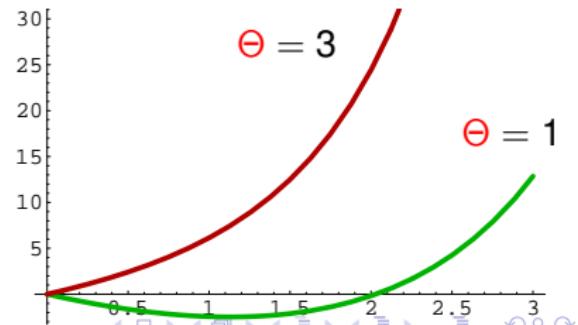
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- asymptotically free for $\Theta > 0$
- nonpolynomial for $\Theta \neq (4 - 2n)$



$$u(\rho \gg 1) \sim \rho^\alpha \exp(\beta\rho)$$

- SSB for $\Theta \in (0, 2)$
- UV → IR:
nonperturbative problem!



Halpern-Huang “Phenomenology”.

(HG'01)

++ from large- N flow

- small masses are “natural”

$$\frac{4}{N} \textcolor{green}{r} = \left(\frac{m_{\textcolor{red}{R}}^2}{\textcolor{violet}{\Lambda}^2} \right)^{\Theta/2} \left[\ln \left(1 + \frac{\textcolor{violet}{\Lambda}^2}{m_{\textcolor{red}{R}}^2} \right) \right]^{(\Theta-2)/2}$$

- hierarchy separation “built in”

$$\rho_{\min}(k) = \frac{N}{32\pi^2} k^2$$

Halpern-Huang “Phenomenology”.

(HG'01)

- # of physical parameters = # of Θ 's > 0

$$\Delta = \dim \mathcal{S} \rightarrow \infty$$

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Asymptotically Safe Fermion Interactions ?

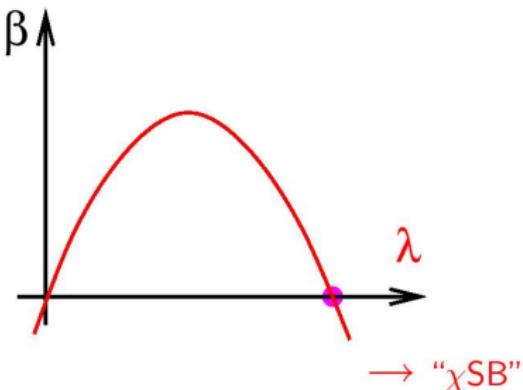
- ▷ Nambu–Jona-Lasinio / Gross-Neveu in 3 dimensions, $[\bar{\lambda}] = -1$:

$$\Gamma_K = \int \bar{\psi} i\partial^\mu \psi + \frac{1}{2} \bar{\lambda} (\bar{\psi} \psi)^2 + \dots$$

- ▷ dim'less coupling $\lambda = k \bar{\lambda}$

$$\partial_t \lambda = \lambda - \kappa \lambda^2$$

- ▷ UV fixed point $\lambda_* = 1/\kappa$
- ▷ critical exponent $\Theta = 1$



⇒ asymptotically safe

(GAWEDZKI, KUPIAINEN'85; ROSENSTEIN, WARR, PARK'89; DE CALAN ET AL.'91)

Towards the standard model . . .

- ▷ $U(1) \times SU(N_c)$ gauge symmetry
+ chiral $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry

(HG, JAECKEL, WETTERICH'04)

$$\Gamma_k = \int \bar{\psi} (iZ_\psi \partial + Z_1 \bar{g} A + Z_1^B \bar{e} B) \psi + \frac{Z_F}{4} F_z^{\mu\nu} F_{\mu\nu}^z + \frac{Z_B}{4} B^{\mu\nu} B_{\mu\nu}$$

$$+ \frac{1}{2} [\bar{\lambda}_- (V-A) + \bar{\lambda}_+ (V+A) + \bar{\lambda}_\sigma (S-P) + \bar{\lambda}_{VA} [2(V-A)^{\text{adj}} + (1/N_c)(V-A)]]$$

- ▷ pointlike four-fermion interactions

$$(V-A) = (\bar{\psi} \gamma_\mu \psi)^2 + (\bar{\psi} \gamma_\mu \gamma_5 \psi)^2$$

$$(V+A) = (\bar{\psi} \gamma_\mu \psi)^2 - (\bar{\psi} \gamma_\mu \gamma_5 \psi)^2$$

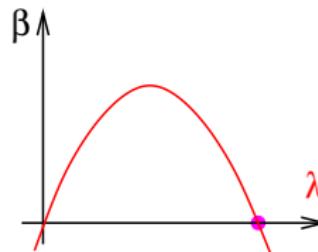
$$(S-P) = (\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2 \equiv (\bar{\psi}_i^a \psi_i^b)^2 - (\bar{\psi}_i^a \gamma_5 \psi_i^b)^2$$

$$(V-A)^{\text{adj}} = (\bar{\psi} \gamma_\mu T^z \psi)^2 + (\bar{\psi} \gamma_\mu \gamma_5 T^z \psi)^2$$

Fixed-Point Structure.

- ▷ flow structure ($e^2, g^2 \rightarrow 0$)

$$\partial_t \lambda_i = (d - 2)\lambda_i + \lambda_k A_i^{kl} \lambda_l$$

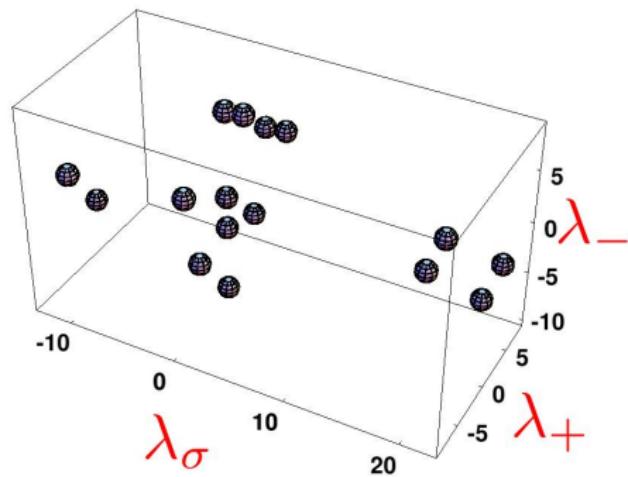


- ▷ 2 FPs per λ

$$\implies 2^4 = 16 \text{ FP}$$

- ▷ in general: 2^n FP's

for $n = \# \text{ of } \lambda \text{'s}$

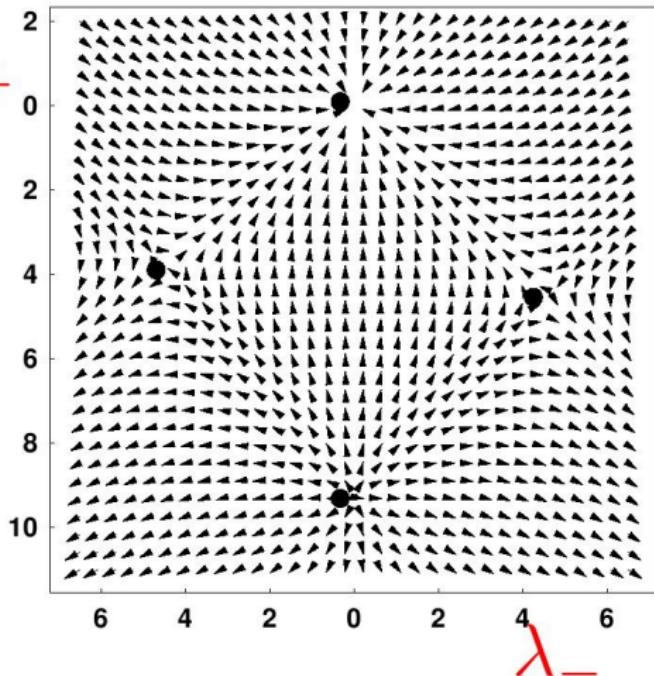


of Physical Parameters.

- ▷ # of FP's with j relevant directions:

$$= \begin{pmatrix} 4 \\ j \end{pmatrix}$$

⇒ 4 FP's with
only 1 parameter



Naturalness ?

- ▷ FP vector λ_{*i} is an eigenvector of the **stability matrix** ($e^2, g^2 \rightarrow 0$):

$$B_i^j \lambda_{*j} = -2 \lambda_{*i}, \quad \Theta_{\lambda_*} = 2 \leq \Theta_{\max}$$

... \geq SM hierarchy problem

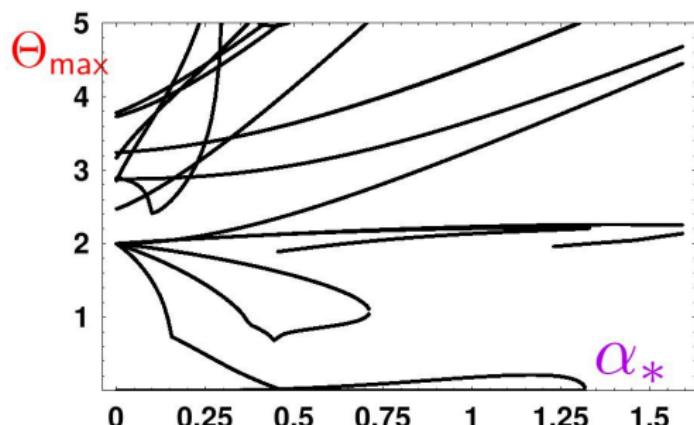
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... \geq SM hierarchy problem

- ▷ + gauge interactions
e.g., U(1) coupling > 0



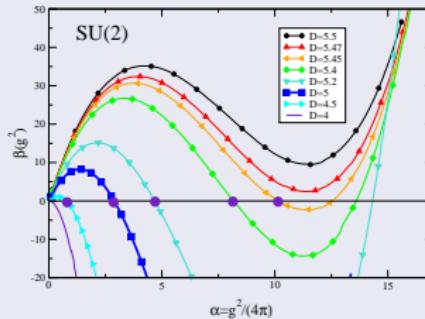
Conclusions

- Asymptotic-Safety scenarios offer new routes to old puzzles
- strong-coupling dynamics can induce UV stability

⇒ shift of critical dimension
e.g., YM in ED

(HG'03)

$$D_{\text{cr}} \simeq 5$$



- more operators \neq more parameters

Outlook

- strongly-coupled Yukawa systems
- large anomalous momentum dependencies

Strategies.

safe route to UV
(UV extension)



IR stable deformation



SM universality class

Strategies.

safe route to UV
(UV extension)

UV stable deformation
(strong dynamics ?)



IR stable deformation



SM universality class