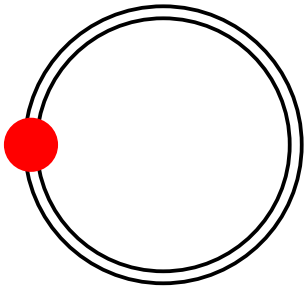
Running coupling and chiral symmetry restauration in QCD

Holger Gies, Heidelberg U.

- many-flavor QCD
- functional RG
- running coupling and χSB

$$\partial_t \Gamma_k = \frac{1}{2}\partial_t R_k$$

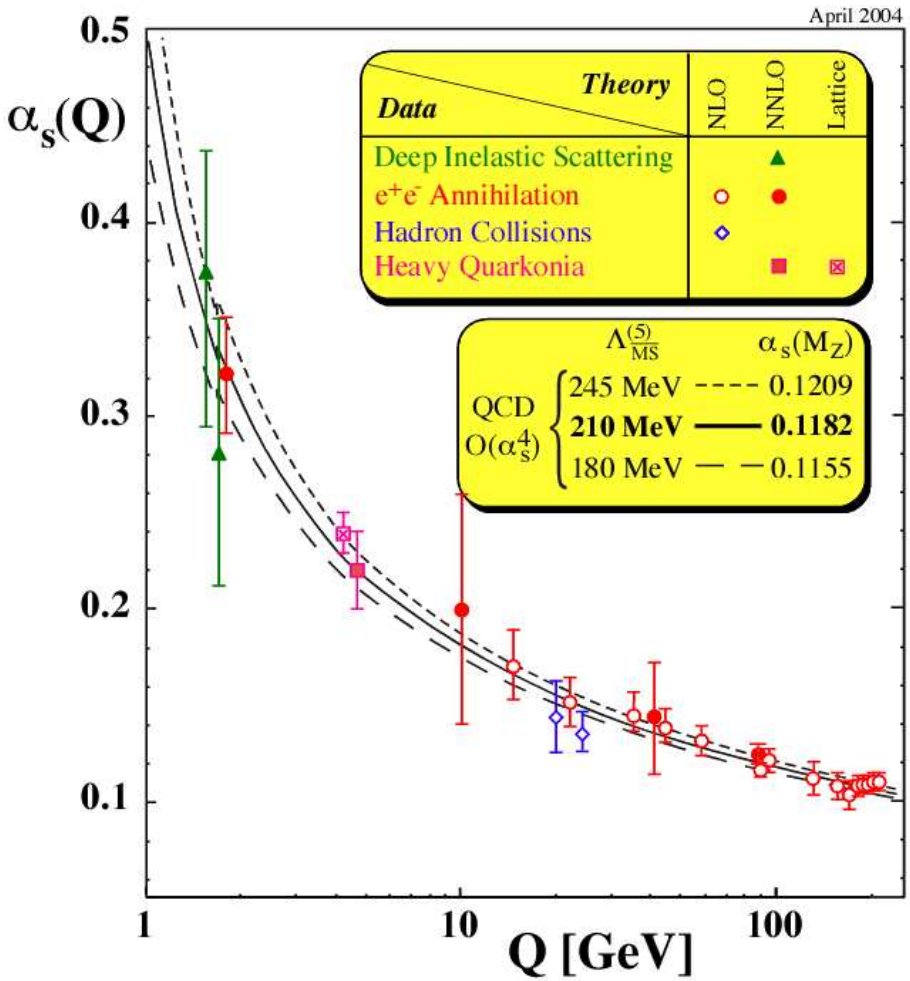


Collaborators: J. Jaeckel, C. Wetterich, J. Braun

(Phys. Rev. D66:025006,2002, Phys.Rev.D69:105008,2004, hep-ph/0507171, hep-ph/05MMXXX)

Aspects of QCD

- running coupling
- mass gap
- confinement
- chiral symmetry breaking
- hadron spectrum
- finite T, μ



(Bethke'04)

asymptotic freedom

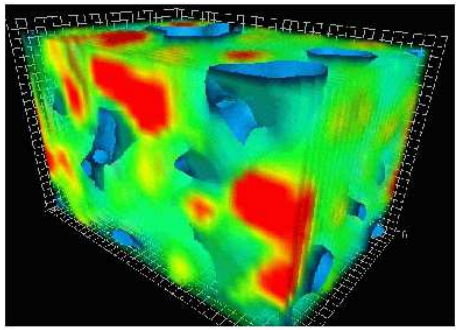
(Gross&Wilczek'73, Politzer'73)

“Learning by D o ing”

"Learning by Deforming"

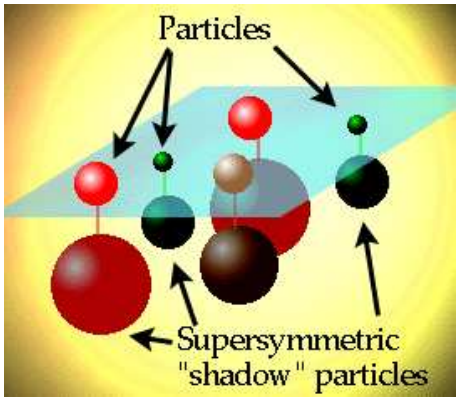
QCD

Lattice QCD



Large N_c

SUSY QCD



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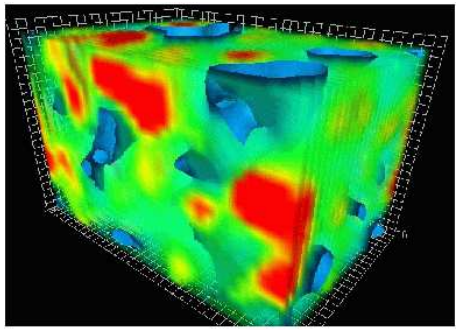

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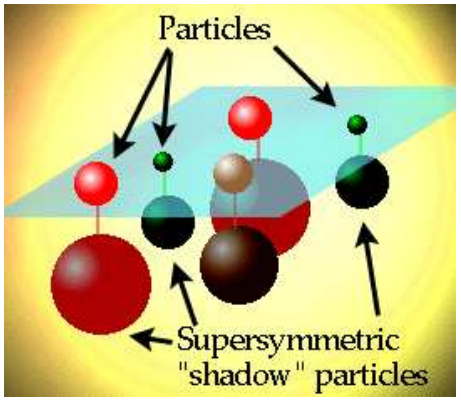
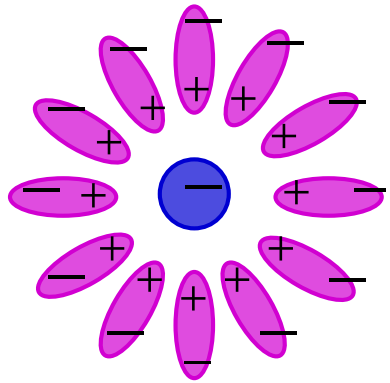


Large N_f

"many flavor QCD"

Many-flavor QCD

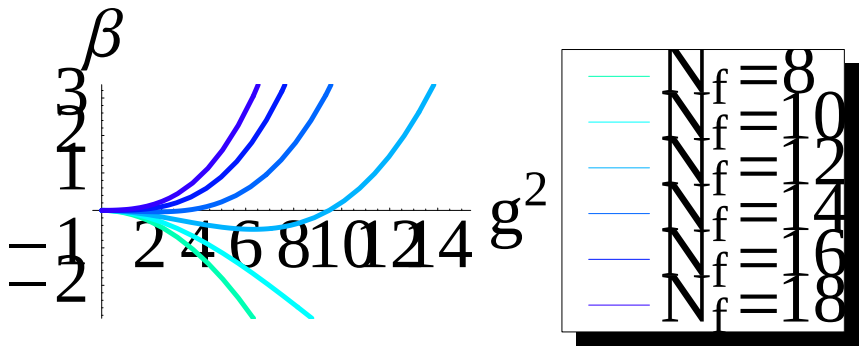


- charge screening:

- β function

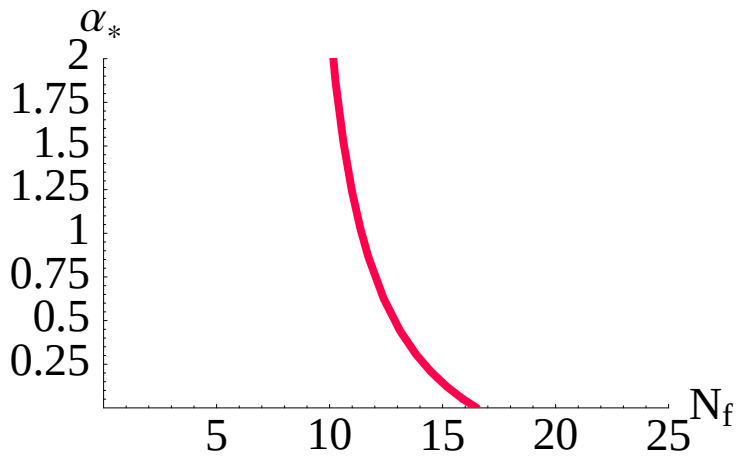
$$\beta = -2\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\frac{g^4}{16\pi^2} - 2\underbrace{\left(\frac{34{N_c}^3 + 3N_f - 13{N_c}^2 N_f}{3N_c}\right)}_{>0 \text{ for } N_f > \frac{34{N_c}^3}{13{N_c}^2-3} \overset{SU(3)}{\simeq} 8.05}\frac{g^6}{(16\pi^2)^2} + \dots$$

- e.g. SU(3): IR fixed point α_*

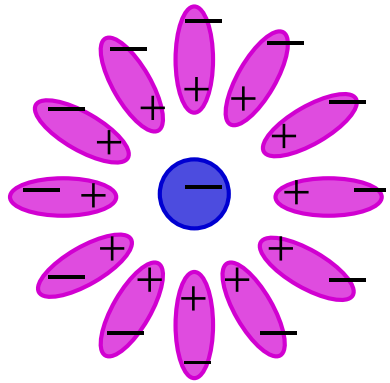


(Banks&Zaks'82)

- N_f dependence of α_*



Many-flavor QCD



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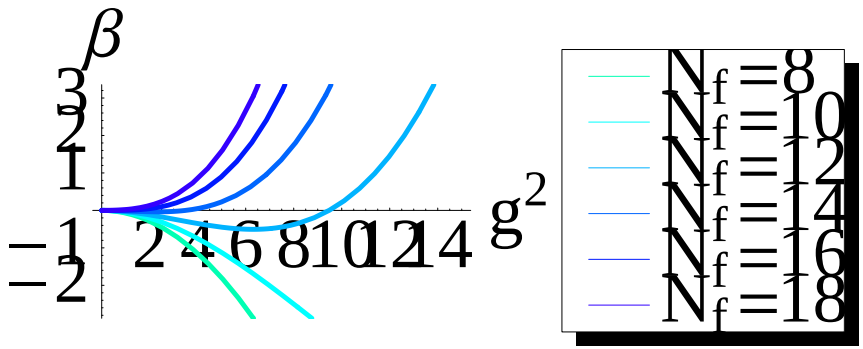
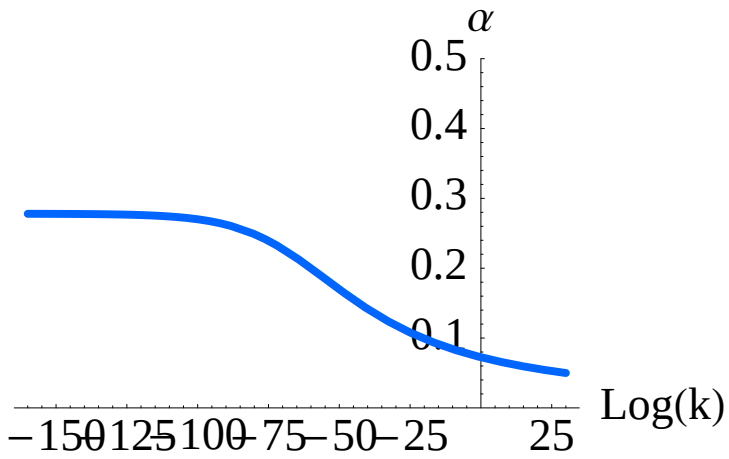


(Banks&Zaks'82)

▷ e.g. $N_f = 14$ $\Longrightarrow$ IR fixed point



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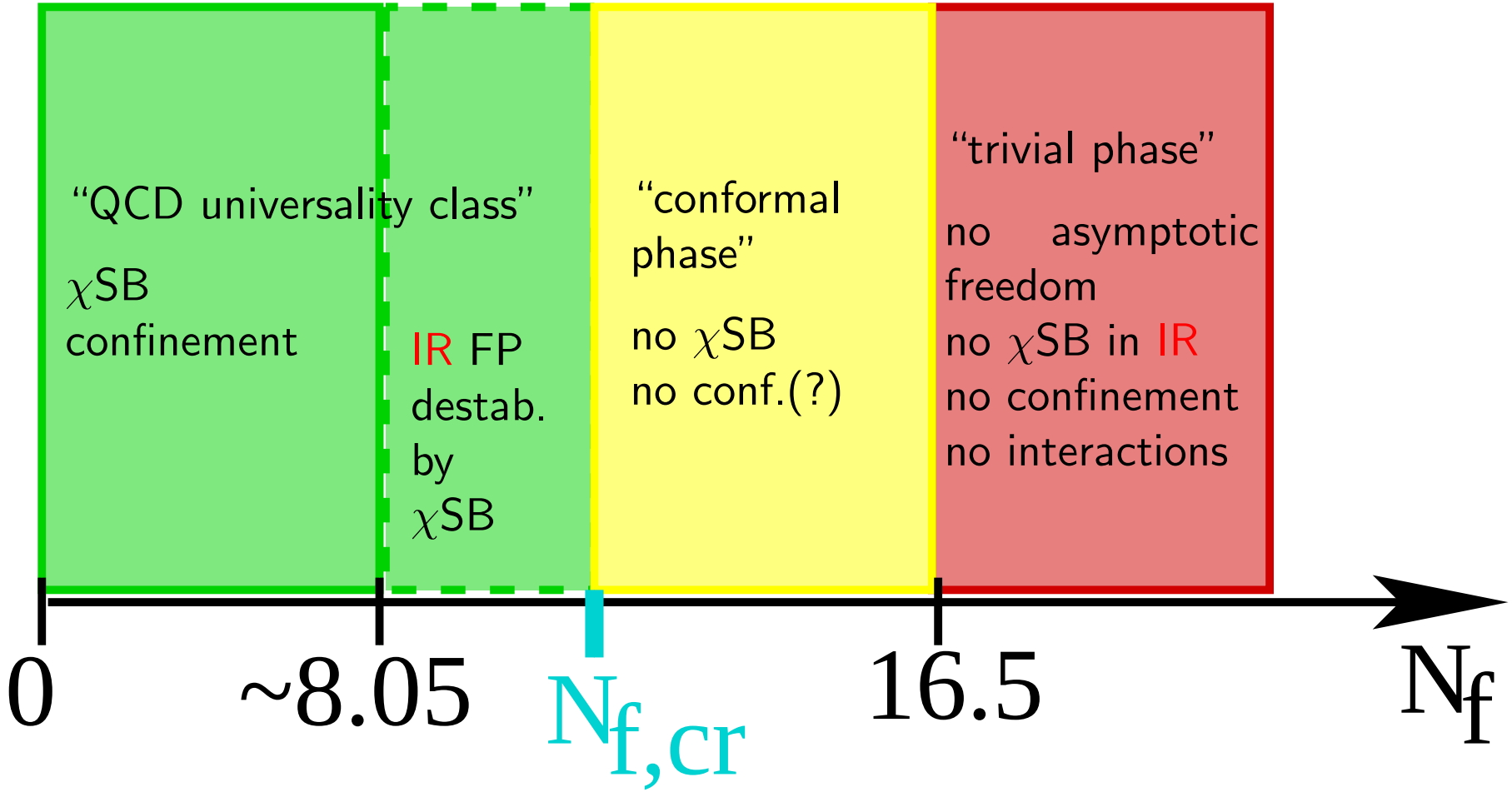
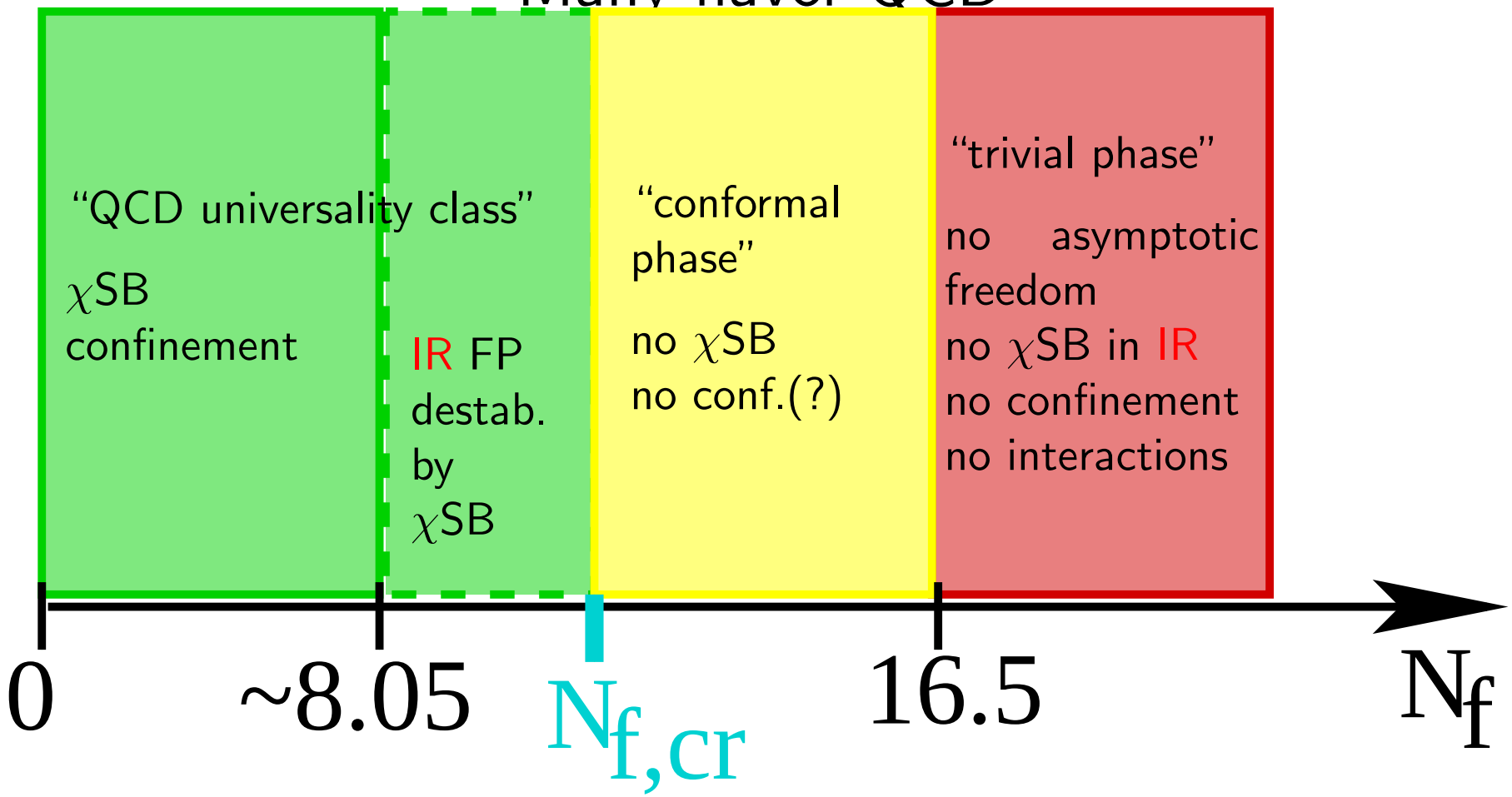


$$N_{f,cr} = ?$$

Many-flavor QCD



$$N_{\mathrm{f,cr}} = \begin{cases} 5 & \text{(Harada\&Yamawaki'00)} \\ 6 & \text{(Iwasaki et al.'03)} \\ \gtrsim 6 & \text{(Velkovsky\&Shuryak'97,Appelquist\&Selipsky'97)} \\ \gtrsim 10 & \text{(Sannino\&Schechter'99)} \\ \simeq 12 & \text{(Miransky\&Yamawaki'96,Appelquist et al.'96)} \end{cases}$$

"Exact" RG

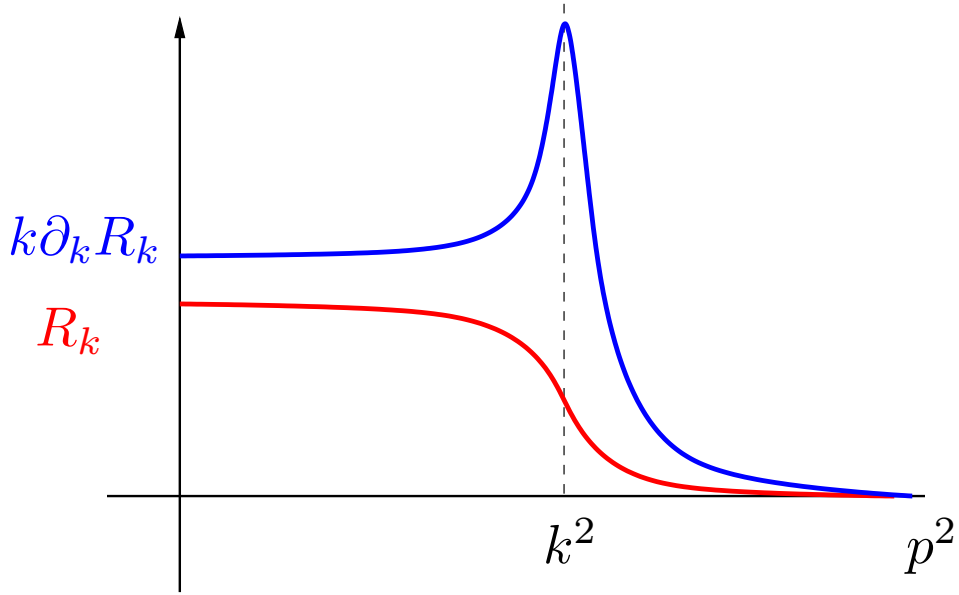
▷ generating functional

$$Z[J] = \int_\Lambda \mathcal{D}\phi\, e^{-S[\phi]+\int J\phi}$$

▷ IR regulator

$$Z_k[J] := e^{-\Delta S_k[\frac{\delta}{\delta J}]}\, Z[J] = \int_\Lambda \mathcal{D}\phi\, e^{-S[\phi]-\Delta S_k[\phi]+\int J\phi}$$

▷ regulator: $\Delta S_k[\phi] = \frac{1}{2}\int \phi R_k \phi$



$\Longrightarrow$ effective average action: $\Gamma_k[\phi] = -\ln Z_k[J] - \int J\phi - \Delta S_k[\phi]$

Functional RG

▷ generating functional

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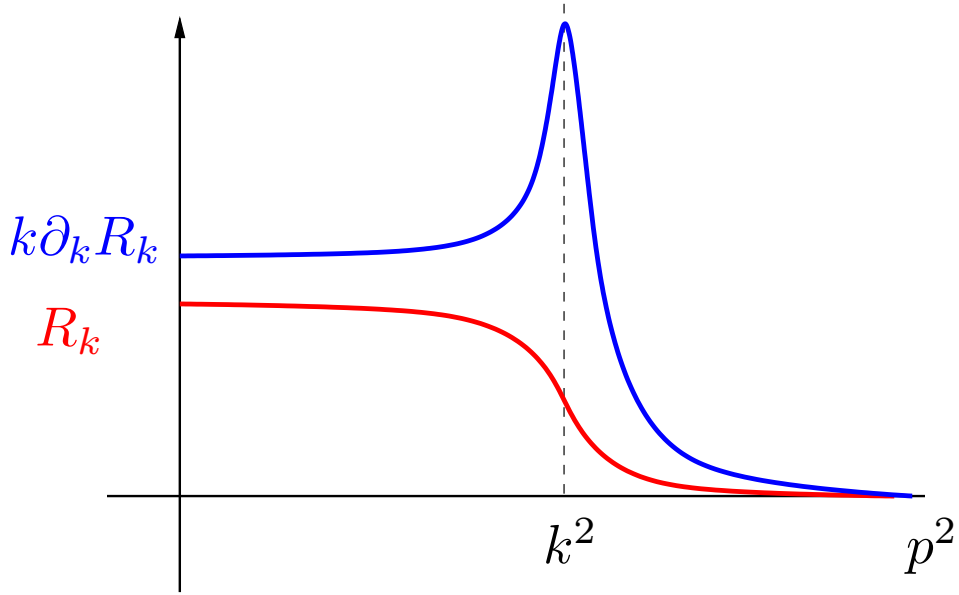

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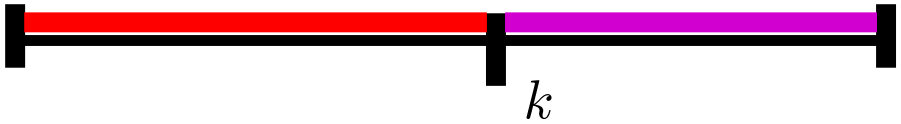
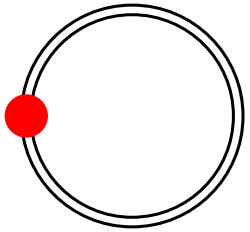
RG Flow Equation

IR: $k \to 0$



UV: $k \to \Lambda$

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\;\partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1} =$$



(Wetterich'93)

$$t = \ln \frac{k}{\Lambda}, \quad \partial_t \equiv k \frac{d}{dk}$$

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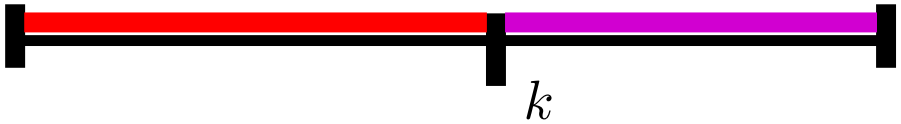

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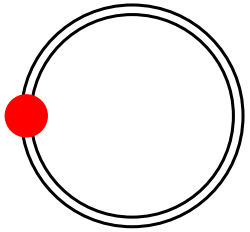
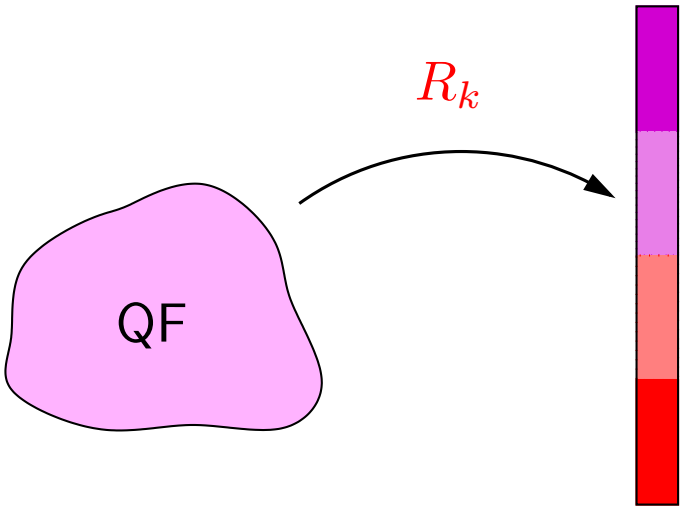


(Wetterich'93)

▷ quantum fluctuations



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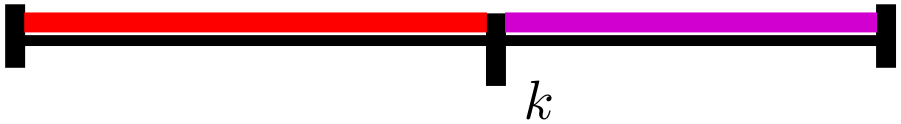

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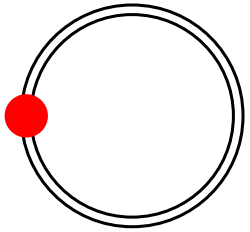

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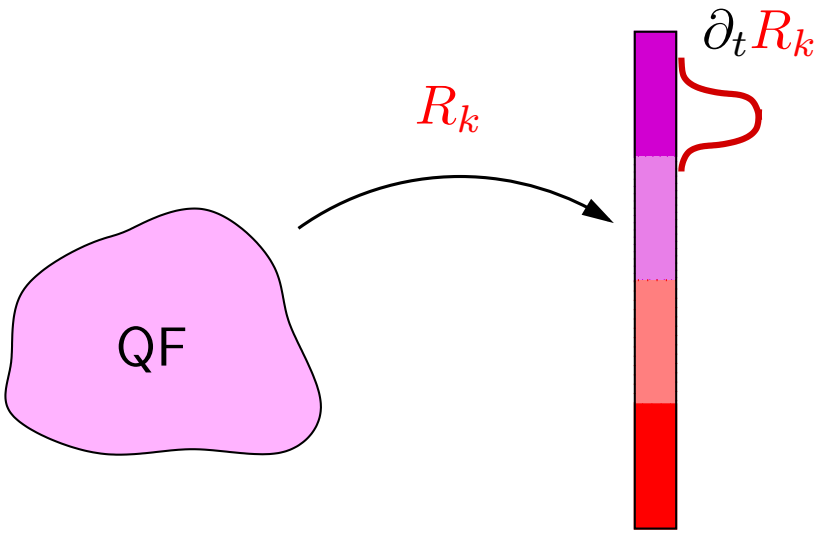

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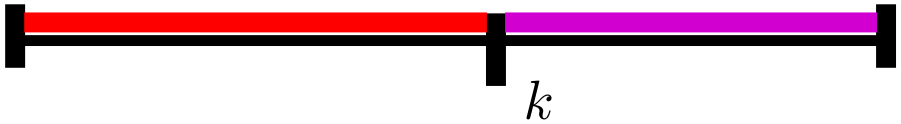

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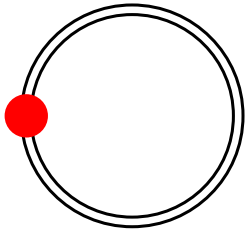

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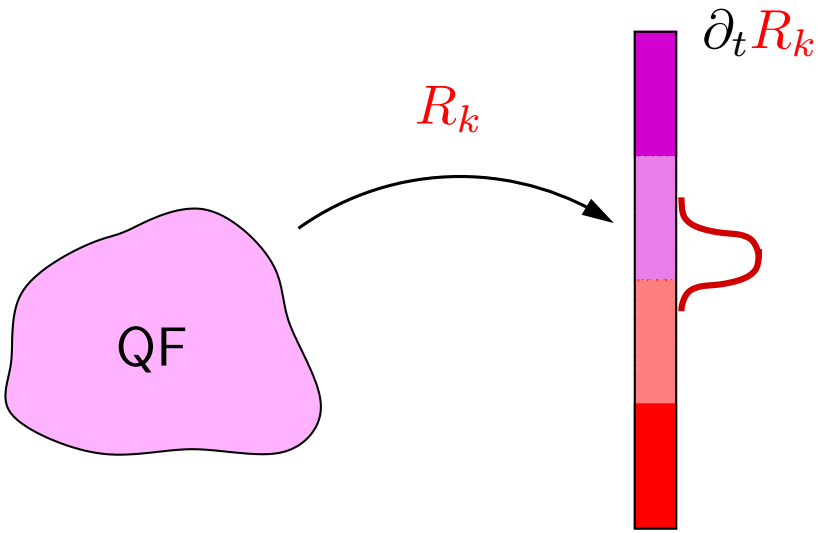

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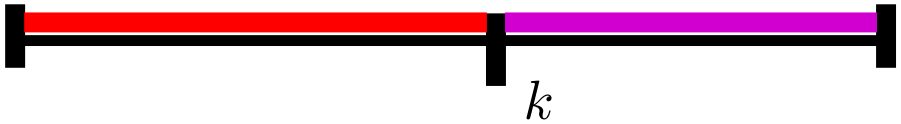

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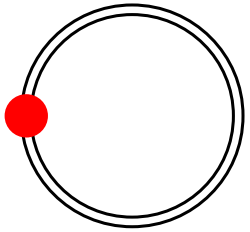

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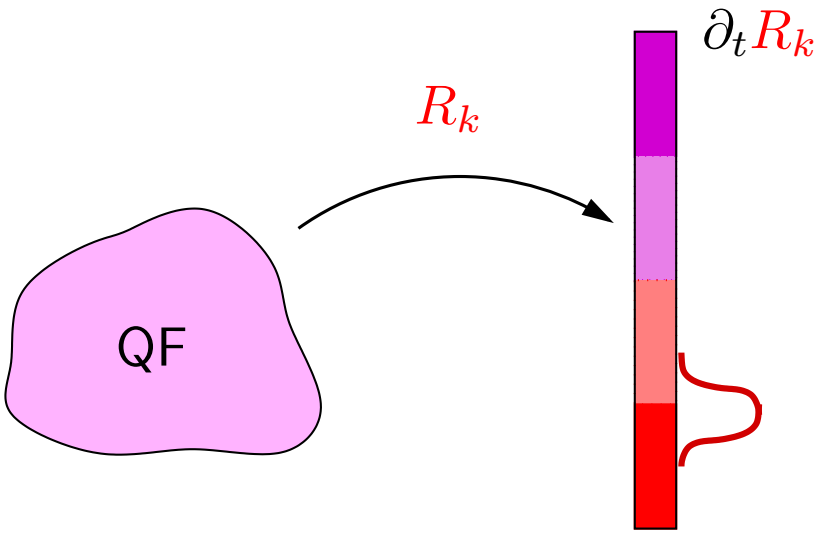

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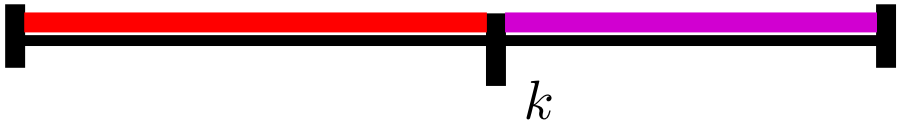

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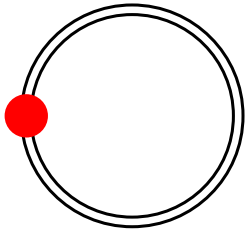

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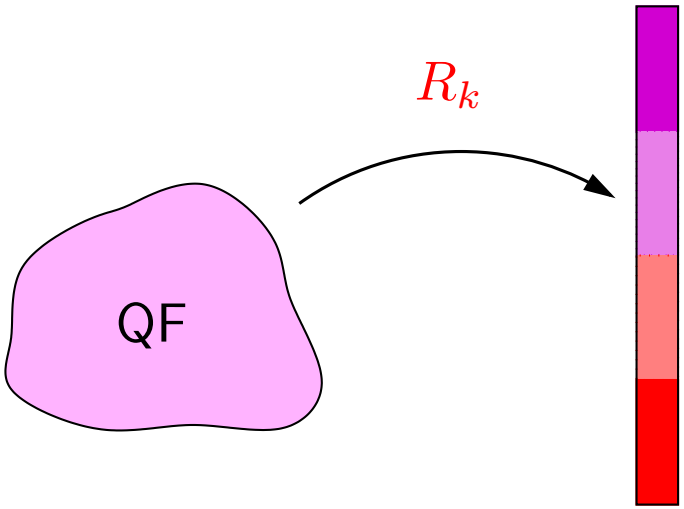
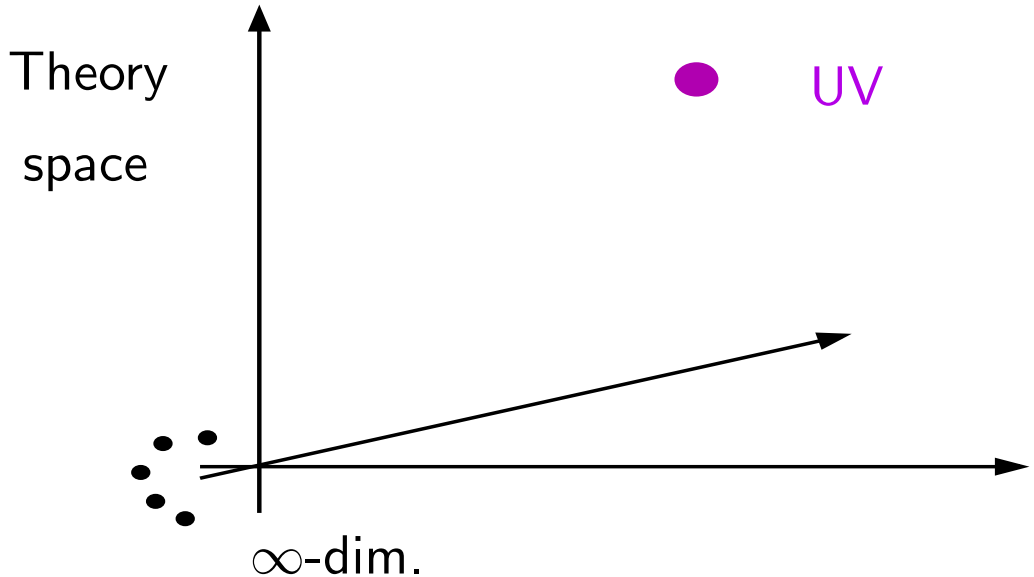


▷ RG trajectory: $\Gamma_{k=\Lambda} = S_{\text{bare}}$



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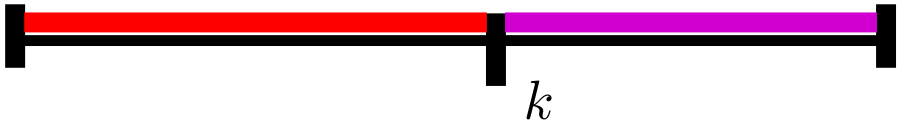

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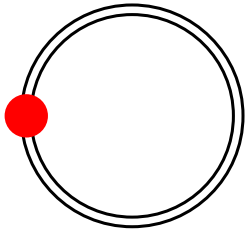

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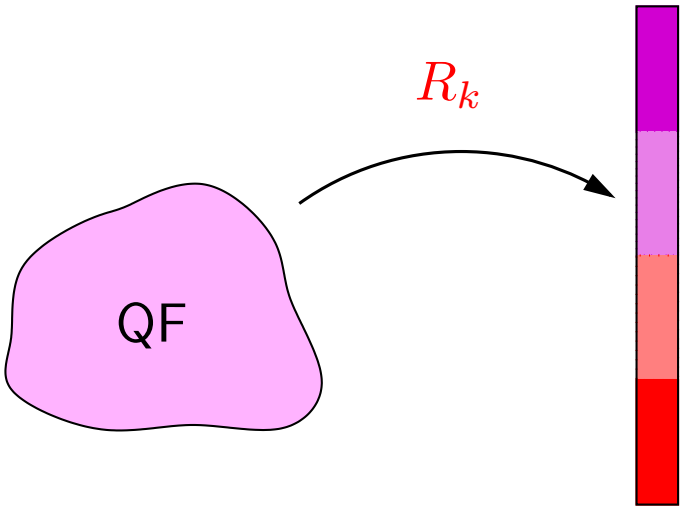

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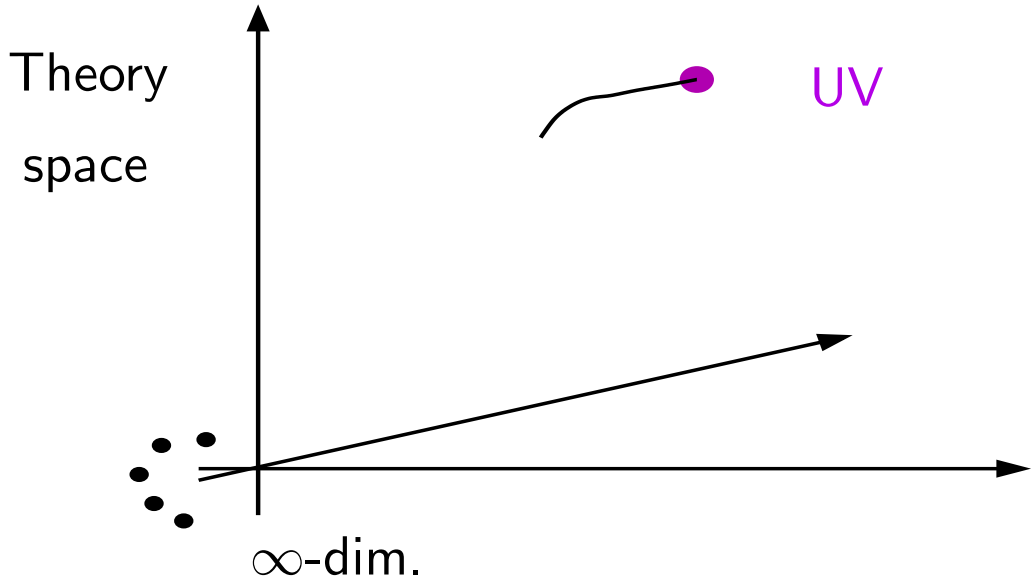

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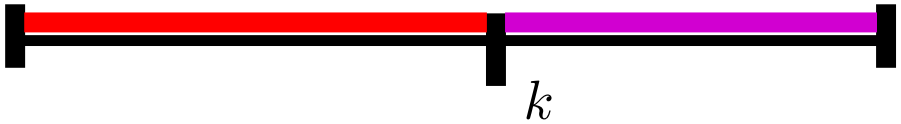

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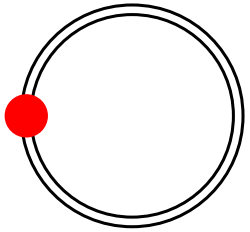

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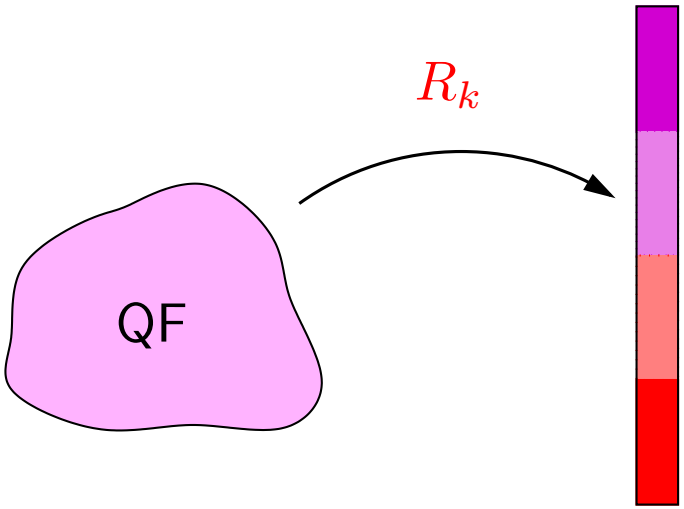

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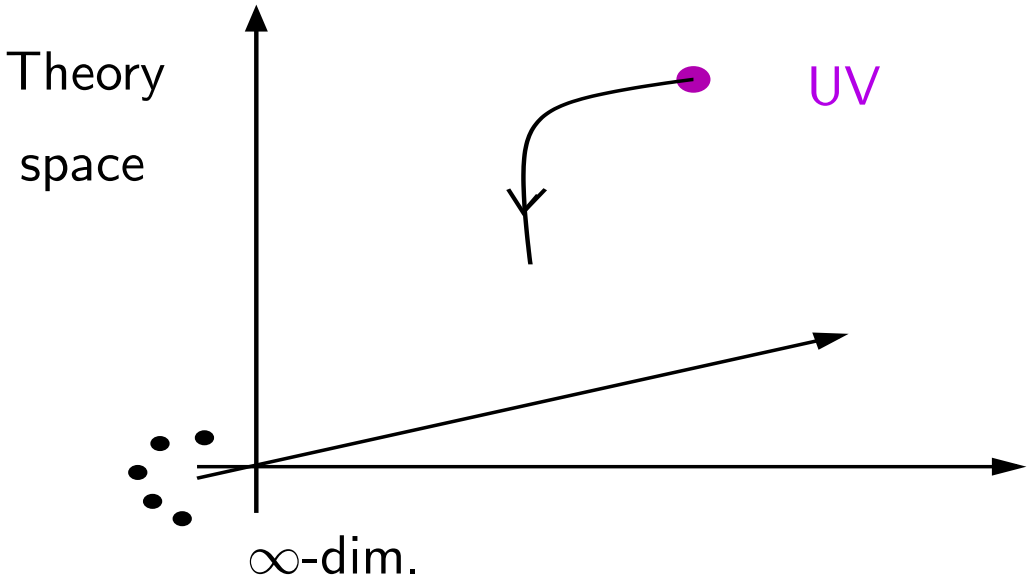

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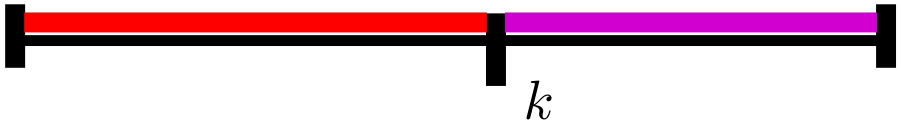

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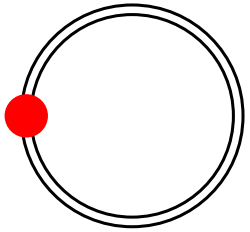

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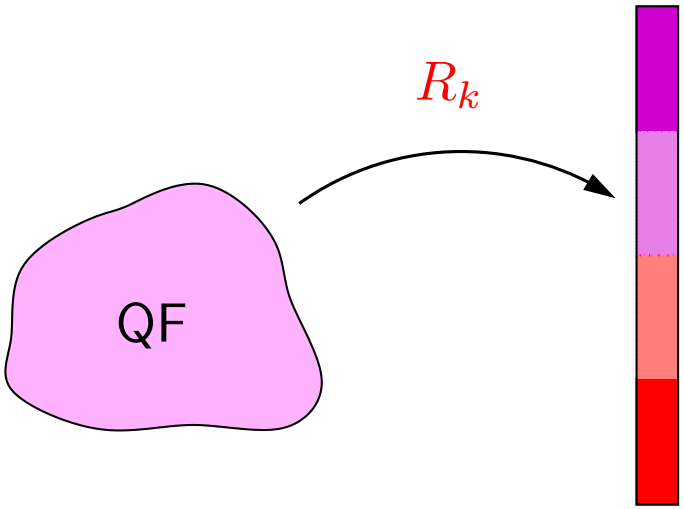
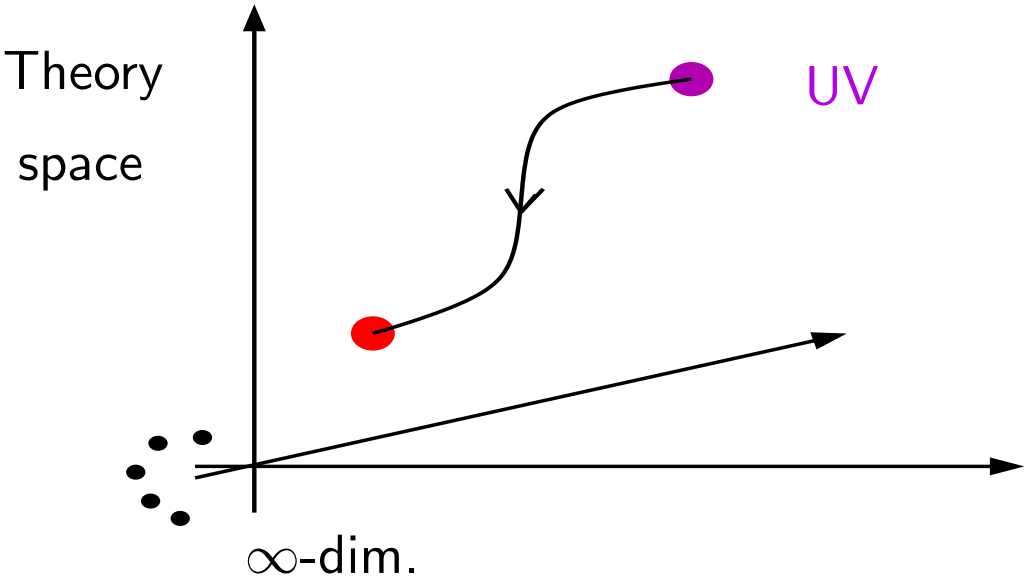


▷ RG trajectory: $\Gamma_{k=0} = \Gamma_{\text{eff}}^{1\text{PI}}$



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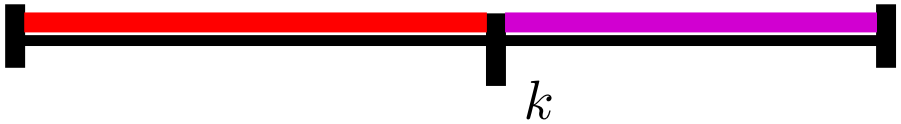
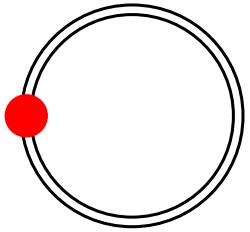


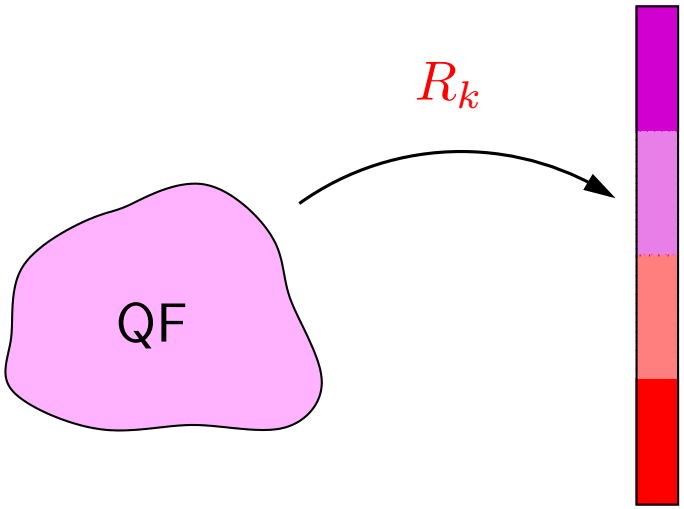
UV: $k \to \Lambda$

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1} =$$

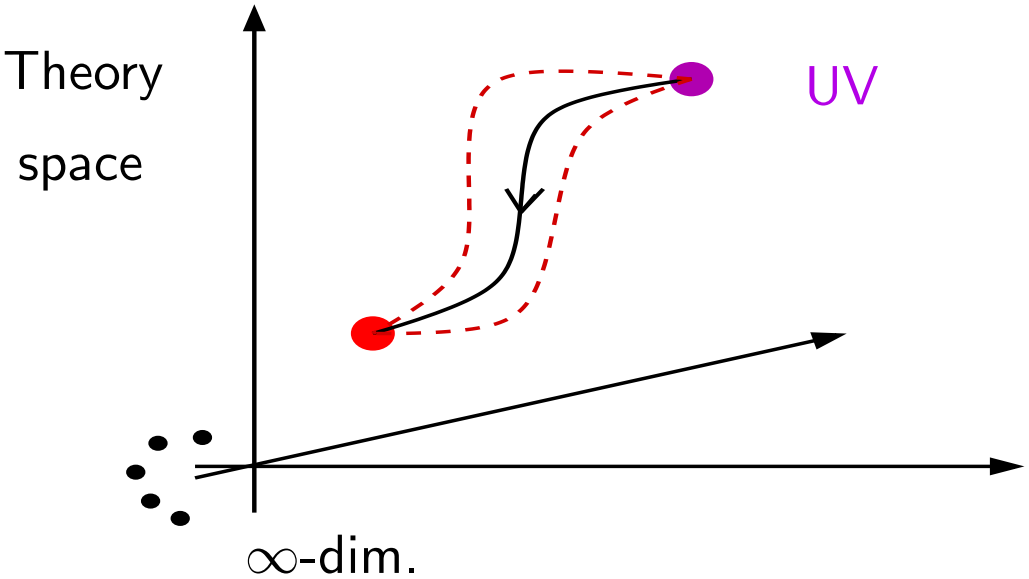


(Wetterich'93)

▷ quantum fluctuations

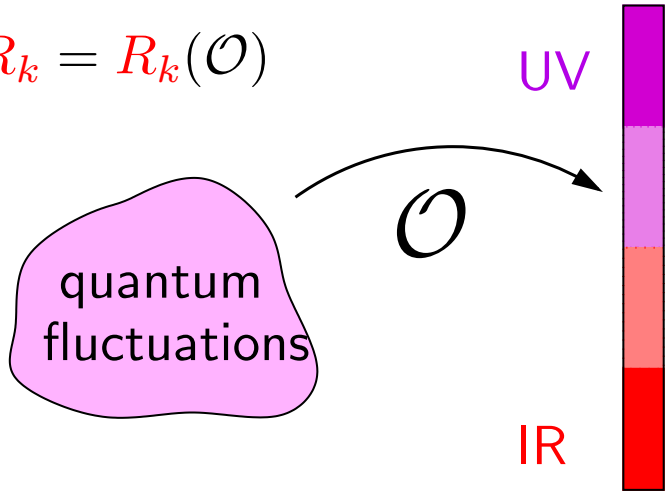


▷ RG trajectory: $\Gamma_{k=0} = \Gamma_{\text{eff}}^{1\text{PI}}$



The role of the regulator

▷ $R_k = R_k(\mathcal{O})$



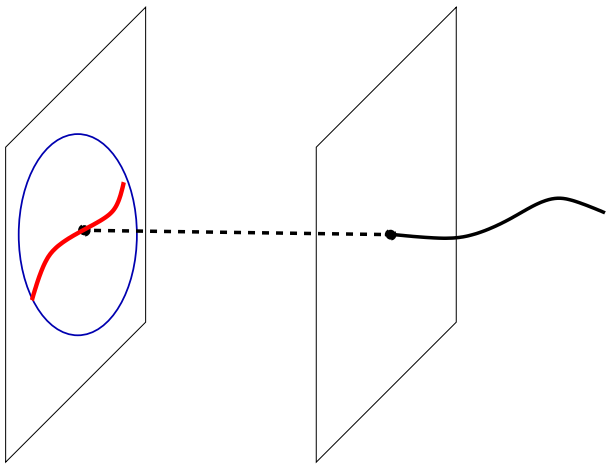
▷ e.g., chiral symmetry OK!

$$R_k{}^{\psi} = R_k{}^{\psi}(\mathrm{i}\bar{\not{D}})$$

▷ gauge symmetry $\longrightarrow$ modified Ward-Takahashi identities (mWTI)

(Ellwanger'94; Reuter,Wetterich'94; Freire,Litim,Pawlowski'00)

$$\mathcal{G}\left(\Gamma_k - S_{\mathrm{gf}}\right) = -\mathrm{i}g\left[R_k, (\Gamma_k^{(2)} + R_k)^{-1}\right]$$



▷ exact flow compatibility: $\partial_t(\text{mWTI}) = 0$

▷ truncation: explicit resolution

Many-flavor QCD

▷ $\mathrm{SU}(N_\mathsf{c})$ gauge symmetry + chiral $\mathrm{SU}(N_\mathsf{f})_\mathrm{L} \times \mathrm{SU}(N_\mathsf{f})_\mathrm{R}$ flavor symmetry

$$\Gamma_{k=\Lambda} = \int \frac{1}{4} F_z^{\mu\nu} F_{\mu\nu}^z + \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/\right)\psi$$

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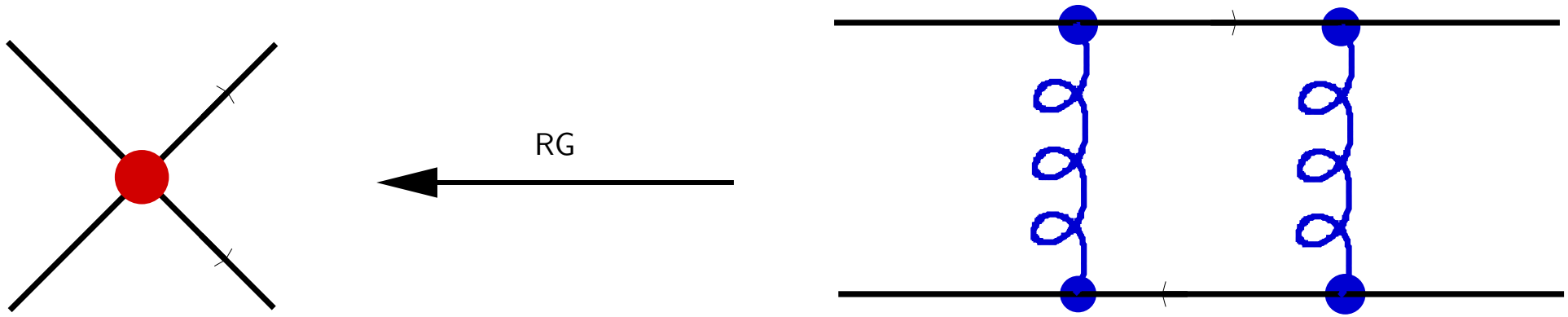
▷ RG flow



Many-flavor QCD: fermion sector. . .

▷ $\mathrm{SU}(N_\mathrm{c})$ gauge symmetry + chiral $\mathrm{SU}(N_\mathrm{f})_\mathrm{L} \times \mathrm{SU}(N_\mathrm{f})_\mathrm{R}$ flavor symmetry

$$\Gamma_k = \int \frac{Z_\mathrm{F}}{4} F_z^{\mu\nu} F_{\mu\nu}^z + \ldots + \bar{\psi}\left(\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar{g} A\!\!\!/\right)\psi$$
$$+\frac{1}{2}\Big[\bar{\lambda}_- \,(\mathrm{V–A}) + \bar{\lambda}_+ \,(\mathrm{V+A}) + \bar{\lambda}_\sigma \,(\mathrm{S–P}) + \bar{\lambda}_\mathrm{VA}\,[2(\mathrm{V–A})^\mathrm{adj} + (1/N_\mathrm{c})(\mathrm{V–A})]\Big]$$

▷ four-fermion interactions, $\bar{\lambda}_i|_{k\to\Lambda} \to 0$

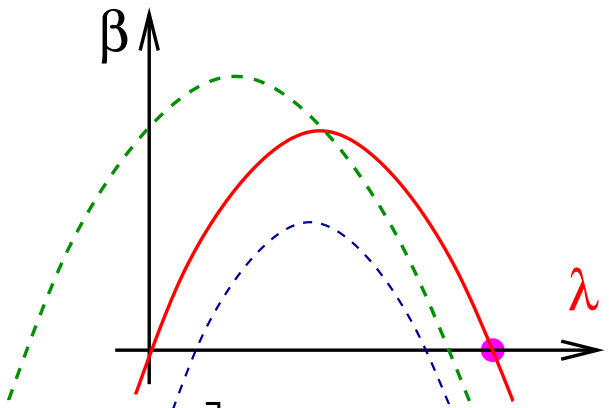
$$\begin{aligned}
(\mathrm{V–A}) &= (\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \\
(\mathrm{V+A}) &= (\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \\
(\mathrm{S–P}) &= (\bar{\psi}^a\psi^b)^2 - (\bar{\psi}^a\gamma_5\psi^b)^2 \equiv (\bar{\psi}_i^a\psi_i^b)^2 - (\bar{\psi}_i^a\gamma_5\psi_i^b)^2 \\
(\mathrm{V–A})^\mathrm{adj} &= (\bar{\psi}\gamma_\mu T^z\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5 T^z\psi)^2
\end{aligned}$$

▷ "point-like" truncation:

$$\bar{\lambda}_i(p_1,p_2,p_3,p_4) \to \bar{\lambda}_i(p_i=0), \quad Z_\psi(p) \to Z_\psi(p=0)$$

λ flow

▷ for instance $\lambda_\sigma = k^2 \bar{\lambda}_\sigma$:



$$\partial_t \lambda_\sigma = 2\lambda_\sigma - \frac{1}{8\pi^2} l_{1,1}^{(\mathrm{FB}),4} \left[3\frac{{N_\mathsf{c}}^2 - 1}{N_\mathsf{c}} g^2 \lambda_\sigma - 6 g^2 \lambda_+ \right]$$
$$- \frac{3}{128\pi^2} l_{1,2}^{(\mathrm{FB}),4} \left[\frac{3{N_\mathsf{c}}^2 - 8}{N_\mathsf{c}} g^4 \right]$$
$$- \frac{1}{4\pi^2} l_1^{(\mathsf{F}),4} \left\{ 2N_\mathsf{c} {\lambda_\sigma}^2 - 2\lambda_- \lambda_\sigma - 2N_\mathsf{f} \lambda_\sigma \lambda_{\mathsf{VA}} - 6\lambda_+ \lambda_\sigma \right\}$$

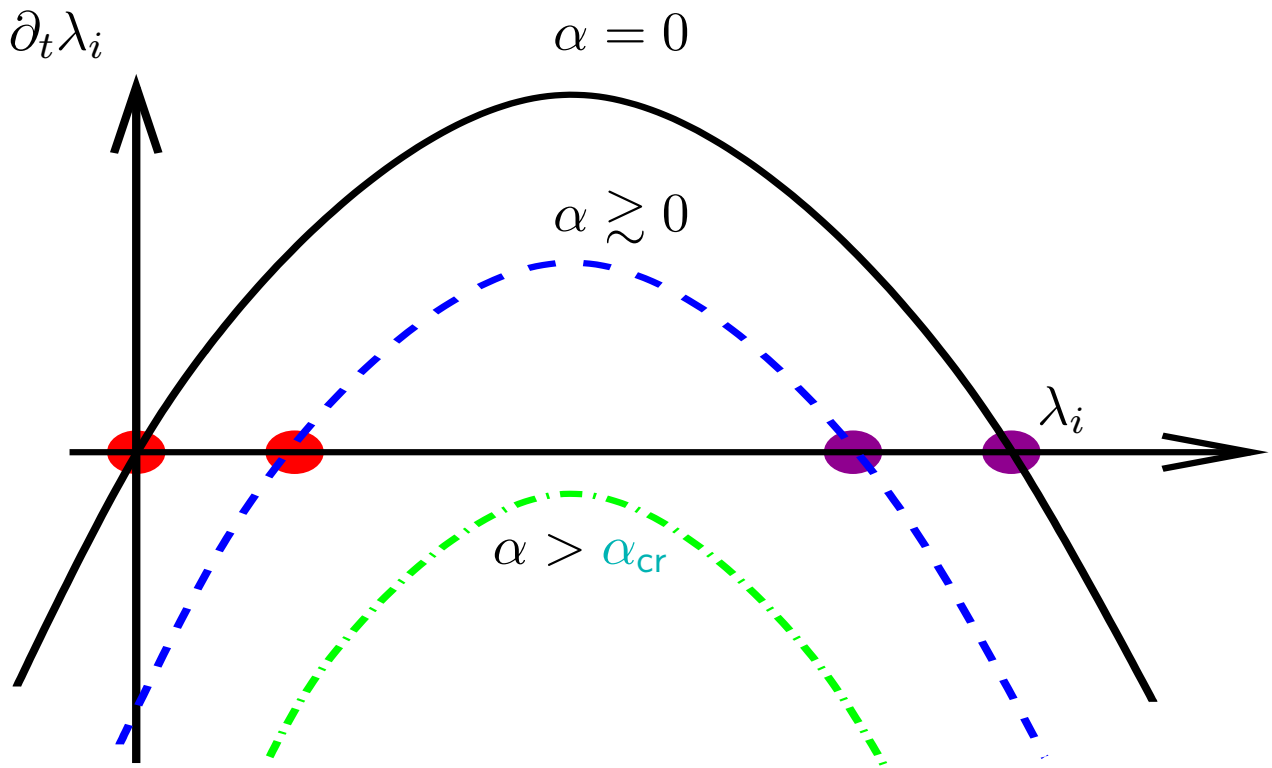
(Gies, Jaeckel, Wetterich'04)

▷ threshold functions:

$$l_1^{(\mathsf{F}),4}, l_{1,2}^{(\mathrm{FB}),4}, l_{1,1}^{(\mathrm{FB}),4} = l_1^{(\mathsf{F}),4}, l_{1,2}^{(\mathrm{FB}),4}, l_{1,1}^{(\mathrm{FB}),4}[R_k]$$

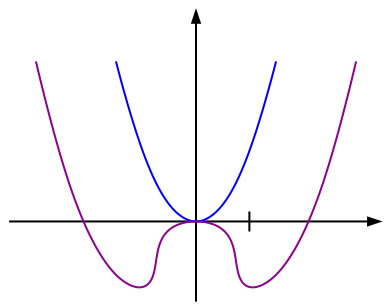
λ flow

$\Longrightarrow$ critical gauge coupling α_{cr}:



$\Longrightarrow$ if $\alpha > \alpha_{\text{cr}}$: $\lambda \to \infty$ (χSB)

$\Longrightarrow$ bosonization: $\lambda \sim \frac{1}{m_\phi^2}$

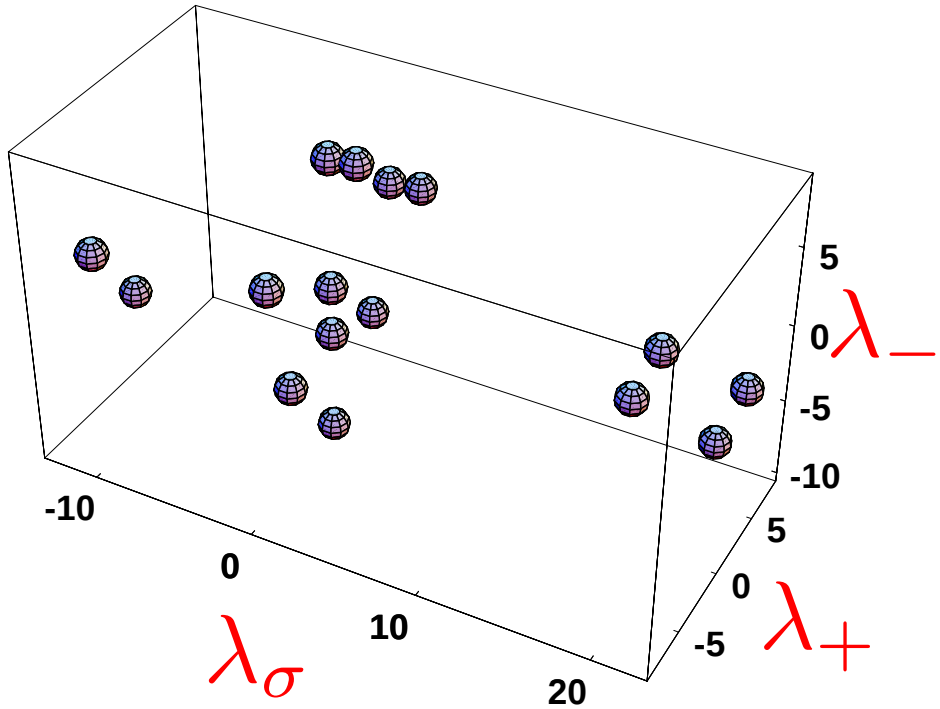


λ flow

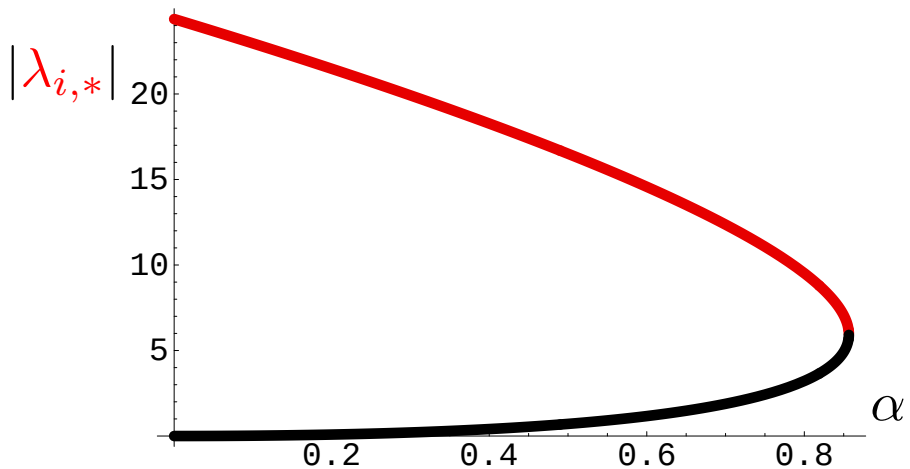
- ▷ 2 fixed points per $\partial_t \lambda$

$\Longrightarrow$ $2^4 = 16$ fixed points

- ▷ in general: 2^n FP's
  for $n = \#$ of λ's

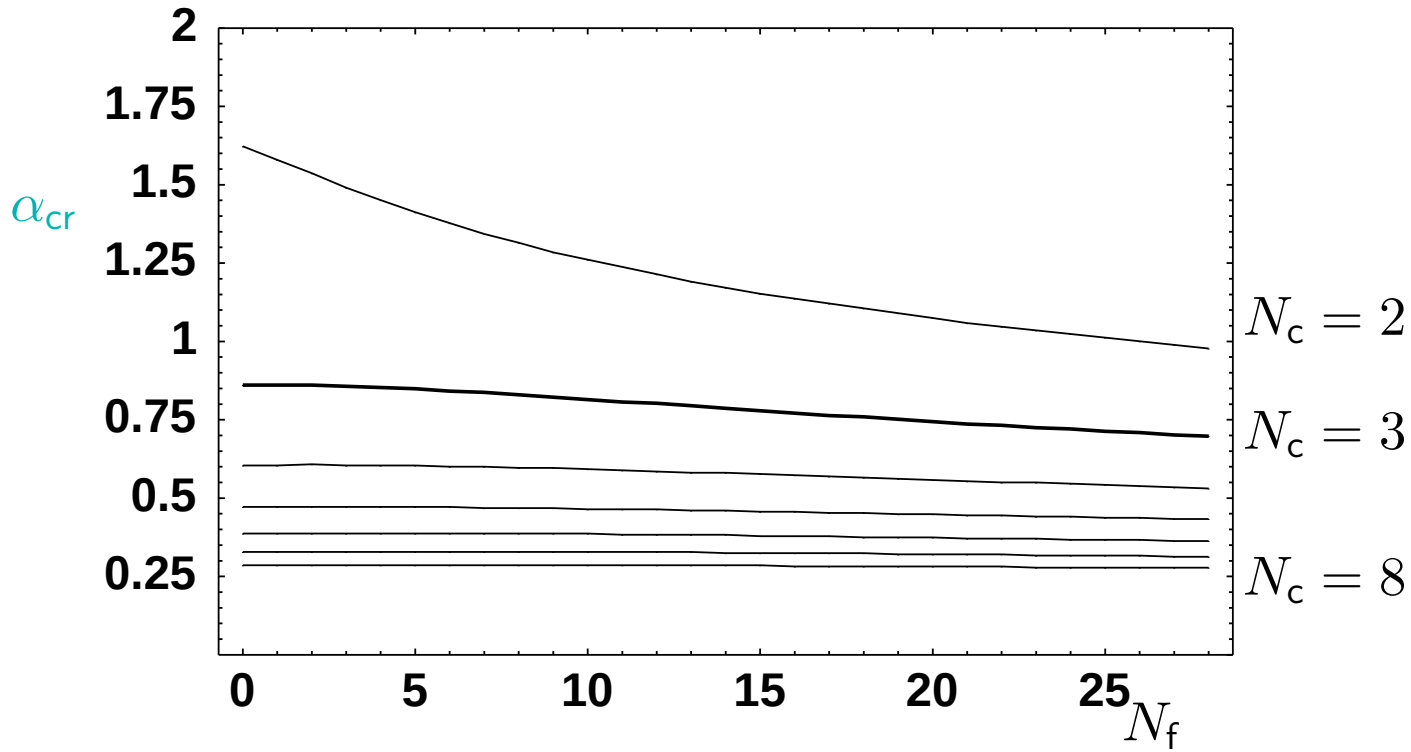


- ▷ fixed-point annihilation

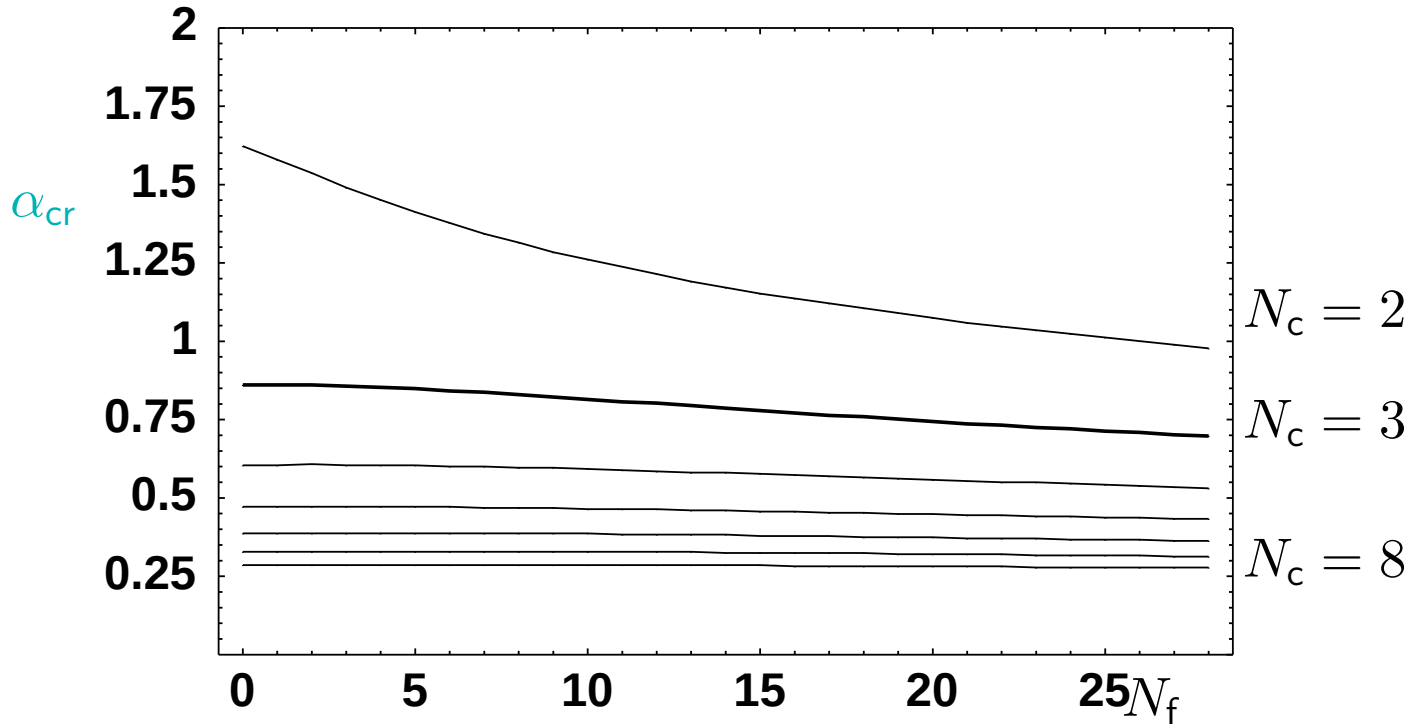


$N_\mathrm{c} = 3,\ N_\mathrm{f} = 3$

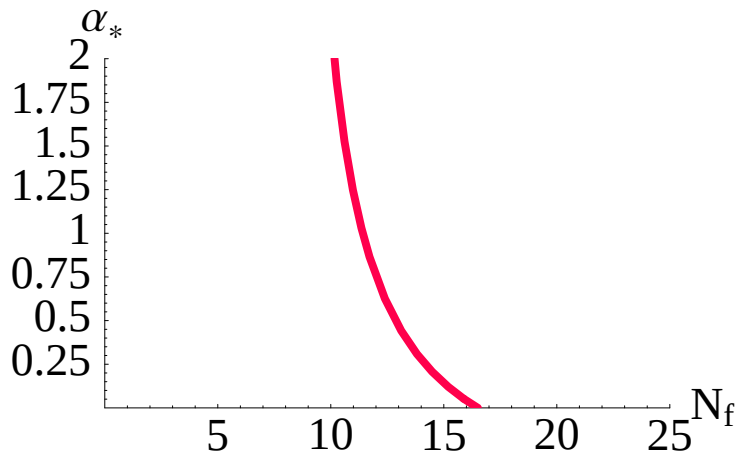
Critical coupling α_{cr} for χSB



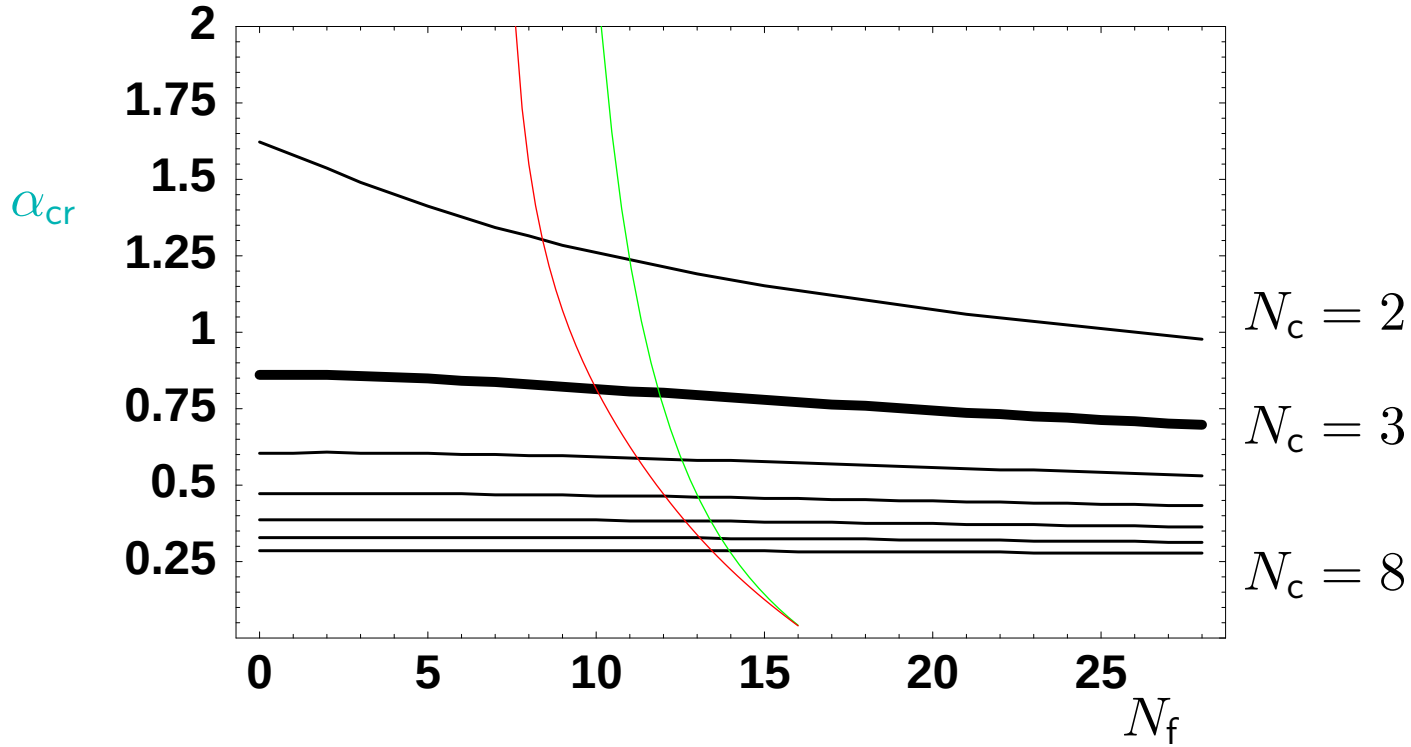
Critical coupling $\alpha_{\rm cr}$ for χSB



▷ compare with Banks-Zaks IR fixed point:



$\Longrightarrow$ Critical number of flavors $N_{\mathrm{f,cr}}$

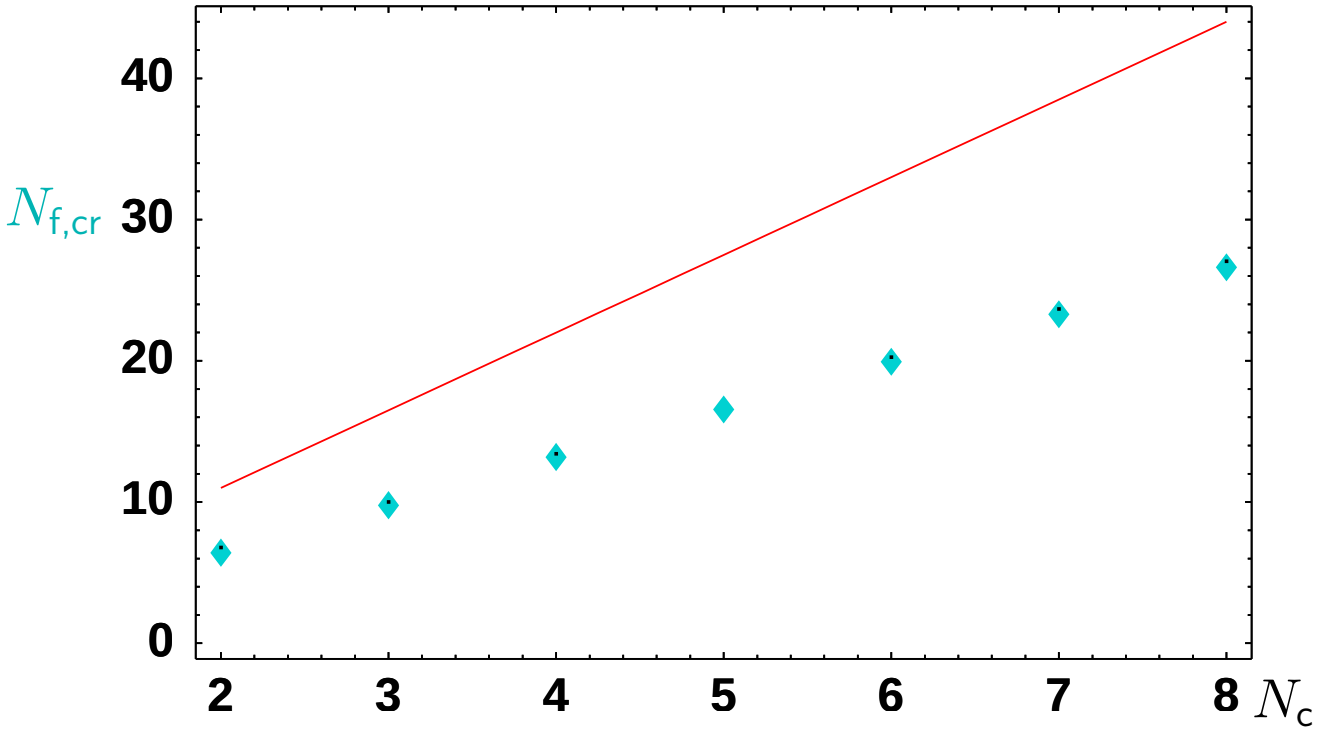


2-loop SU(3) β function in $\overline{\mathrm{MS}}$ scheme

4-loop SU(3) β function in $\overline{\mathrm{MS}}$ scheme $\Longrightarrow$ $N_{\mathrm{f,cr}} \simeq 10.0$ for SU(3)

(Ritbergen et al.'97)

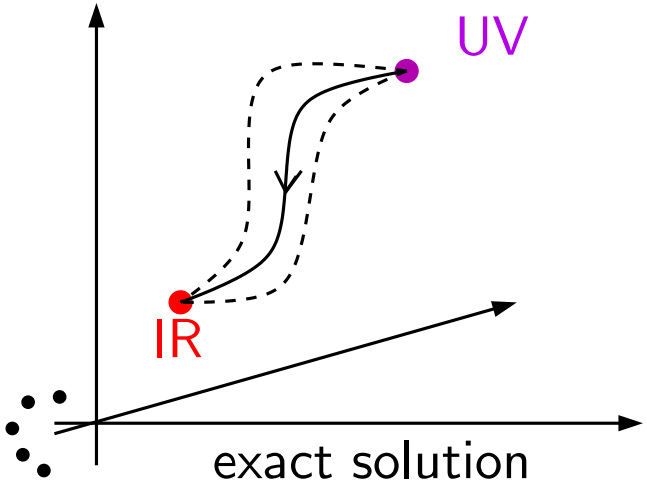
Critical number of flavors $N_{\mathrm{f,cr}}$

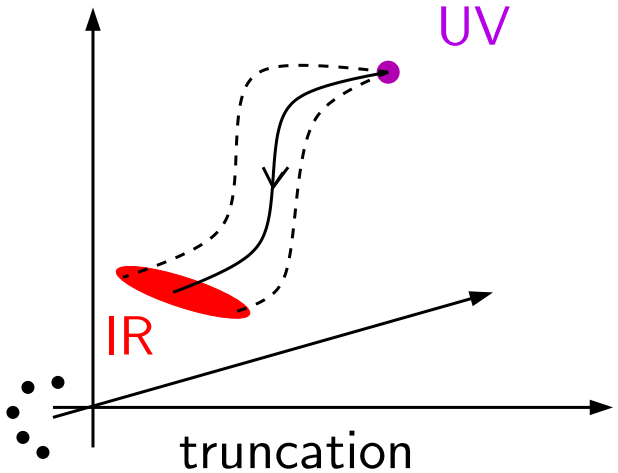


$\Longrightarrow$ "conformal phase" for $10.0 \lesssim N_{\mathrm{f}} < 16.5$ for SU(3)

Error estimate

▷ regulator dependence





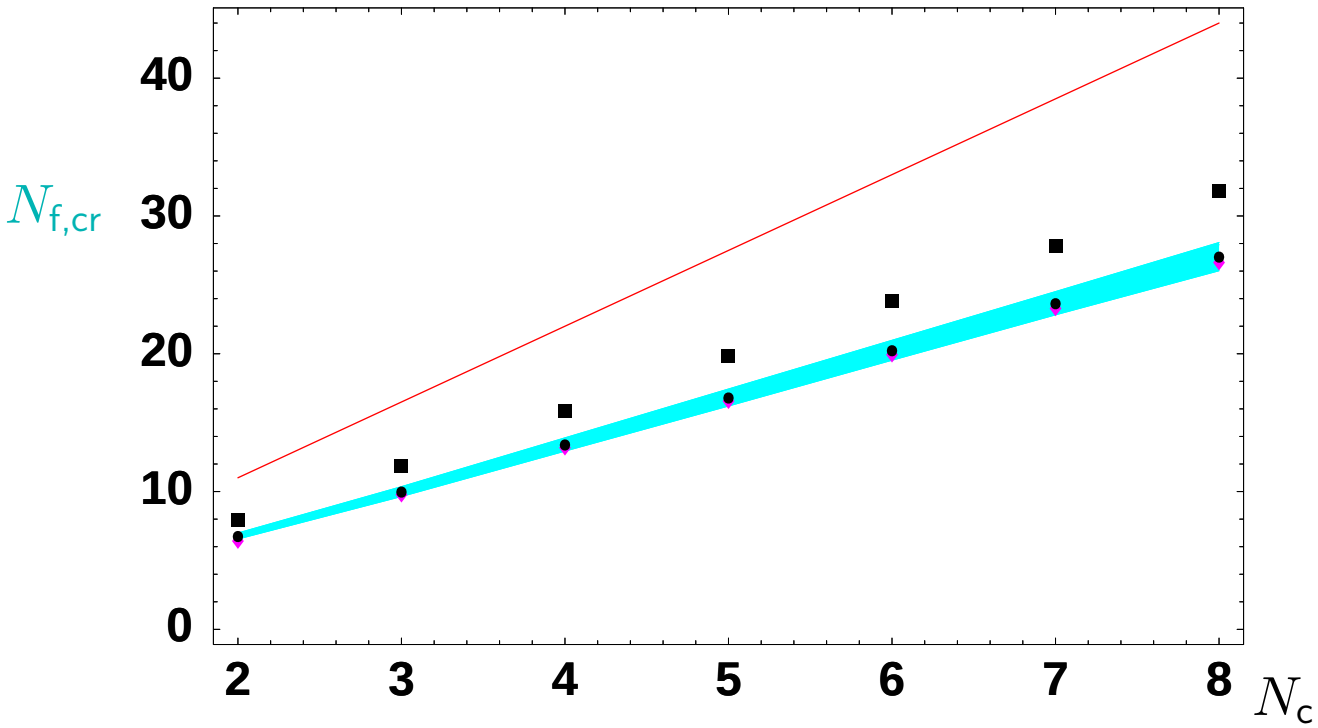
▷ fermion sector: "optimized" regulator vs. "sharp cutoff" (Litim'01)

$$l_1^{(\mathrm{F}),4} = \frac{1}{2},\ l_{1,1}^{(\mathrm{FB}),4} = 1,\ l_{1,2}^{(\mathrm{FB}),4} = \frac{3}{2} \qquad \text{vs.} \qquad l_1^{(\mathrm{F}),4} = l_{1,1}^{(\mathrm{FB}),4} = l_{1,2}^{(\mathrm{FB}),4} = 1$$

▷ gauge sector: 2-loop, 3-loop, 4-loop β function

▷ gauge sector: $\overline{\mathrm{MS}}$ scheme vs. RG scheme (∼10% variation (?))

Critical number of flavors $N_{\mathrm{f,cr}}$



$\Longrightarrow$ "conformal phase" for $10.0 \pm 0.4 \lesssim N_{\mathrm{f}} < 16.5$ for SU(3)

(HG&Jaeckel'05)

Lessons to be learned for "real QCD"

- fermionic screening is rather weak
- point-like four-fermion truncation (surprisingly) stable in χ symmetric phase
- phase boundary detectable with "derivative expansion"
- "real QCD" requires nonperturbative estimate of β_{g^2}

Lessons to be learned for "real QCD"

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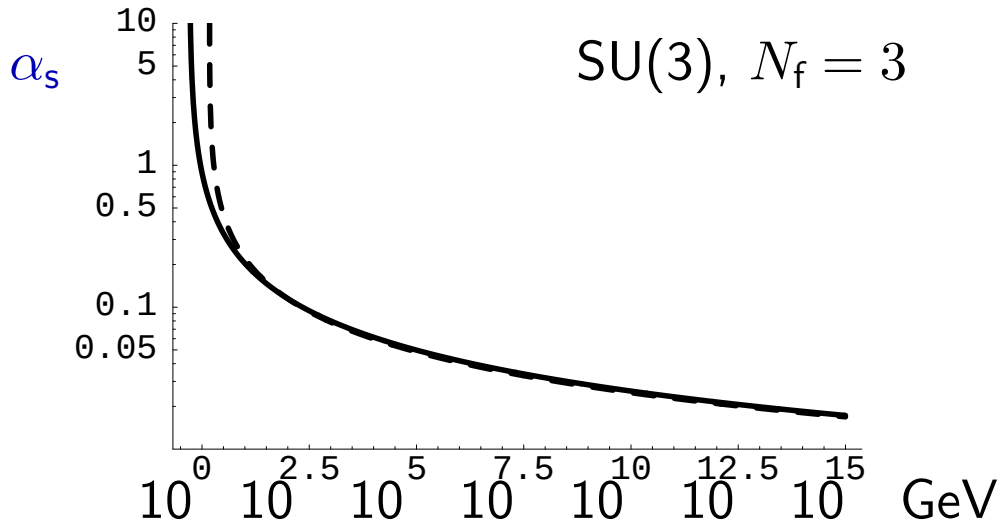
. . . χ phase transition at finite T

- "real QCD" requires nonperturbative estimate of β_{g^2}

$$\Gamma_k = \int \frac{Z_{\mathsf{F}}}{4} F_z^{\mu\nu} F_{\mu\nu}^z \underbrace{+ \quad . \quad . \quad .}$$

Running coupling

▷ perturbation theory



$\Longrightarrow$ Landau pole:
"artifact" of perturbation theory

▷ beyond perturbation theory . . . ?

nonrenormalization properties:

<u>ghost-gluon vertex:</u> (Taylor'71)

(Landau gauge)

$$Z_g \, (Z_{\text{gluon}})^{1/2} \, Z_{\text{ghost}} \equiv 1$$

full propagators $\longrightarrow$ running g

<u>gauge connection:</u> (Abbott'82)

(background gauge)

$$\bar{D}_\mu = \partial_\mu - \mathrm{i}\, \bar{g}\, \bar{A}_\mu \equiv \partial_\mu - \mathrm{i}\, g\, \bar{A}^{\mathrm{R}}_\mu$$

running Z_{B} $\longrightarrow$ running g

RG flow of gluodynamics

▷ Operator expansion in background gauge

(Reuter, Wetterich'94)

(Freire, Litim, Pawlowski'00)

$$\Gamma_k[A] = \int d^d x \; W_k(F^2), \quad F^2 \equiv F^a_{\mu\nu} F^a_{\mu\nu}$$

$$W_k(F^2) = \frac{Z_{\mathsf{B}}}{4} F^2 + \frac{W_2}{16} (F^2)^2 + \frac{W_3}{3!4^3} (F^2)^3 + \frac{W_4}{4!4^4} (F^2)^4 + \ldots$$

(cf. Savvidy model of confinement)

▷ spectrally adjusted flow equation:

(HG'02)

$$\partial_t Z_{\mathsf{B}} \curvearrowleft \partial_t W_2 \curvearrowleft \partial_t W_3 \curvearrowleft \partial_t W_4 \curvearrowleft \partial_t W_5 \ldots$$

▷ running coupling: $g^2 = Z_{\mathsf{B}}^{-1} \, \bar{g}^2$

▷ β function: $\partial_t g^2 \equiv \beta_{g^2}$

β function

- asymptotic series

$$\beta = g^2 \sum_{m=1}^{\infty} a_m \left(\frac{g^2}{(4\pi)^2} \right)^m$$

- perturbative beta function, SU(N_c):

$$\begin{aligned} \beta(g^2) \quad = \quad & -\frac{22 N_\mathsf{c}}{3} \frac{g^4}{(4\pi)^2} \\ & - \left(\frac{77 N_\mathsf{c}^2}{3} - \frac{127(3N_\mathsf{c}^2 - 2)}{45} f[R_k] \right) \frac{g^6}{(4\pi)^4} + \dots \end{aligned}$$

- 1 loop: <u>exact</u>

  2 loop: <u>99%</u> for SU(2), 95% for SU(3),
  (for exponential regulator)

β function

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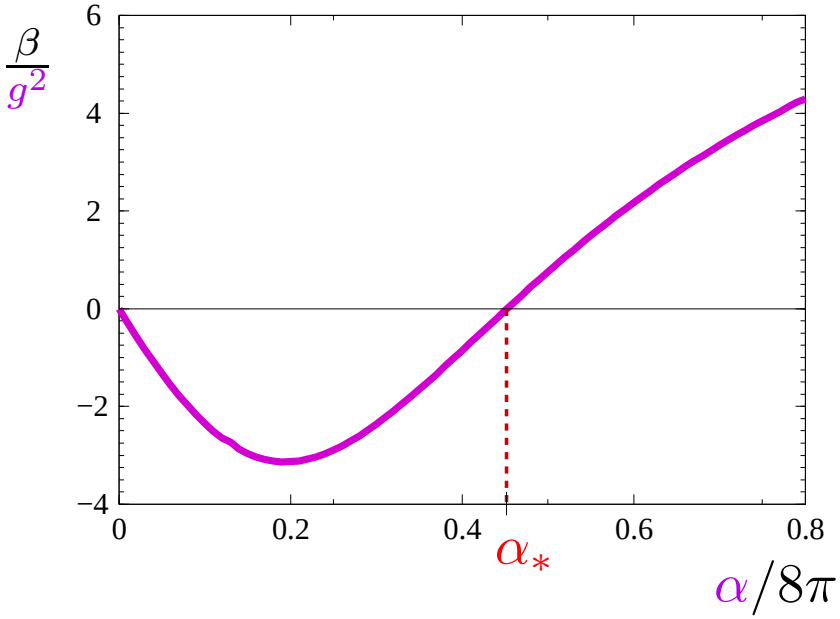
- 1 loop: <u>exact</u>

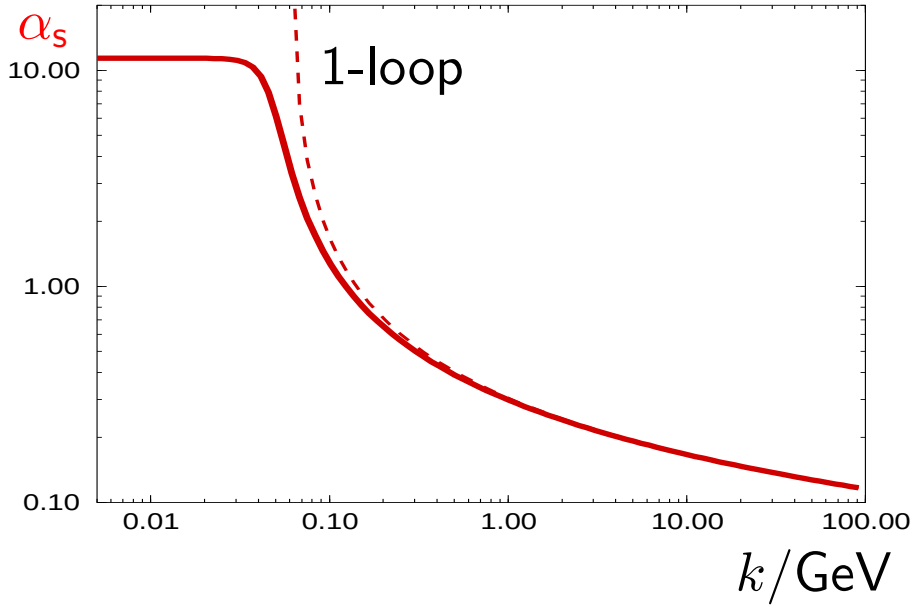
  2 loop: <u>99%</u> for SU(2), 95% for SU(3),
  (for exponential regulator)

1	-29.3333
2	-357.83
3	-191.32
4	15499.6
5	$-1.88776 \cdot 10^{6}$
6	$1.65315 \cdot 10^{7}$
7	$2.79324 \cdot 10^{9}$
8	$-1.37622 \cdot 10^{11}$
9	$-4.21715 \cdot 10^{12}$
10	$8.60663 \cdot 10^{14}$
11	$-8.05611 \cdot 10^{16}$
12	$5.21052 \cdot 10^{19}$
13	$-6.30043 \cdot 10^{22}$
14	$9.35648 \cdot 10^{25}$
15	$-1.78717 \cdot 10^{29}$
16	$4.35314 \cdot 10^{32}$
17	$-1.33397 \cdot 10^{36}$
18	$5.08021 \cdot 10^{39}$
19	$-2.37794 \cdot 10^{43}$
20	$1.35433 \cdot 10^{47}$

β function

▷ SU(2) gluodynamics:





$\Longrightarrow$ IR fixed points: SU(2): $\alpha_* \simeq 11.3$

SU(3): $\alpha_* \simeq 7.7$

SU(3),($N_\mathrm{f} = 1$): $\alpha_* \simeq 3.44$

. . . cf. vertex expansion in Landau gauge QCD

(v.Smekal,Alkofer&Hauck'97;Fischer&Alkofer'02)

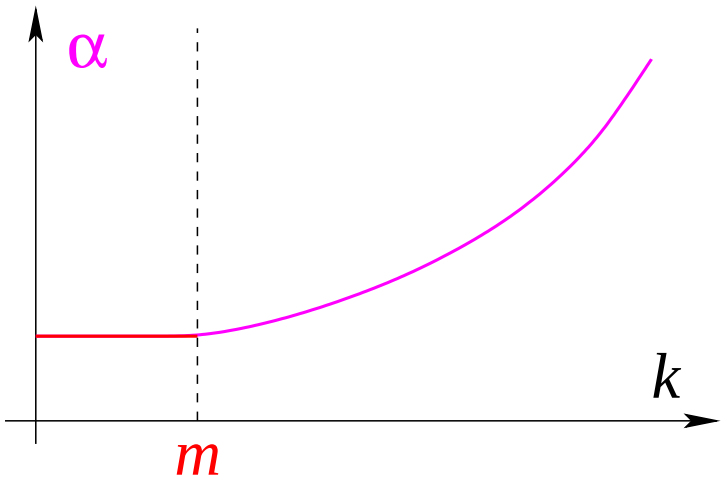
(Pawlowski et al.'04,Fischer&HG'04)

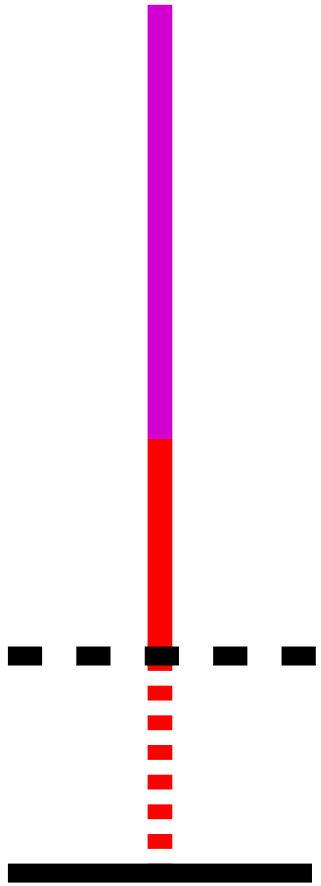
Running coupling and mass gap

- ▷ mass-dependent renormalization scheme

  $\longleftrightarrow$ threshold behavior

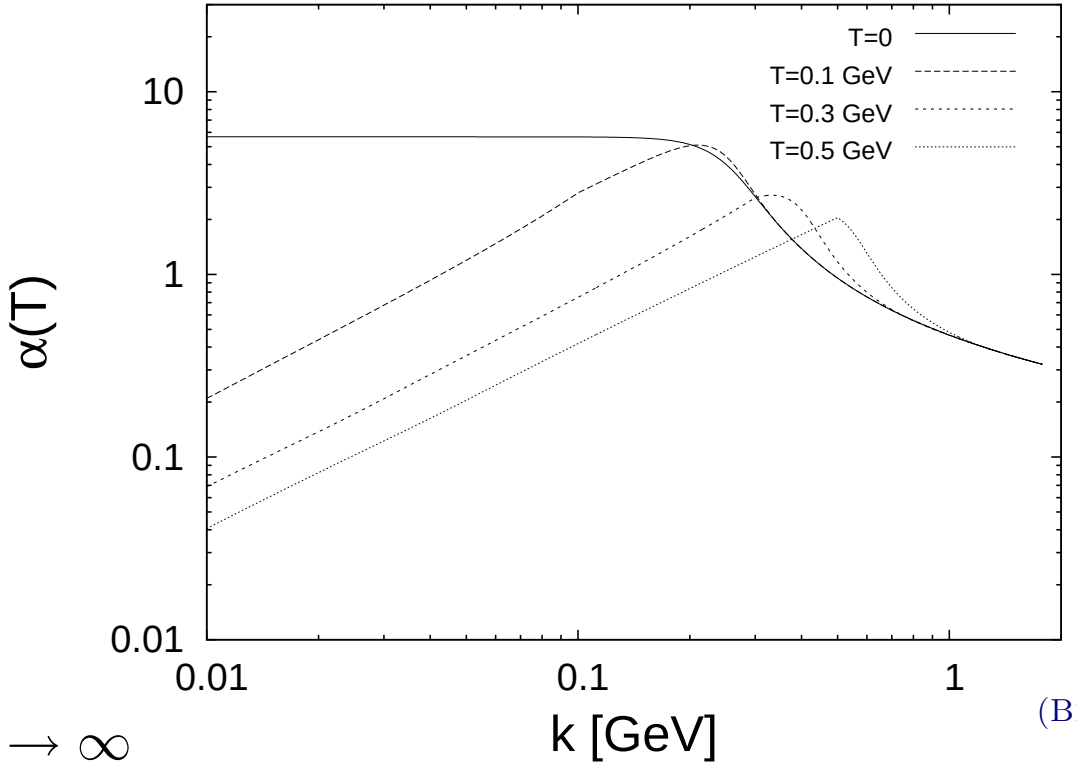
- ▷ e.g., QED: $\alpha \to \alpha_* \simeq \frac{1}{137}$





$\Longrightarrow$ IR fixed point compatible with mass gap

Running coupling at finite T



(Braun&HG'05: preliminary)

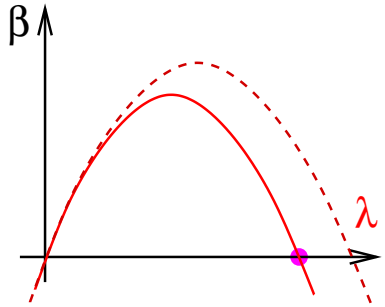
▷ $3D$ limit for $T/k \to \infty$

$$\alpha \quad \to \quad \frac{k}{T}\alpha_{3D}, \qquad \alpha_{3D} \to \alpha_{3D,*} \simeq 2.43 \text{ for SU(3)}$$

$\Longrightarrow$ $3D$ theory is strongly interacting

(▷ problem: Nielsen-Olsen unstable mode requires thermal screening)

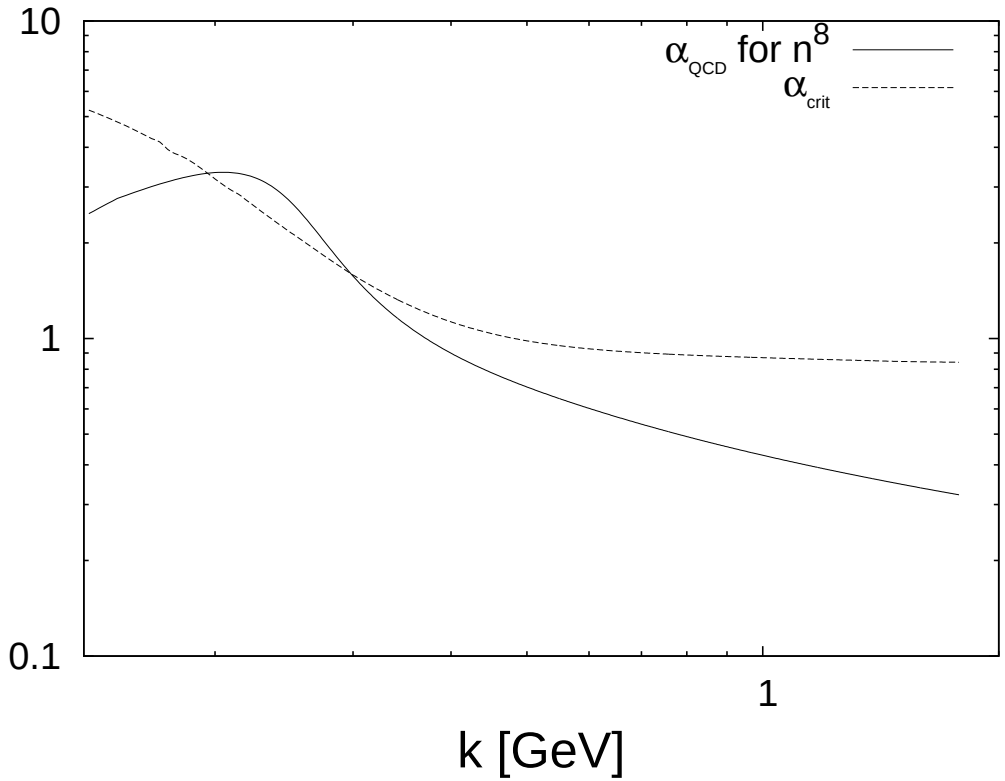
Synthesis: chiral symmetry restauration at finite T



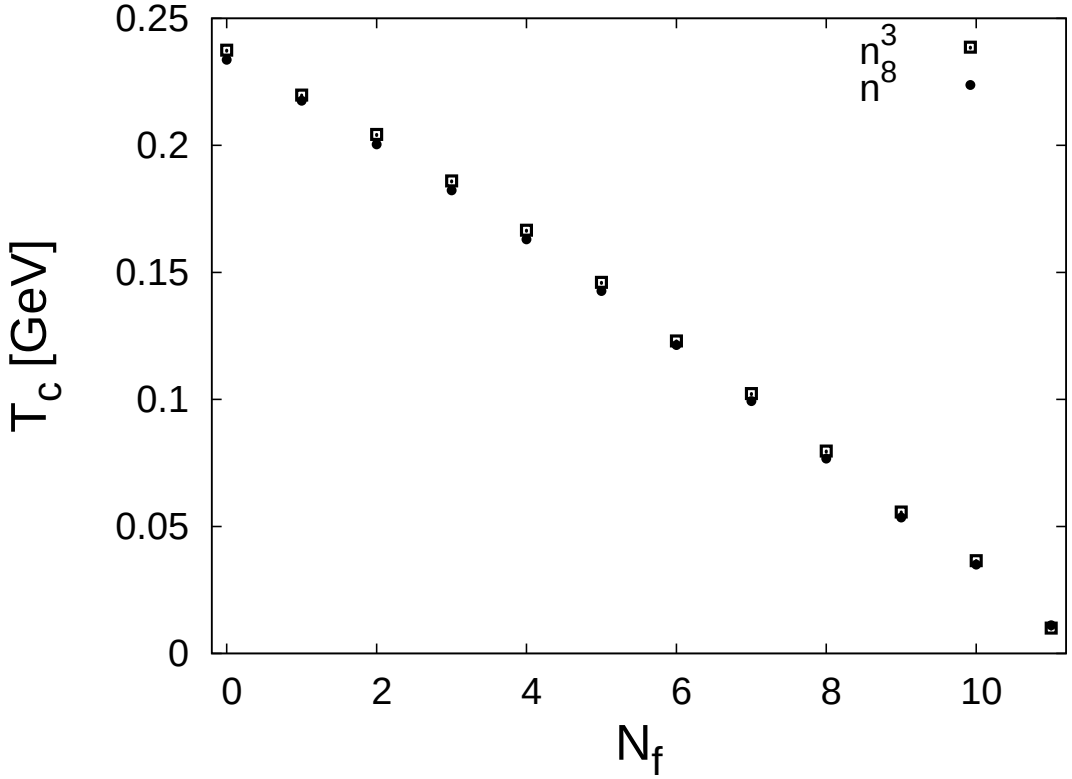
- fermion sector: $\alpha_{\text{cr}}(T/k) > \alpha_{\text{cr}}(T=0)$

- gauge coupling: $\beta \simeq \beta_{\text{gluodyn}} + \beta_{\text{f}}^{\text{1-loop}}(g^2, T, m_{\text{f}})$

- e.g., SU(3)
  $N_{\text{f}} = 3$ massless quarks
  $T = 150$MeV



Critical temperature T_{cr}



- e.g. SU(3), $N_{\text{f}} = 3$ massless quark flavors: $T_{\text{cr}}[R_k] \lesssim 181$MeV (upper bound)
- $T_{\text{cr}}|_{N_{\text{f}}=2} - T_{\text{cr}}|_{N_{\text{f}}=3} \simeq 20$MeV
- lattice: $T_{\text{cr}}|_{N_{\text{f}}=2} = 173 \pm 8$MeV, $T_{\text{cr}}|_{N_{\text{f}}=3} = 154 \pm 8$MeV (Karsch, Laermann, Peikert'01)

Conclusions

- ▷ functional RG:

  systematic and consistent expansion scheme for strongly coupled QFTs

- ▷ calculations from "first principles"

- ▷ operator expansion: promising at least in χ symmetric phase