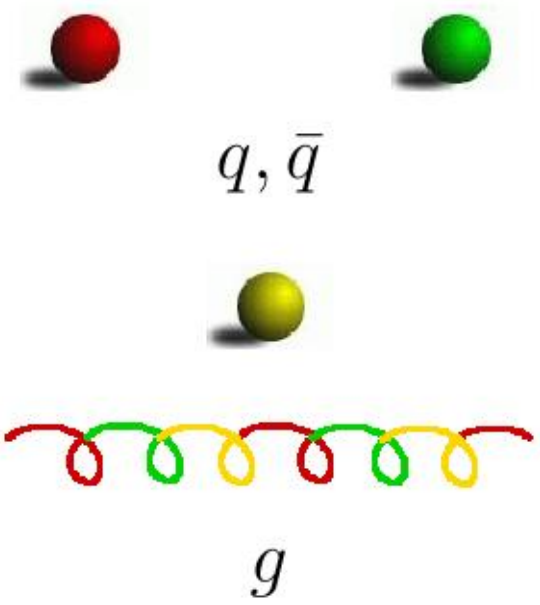
QCD chiral phase boundary from RG flows

Holger Gies

Heidelberg U.

From Micro to Macro DoF



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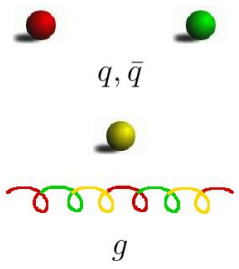
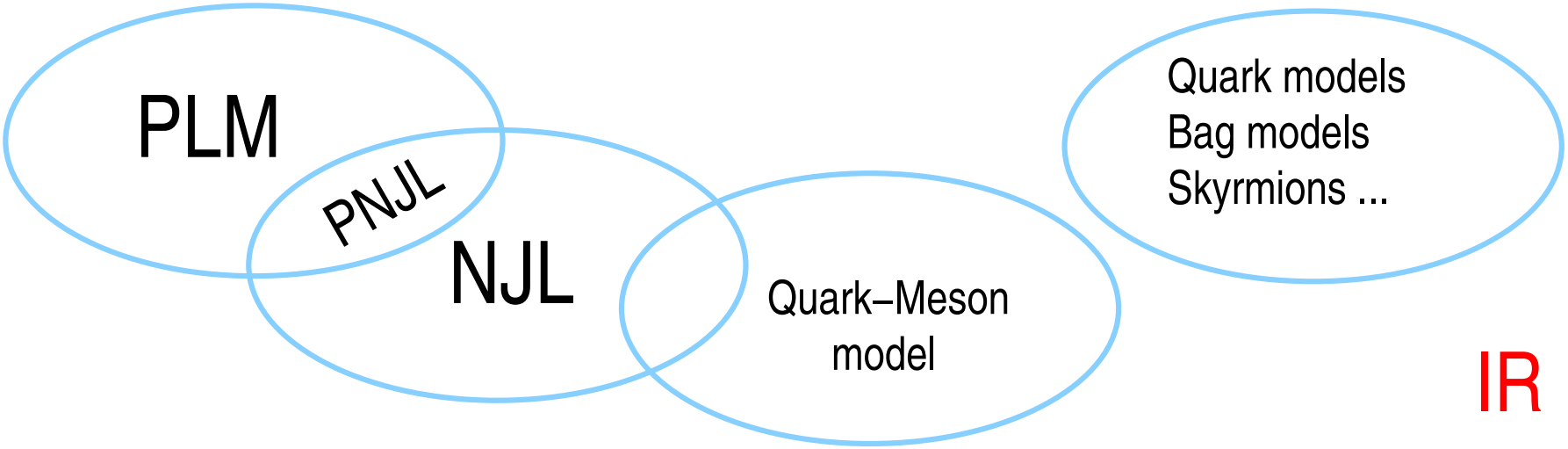
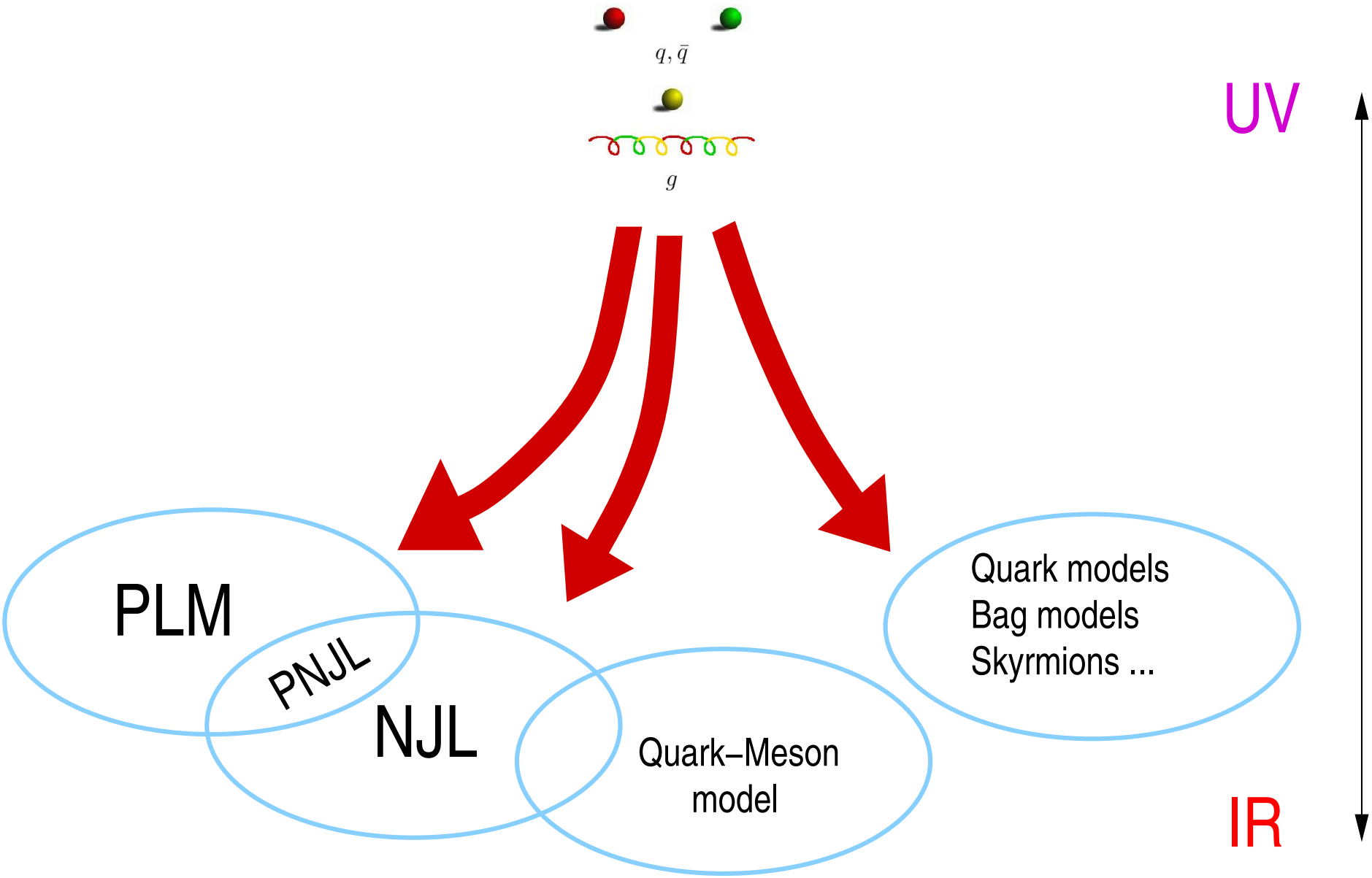


UV



IR

From Micro to Macro DoF



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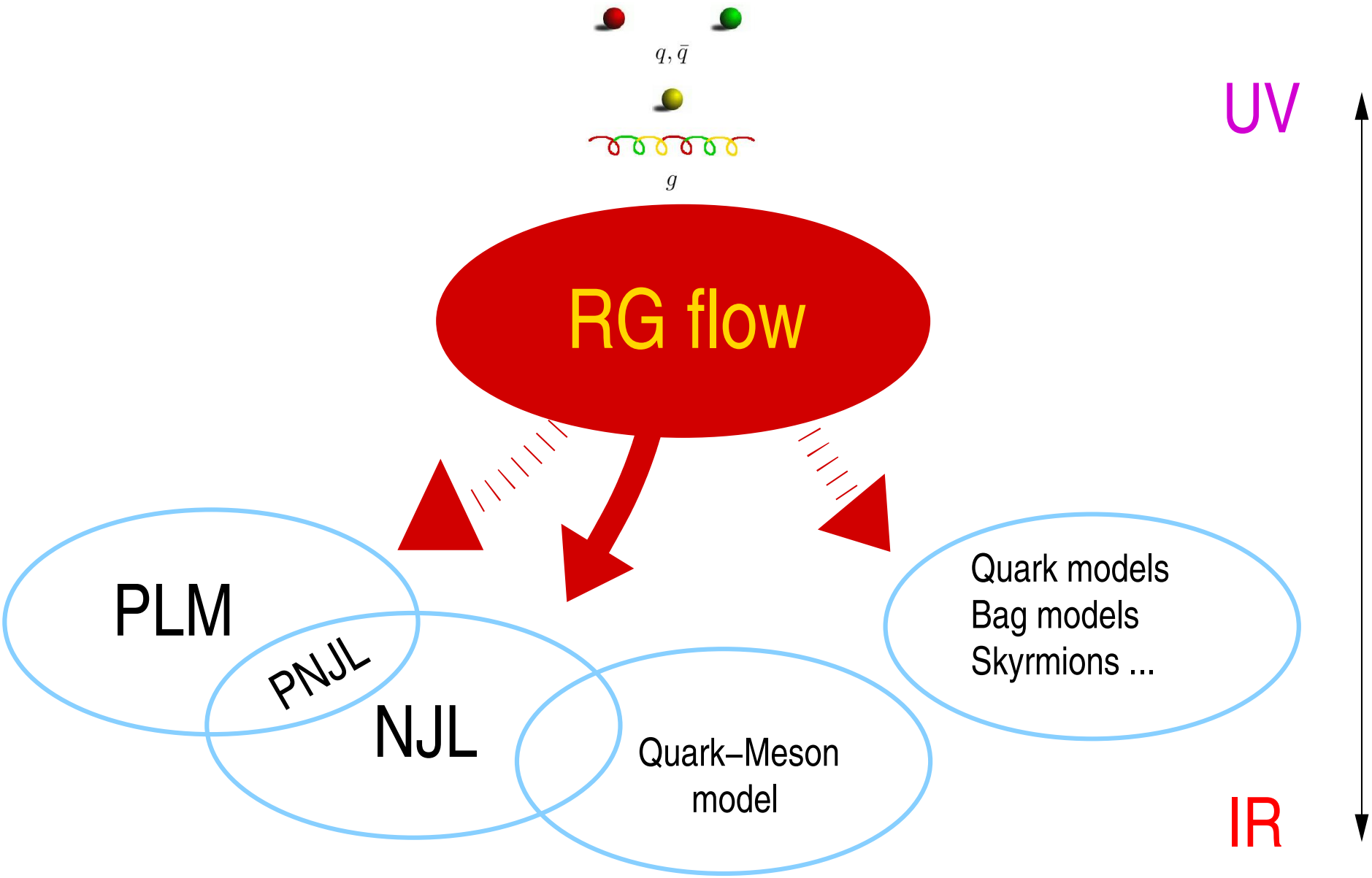


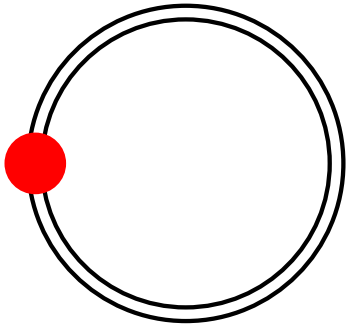
Effective Action

- universal tool:

effective action $\Gamma[\phi]$

Functional RG Flow Equation

$$\partial_t \Gamma_k \equiv k \partial_k \Gamma_k \;=\; \frac{1}{2}\,\mathrm{Tr}\;\partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

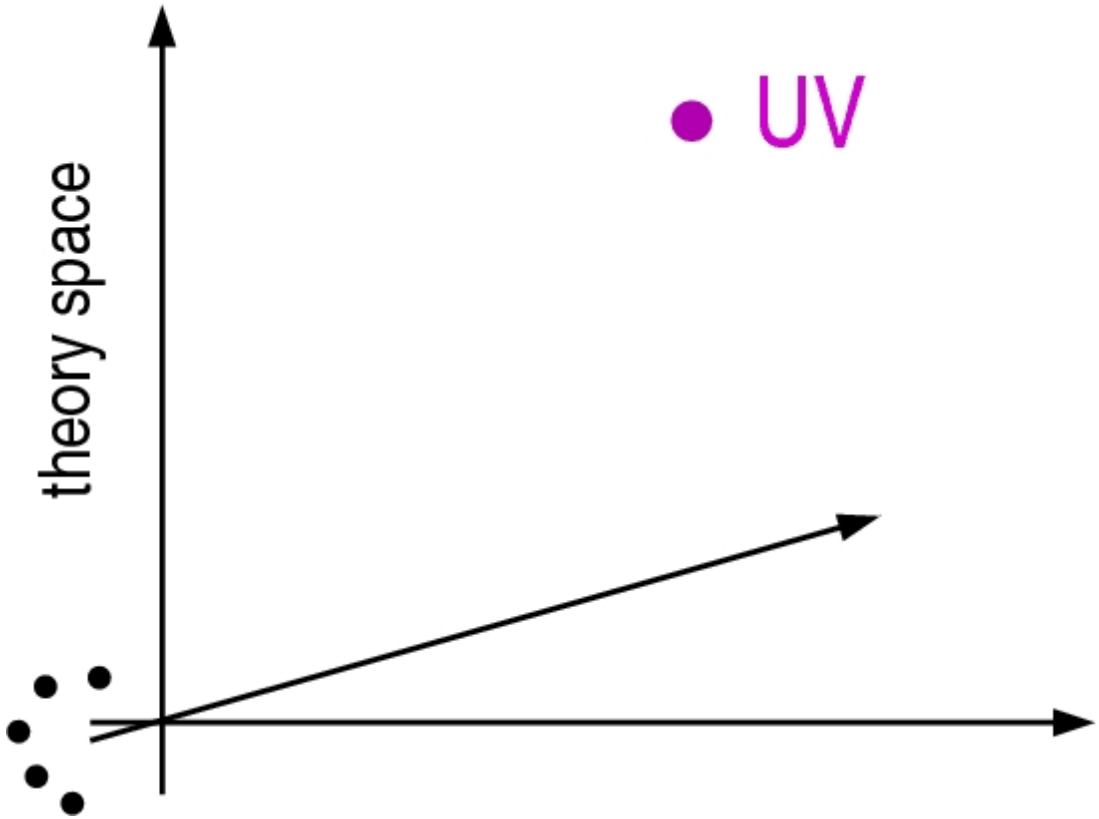
$=$ 

(WEGNER&HOUGHTON'73; WETTERICH'93)

Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

▷ RG trajectory: $\Gamma_{k=\Lambda} = S_{\text{bare}} = \int \frac{1}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \bar{\psi} \left(\mathrm{i} \partial\!\!\!/ + g A\!\!\!/ \right) \psi$



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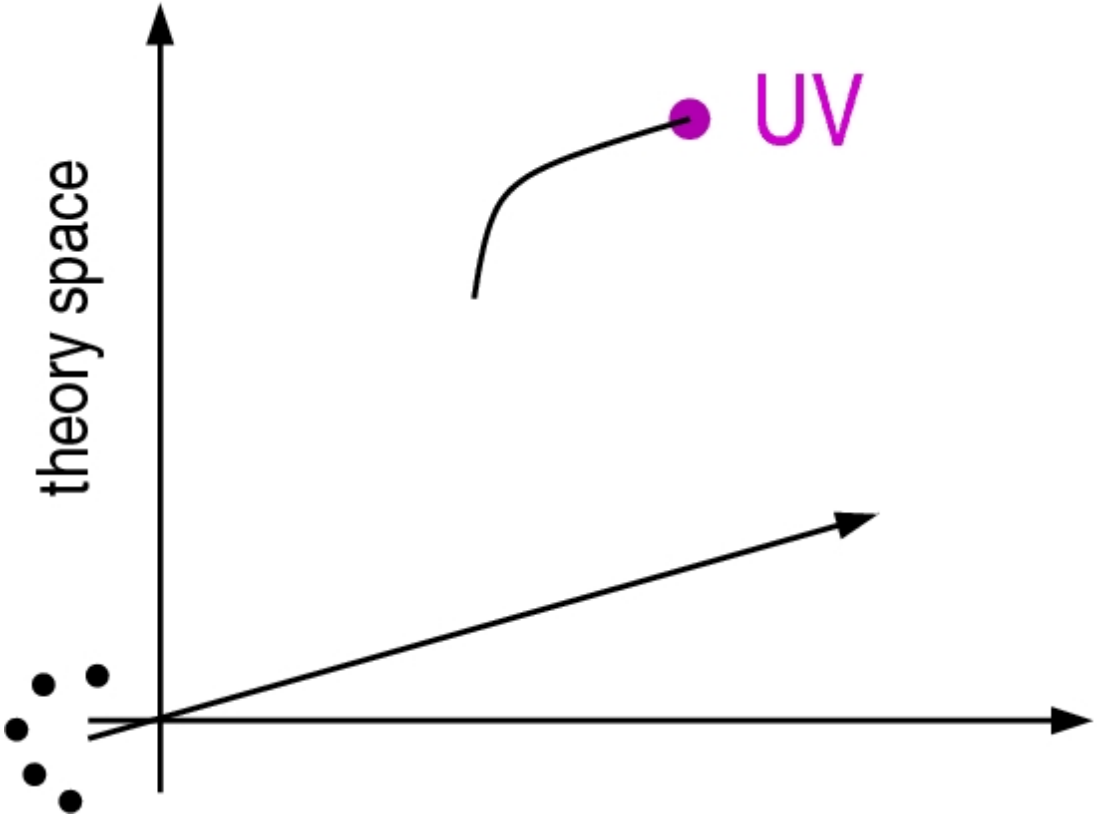

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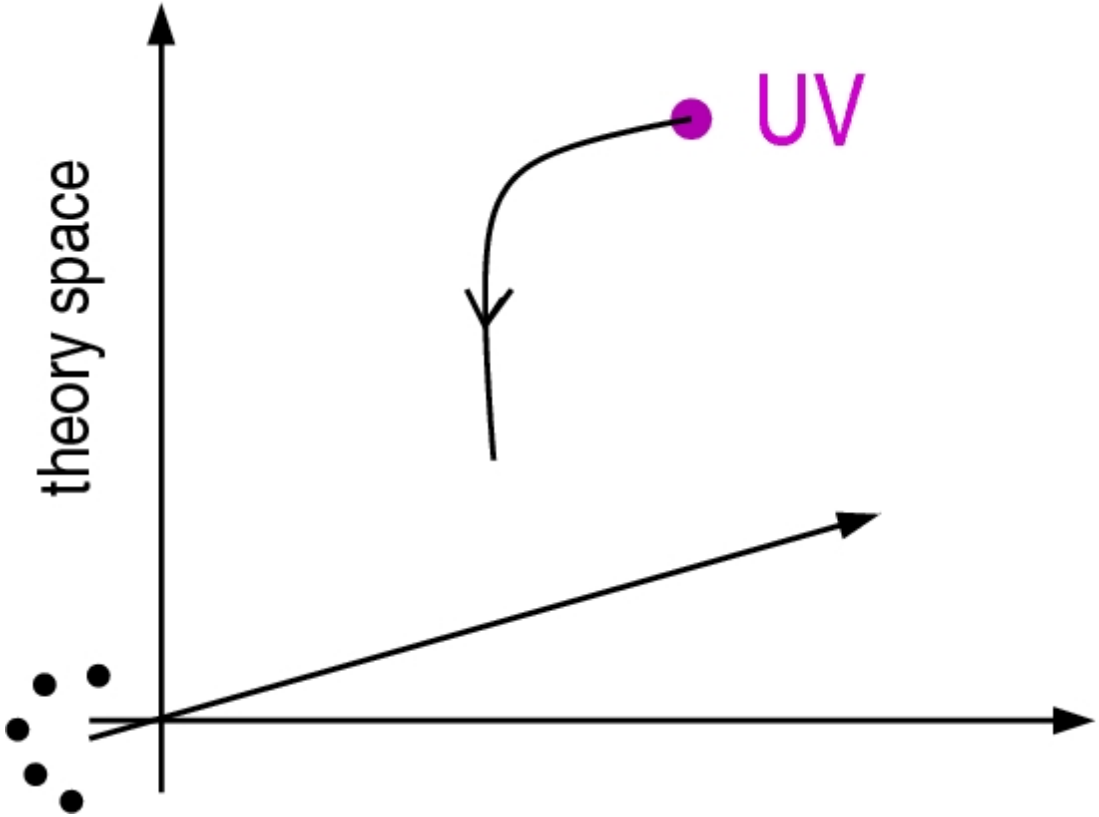
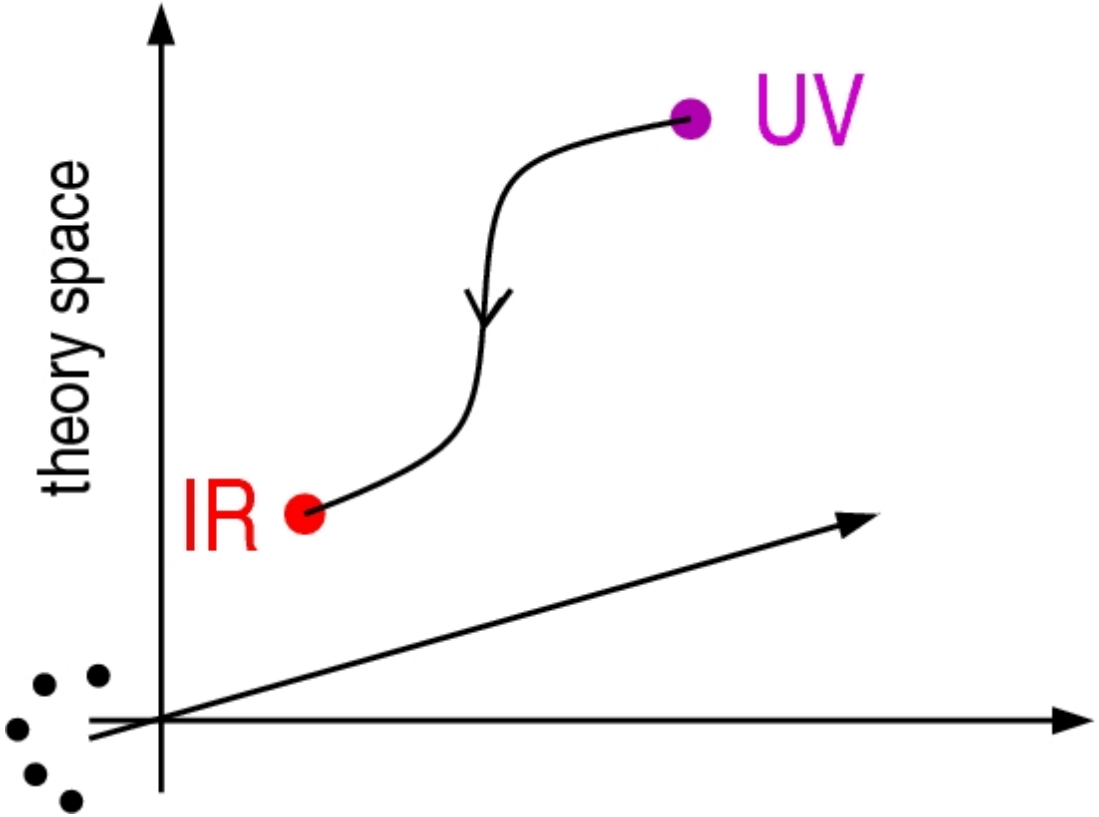


Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \, \mathrm{Tr} \, \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

▷ RG trajectory: $\Gamma_{k \to 0} = \Gamma$



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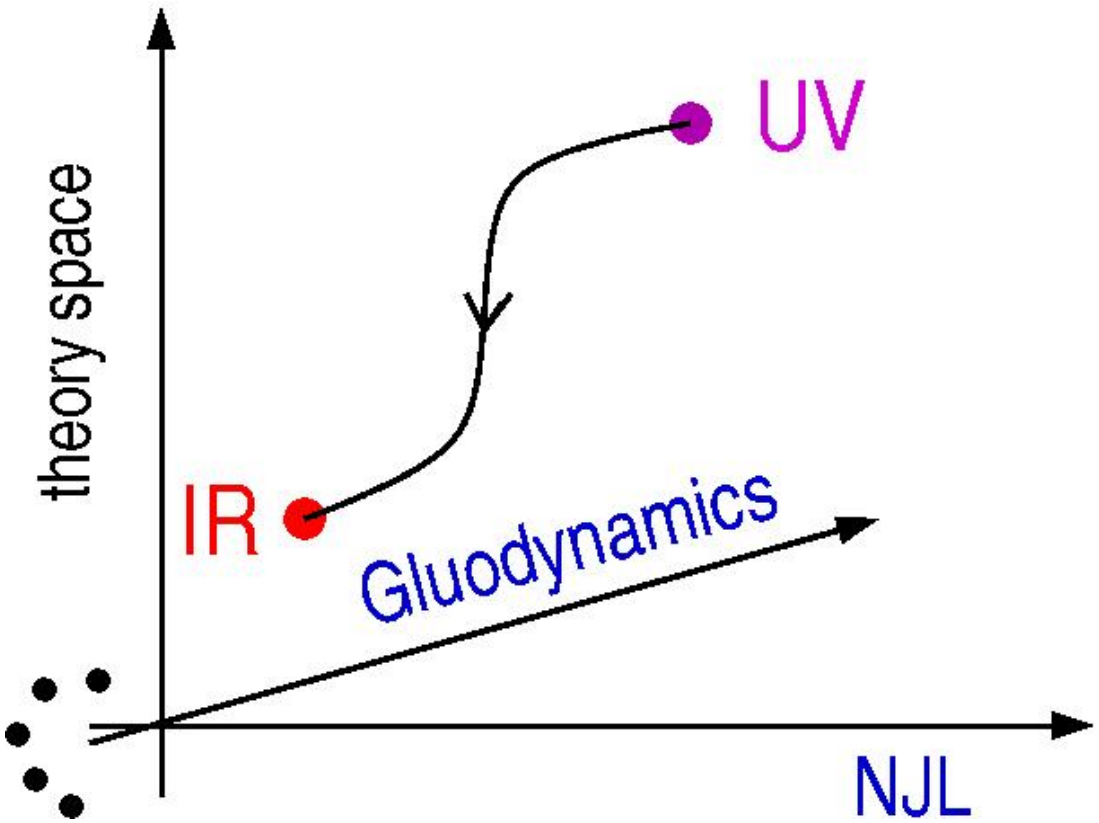


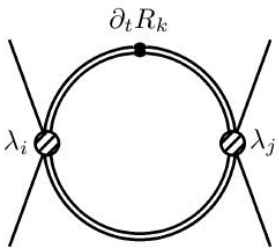
RG Flow of the Chiral Sector

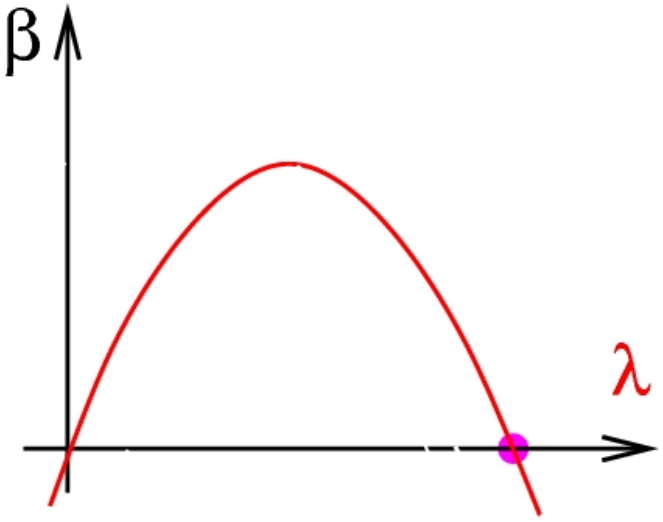
▷ effective action:

$$\Gamma_k = \int \frac{1}{2}\frac{\lambda_\sigma}{k^2}\left[(\bar\psi^a\psi^b)^2 - (\bar\psi^a\gamma_5\psi^b)^2\right]$$

▷ RG flow

$$\partial_t\lambda_\sigma = 2\lambda_\sigma - \frac{1}{4\pi^2} l_1^{(F)}\, 2N_c\, {\lambda_\sigma}^2$$





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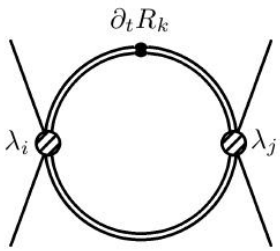
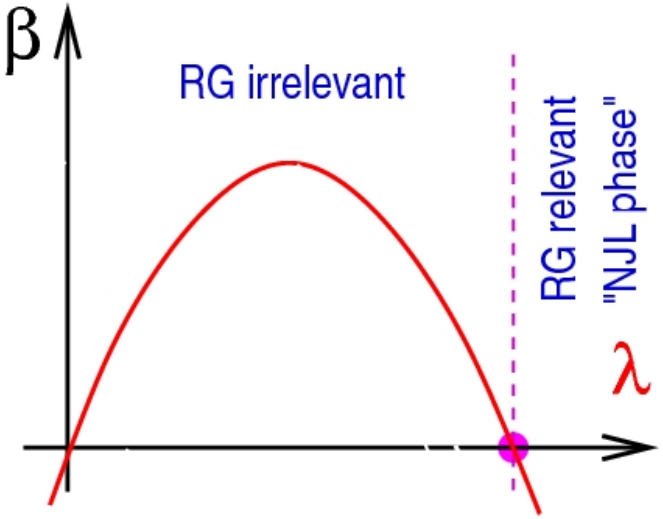




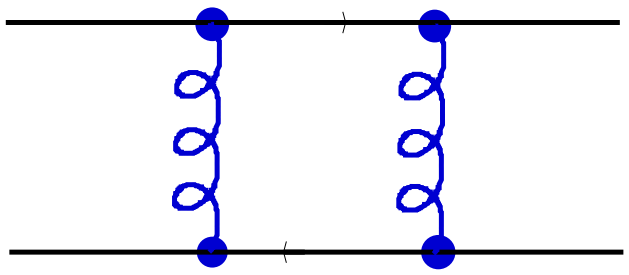
RG Flow of the Chiral Sector

▷ effective action:

$$\Gamma_k = \int \frac{Z_F}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \ldots + \bar\psi \left(\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar g A\!\!\!/ \right) \psi + \frac{1}{2} \frac{\lambda_\sigma}{k^2} \left[(\bar\psi^a \psi^b)^2 - (\bar\psi^a \gamma_5 \psi^b)^2 \right]$$

▷ RG flow

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - \frac{1}{4\pi^2} l_1^{(F)} 2N_c \, {\lambda_\sigma}^2 - \frac{1}{8\pi^2} l_{1,1}^{(FB)} 3 \frac{{N_c}^2 - 1}{N_c} g^2 \lambda_\sigma - \frac{3}{128\pi^2} l_{1,2}^{(FB)} \frac{3{N_c}^2 - 8}{N_c} g^4$$



RG Flow of the Chiral Sector

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$$\Gamma_k = \int \frac{Z_F}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \ldots + \bar\psi \left(i Z_\psi \partial\!\!\!/ + Z_1 \bar g A\!\!\!/ \right) \psi + \frac{1}{2} \frac{\lambda_\sigma}{k^2} \left[(\bar\psi^a \psi^b)^2 - (\bar\psi^a \gamma_5 \psi^b)^2 \right]$$

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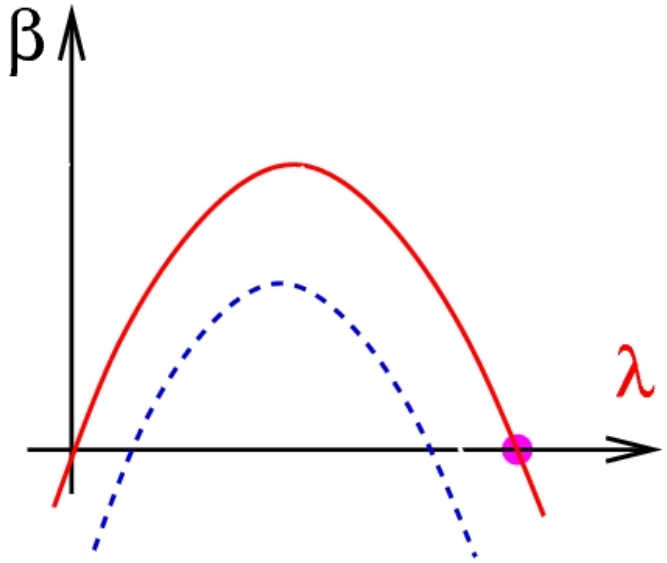

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$$+ \frac{1}{2}\frac{\lambda_\sigma}{k^2}\,(\text{S-P}) + \frac{1}{2}\frac{\lambda_{\text{VA}}}{k^2}\left[2(\text{V-A})^{\text{adj.}} + (1/N_c)(\text{V-A})\right]$$
$$+ \frac{1}{2}\frac{\lambda_+}{k^2}\,(\text{V+A}) + \frac{1}{2}\frac{\lambda_-}{k^2}\,(\text{V-A})$$

▷ RG flow

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - \frac{1}{4\pi^2} l_1^{(F)} \left\{ 2N_c\, {\lambda_\sigma}^2 - 2\lambda_- \lambda_\sigma - 2N_f \lambda_\sigma \lambda_{\text{VA}} - 6\lambda_+ \lambda_\sigma \right\}$$
$$- \frac{1}{8\pi^2} l_{1,1}^{(FB)} \left[3\frac{{N_c}^2 - 1}{N_c} g^2 \lambda_\sigma - 6 g^2 \lambda_+ \right]$$
$$- \frac{3}{128\pi^2} l_{1,2}^{(FB)} \frac{3{N_c}^2 - 8}{N_c} g^4$$

(HG, JAECKEL, WETTERICH'04)

Chiral Criticality

$\Longrightarrow$ critical gauge coupling $\alpha_{\rm cr}$:

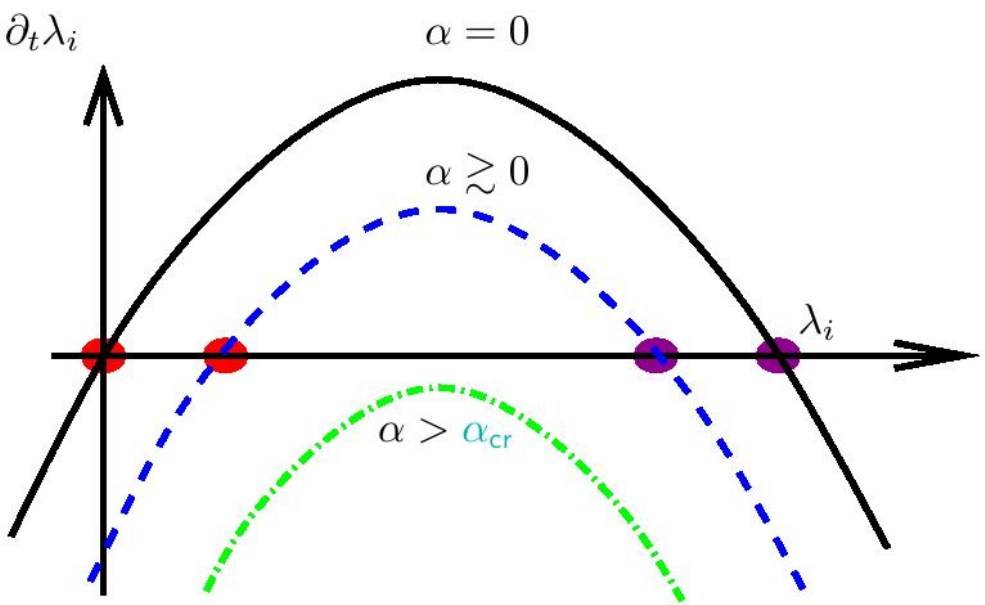
$\Longrightarrow$ if $\alpha > \alpha_{\rm cr}$: $\lambda \to \infty$

(χSB)

▷ $N_{\rm f} = 3 = N_{\rm c}$:

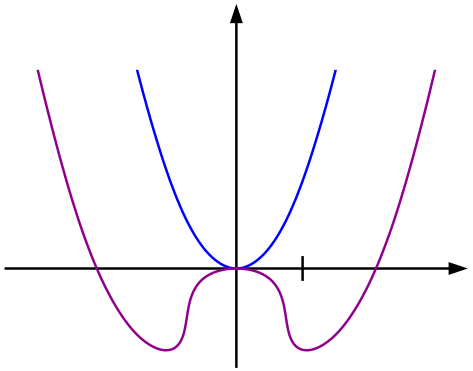
$$\alpha_{\rm cr} \simeq 0.85$$

(HG,JAECKEL'05)



$\Longrightarrow$ bosonization:

$$\lambda \sim \frac{1}{m_\phi^2}$$



Chiral Criticality at Finite Temperature

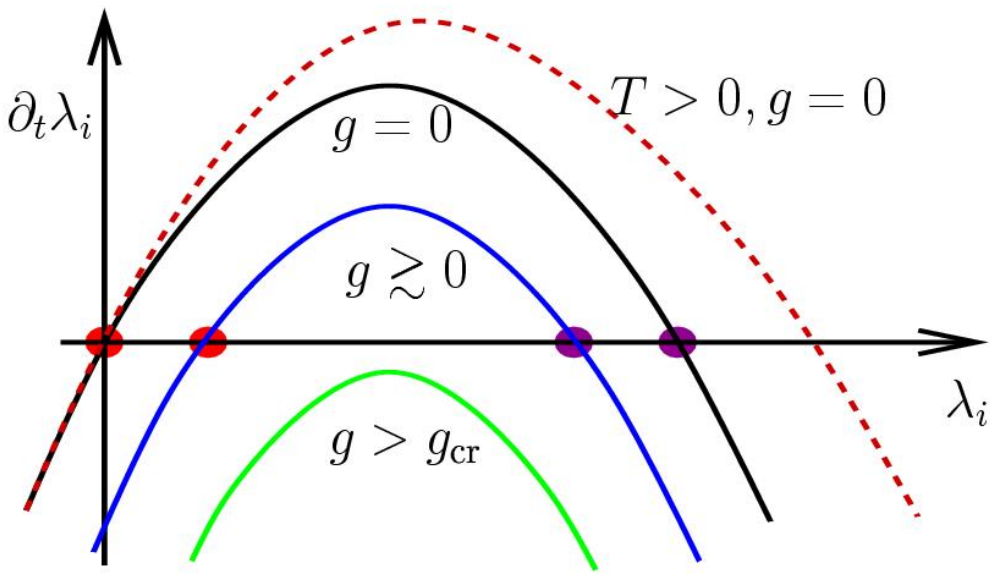
▷ quark modes:

$$m_T^2 = m_f^2 + (2\pi T(n + \frac{1}{2}))^2$$

$\Longrightarrow$ T-dependent critical coupling:

$$\alpha_{cr}(T) \gtrsim \alpha_{cr} \simeq 0.85$$

(BRAUN,HG'05)



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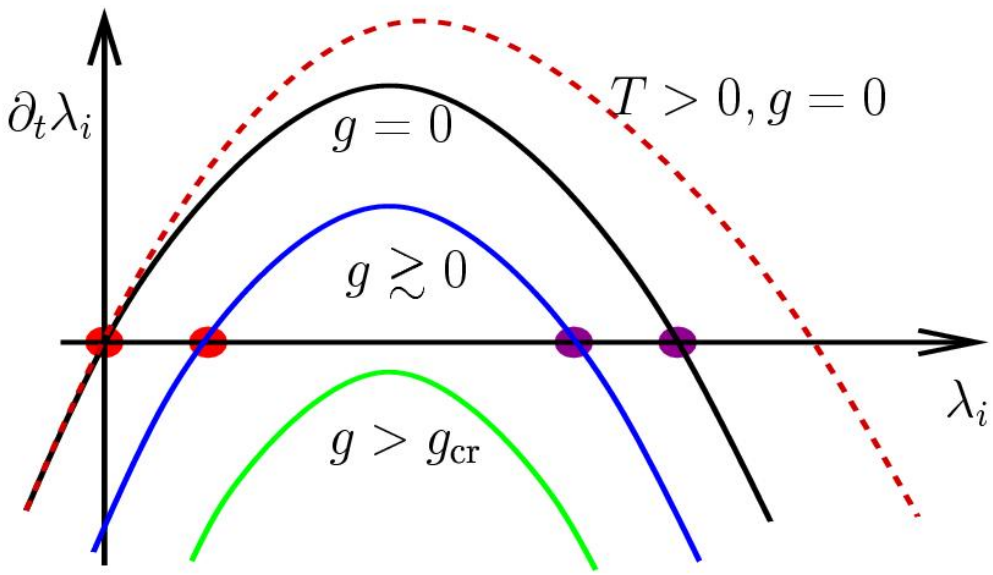


implications for low-energy QCD models:

$$\lambda_{init} = \lambda_{init}(T, \mu, \ldots)$$

RG Flow of Gluodynamics

- Operator expansion with the background-field method

(REUTER,WETTERICH'94; FREIRE,LITIM,PAWLOWSKI'00)

$$\Gamma_k[A] = \int d^d x \; W_k(F^2), \quad F^2 \equiv F^a_{\mu\nu} F^a_{\mu\nu}$$

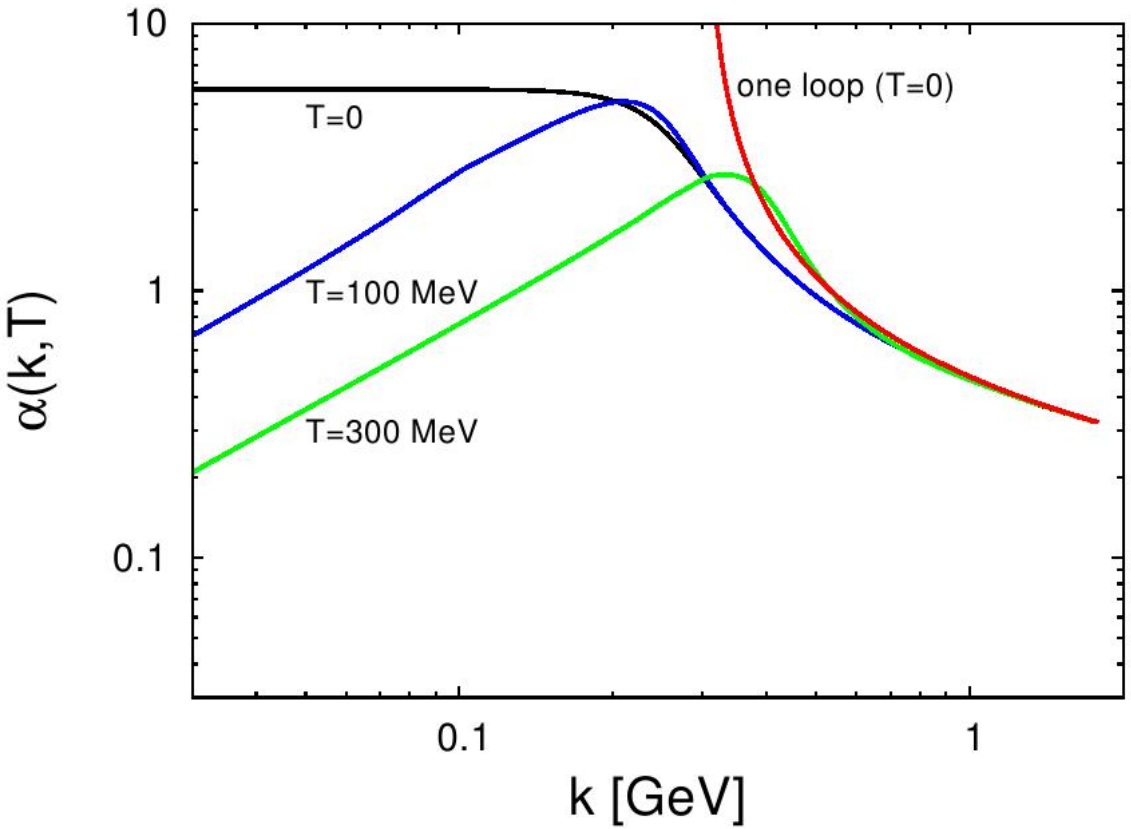
$$W_k(F^2) = \frac{Z_F}{4} F^2 + \frac{W_2}{16} (F^2)^2 + \frac{W_3}{3!4^3} (F^2)^3 + \frac{W_4}{4!4^4} (F^2)^4 + \dots$$

- spectrally adjusted flow equation: (HG'02)

$$\partial_t Z_F \curvearrowleft \partial_t W_2 \curvearrowleft \partial_t W_3 \curvearrowleft \partial_t W_4 \curvearrowleft \partial_t W_5 \dots$$

- running coupling: $g^2 = Z_F^{-1} \bar{g}^2$ (ABBOTT'82)
- β function: $\partial_t g^2 \equiv \beta_{g^2}$

Running Gauge Coupling



(HG'02)

(BRAUN,HG'05)

▷ $T = 0$: IR fixed point α_*

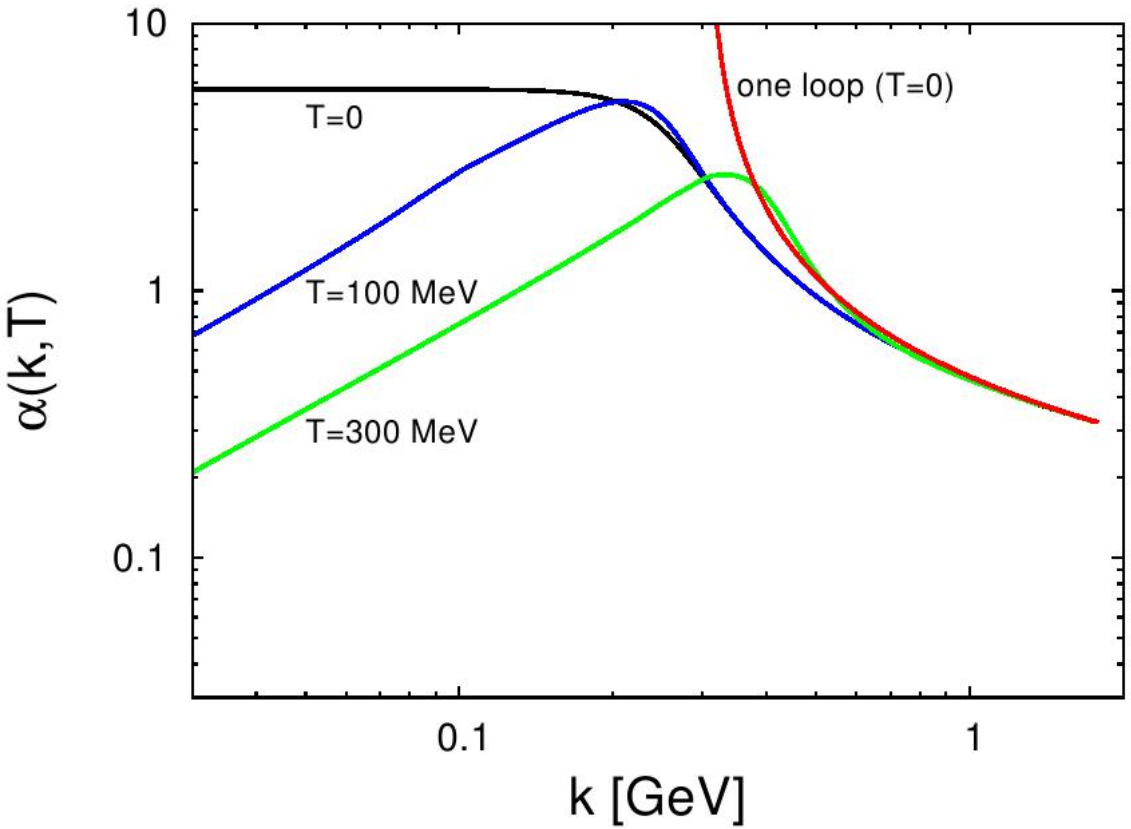
cf. vertex expansion in the Landau gauge

(V.SMEKAL,ALKOFER,HAUCK'97; FISCHER,ALKOFER'02; ZWANZIGER'02)

Running Gauge Coupling

(HG'02)

(BRAUN,HG'05)



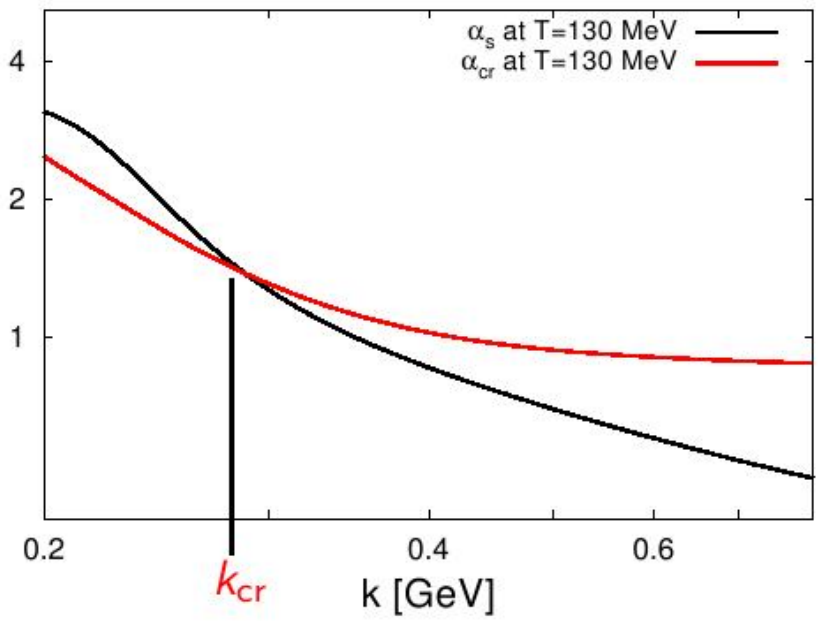
▷ $T/k \to \infty$: strongly interacting 3D theory

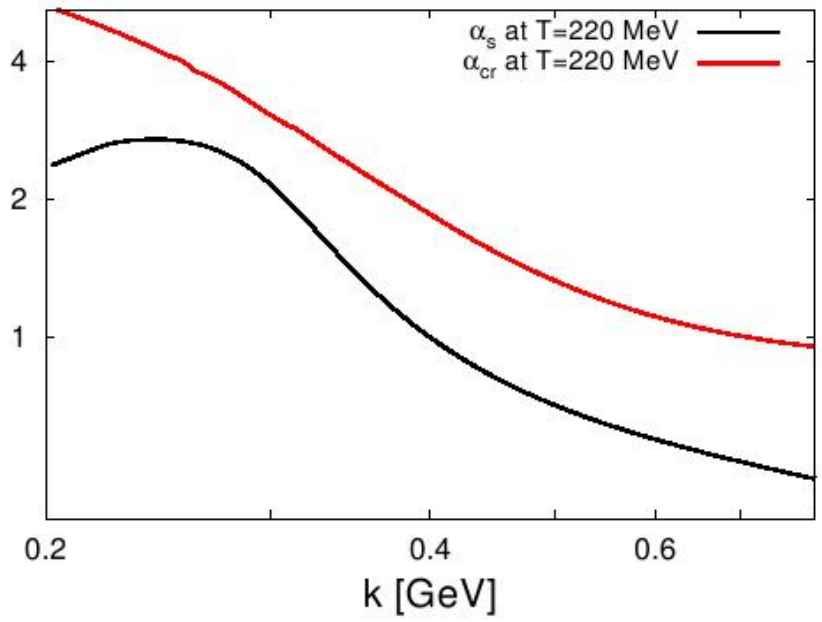
$$\alpha \to \frac{k}{T}\,\alpha_{3D}, \quad \alpha_{3D} \to \alpha_{3D,*} \simeq 2.7, \quad \eta_{3D} \to 1$$

cf. Landau-gauge vertex expansion: (MAAS,WAMBACH,ALKOFER'05)

Chiral Phase Transition

▷ $\alpha(k,T)$ vs. $\alpha_{\rm cr}(T/k)$





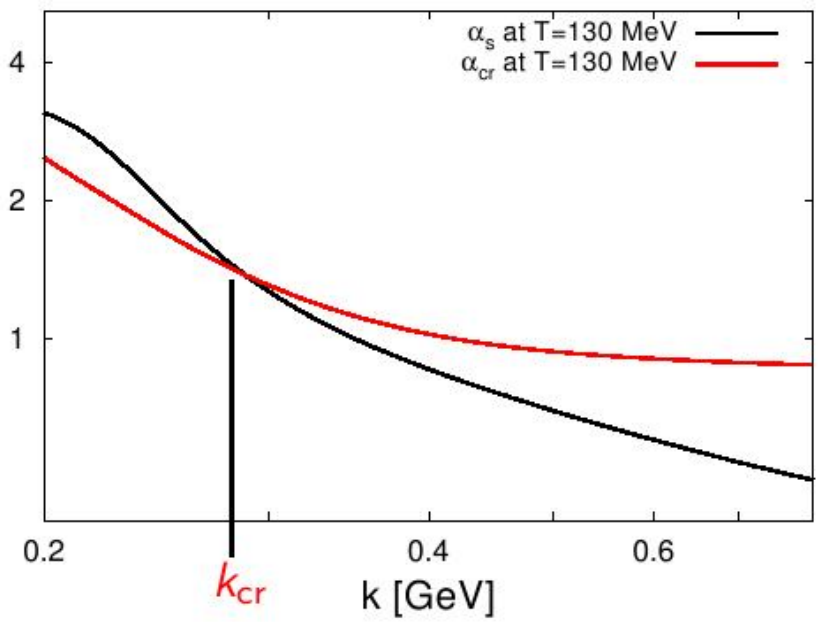
χSB triggered by α_s — single input: $\alpha_s(m_\tau) = 0.322$

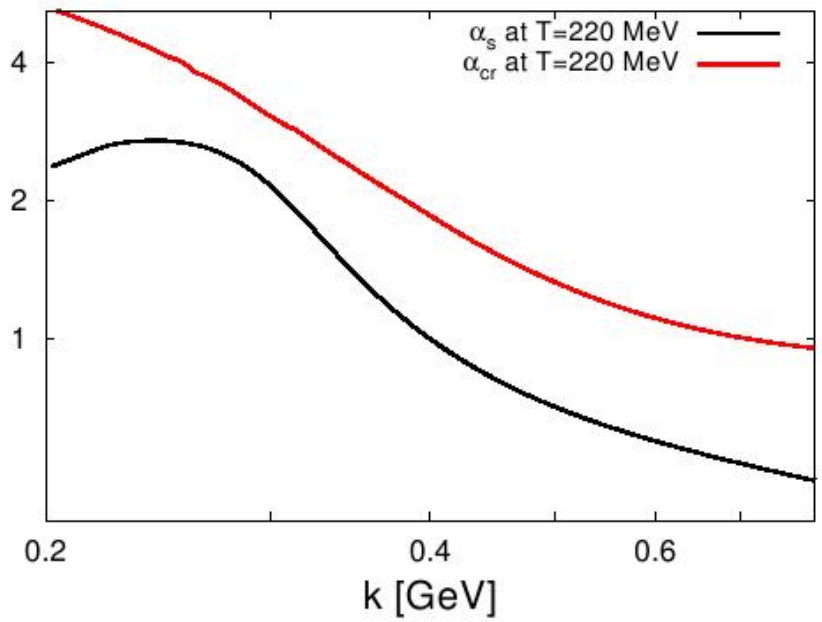
N_f	$T_{\rm cr}$	$T_{\rm cr}$ (lattice) (Karsch et al. 03)
2	172 MeV	175 ± 8 MeV
3	148 MeV	155 ± 8 MeV

(BRAUN,HG'06)

Chiral Phase Transition

▷ $\alpha(k,T)$ vs. $\alpha_{\rm cr}(T/k)$



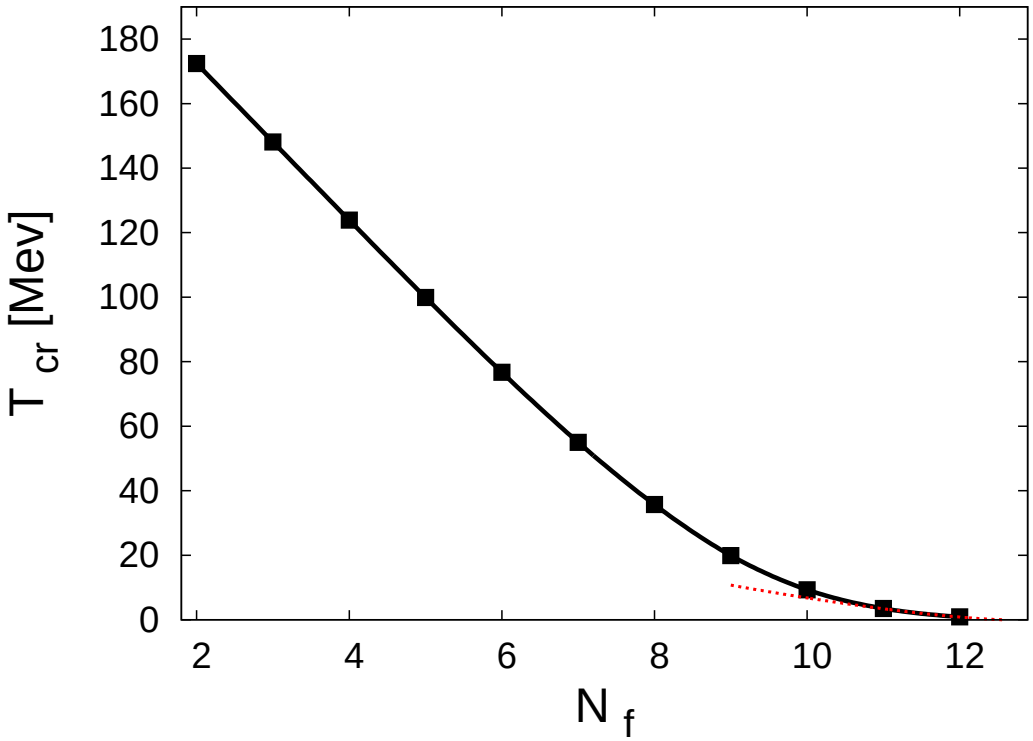


implications for low-energy QCD models:

Mind the glue!

$$\partial_t \Gamma_{\rm model} = \partial_t \Gamma_{\rm model}(\alpha, \dots)$$

Chiral Phase Boundary $T - N_f$



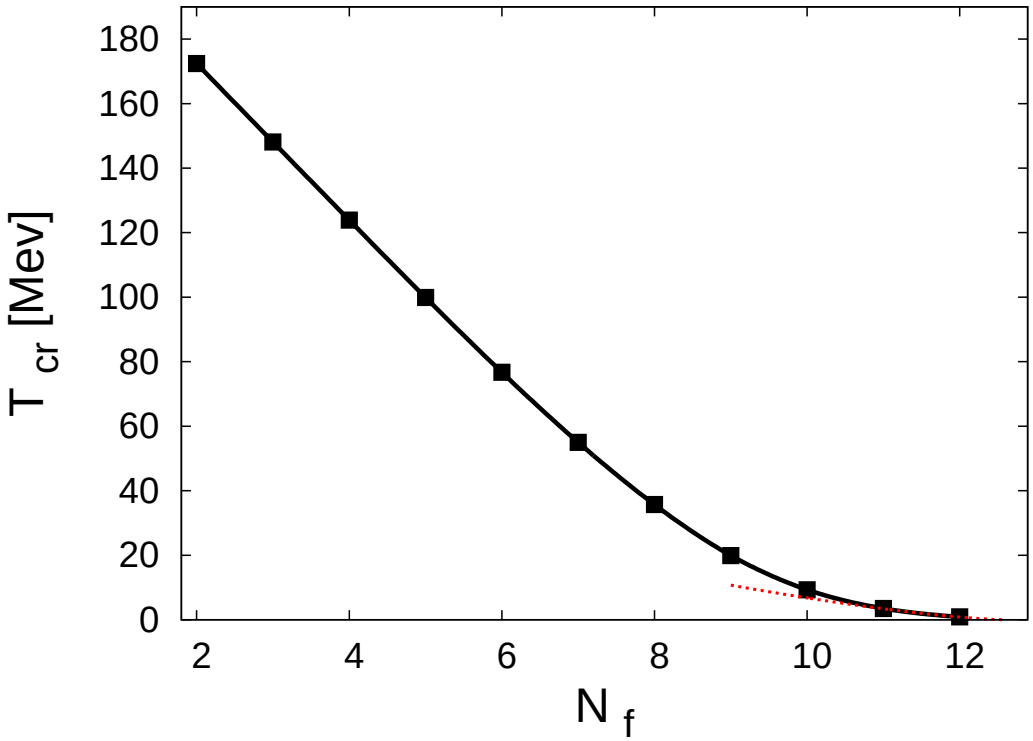
(BRAUN,HG'05,'06)

- small N_f: fermionic screening, $\beta_{\text{quark}} = \frac{2}{3} N_f \frac{g^4}{8\pi^2}$
- critical flavor number:

$$N_f^{cr} \simeq 12$$

(CF. APPELQUIST ET AL.'96; MIRANSKI,YAMAWAKI'96; HG,JAECKEL'05)

Chiral Phase Boundary $T - N_f$



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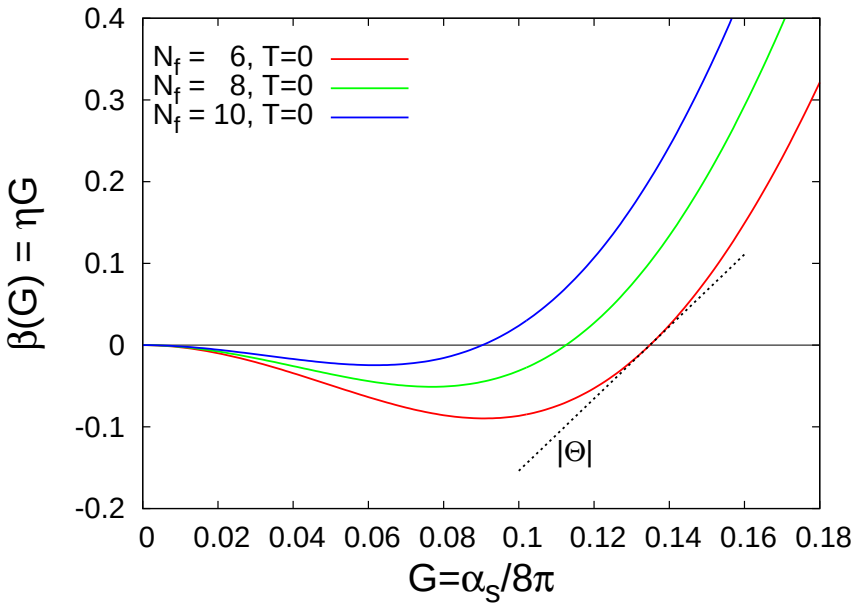
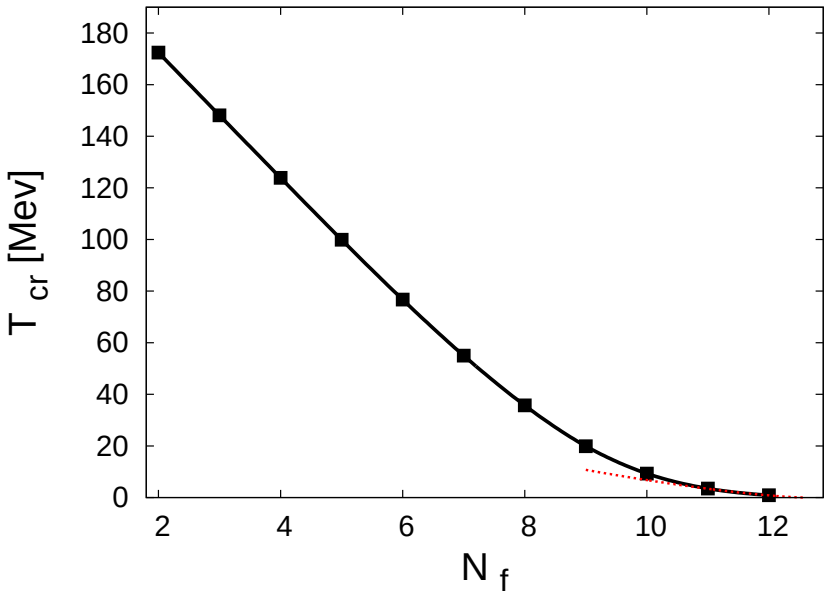
low-energy models:

$$T_{cr} = T_{cr}(N_f, m_f, \dots)$$

Chiral Phase Boundary $T - N_f$

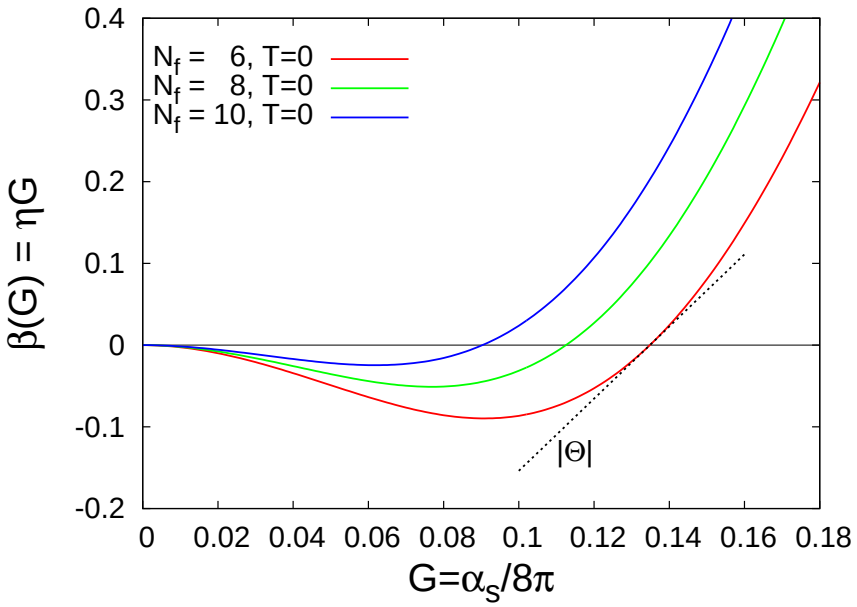


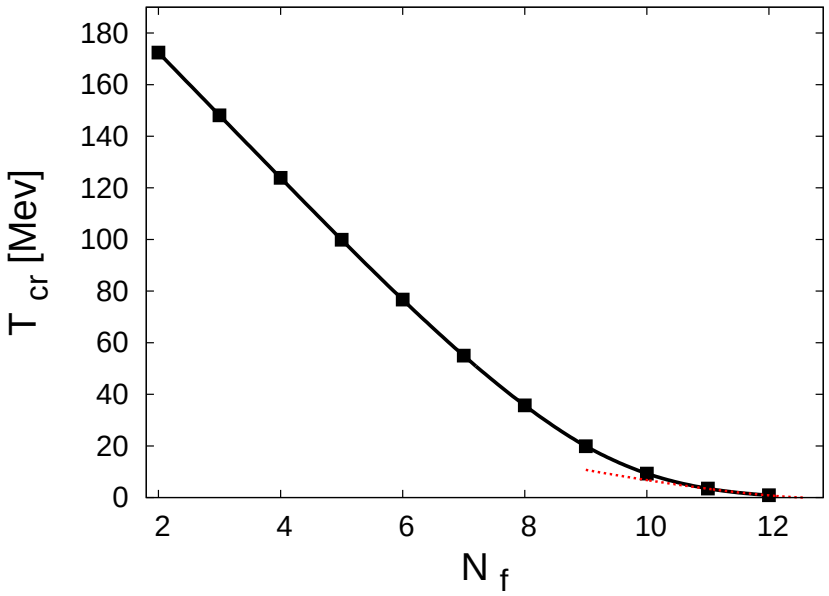


▷ fixed-point regime: critical exponent Θ

$$\beta_{g^2} \simeq -\Theta\,(g^2 - g_*^2)$$

Chiral Phase Boundary $T - N_f$





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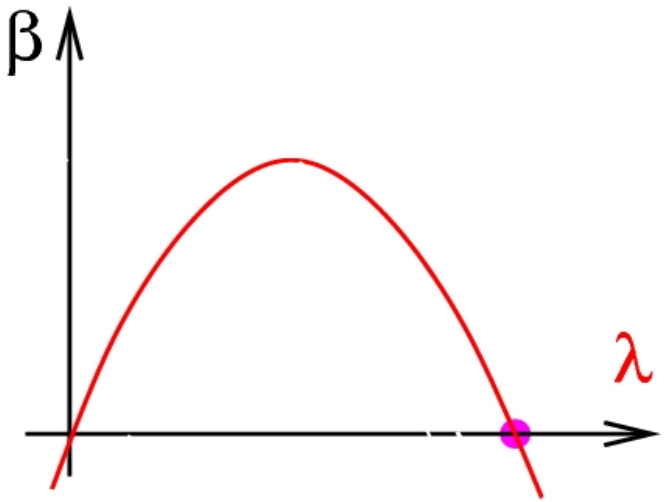
▷ shape of the phase boundary for $N_f \simeq N_f^{cr}$: (BRAUN,HG'05,'06)

$$T_{cr} \sim k_0\,|N_f - N_f^{cr}|^{\frac{1}{|\Theta|}}, \quad \Theta \simeq -0.71$$

Quark Mass Dependence

- NJL, quark-meson model:



- critical initial conditions

$$\frac{T_{\rm cr}(m) - T_{\rm cr}(0)}{T_{\rm cr}(0)} \simeq 0.3$$

for $m \lesssim 10$eV, $m_\pi \lesssim 200$MeV

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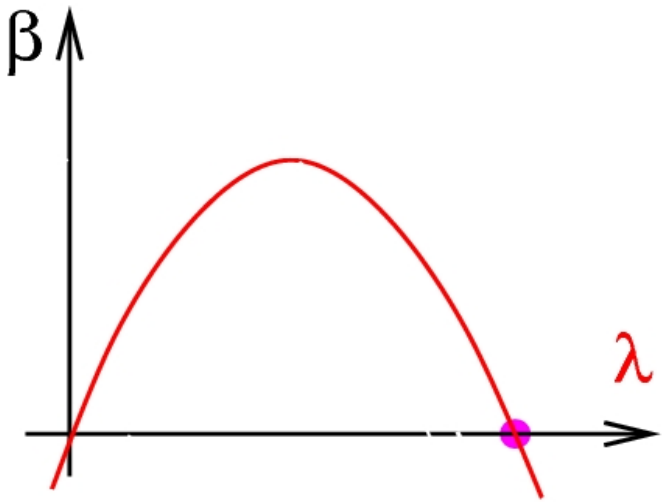

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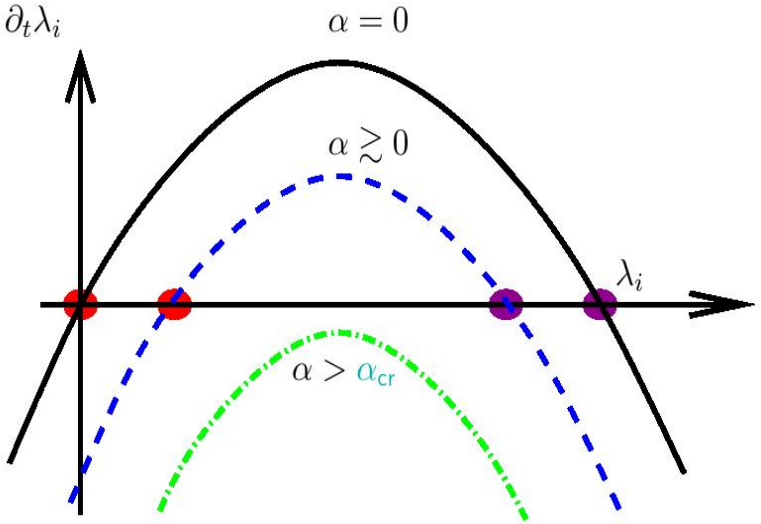
▷ QCD RG flow (BRAUN'06)



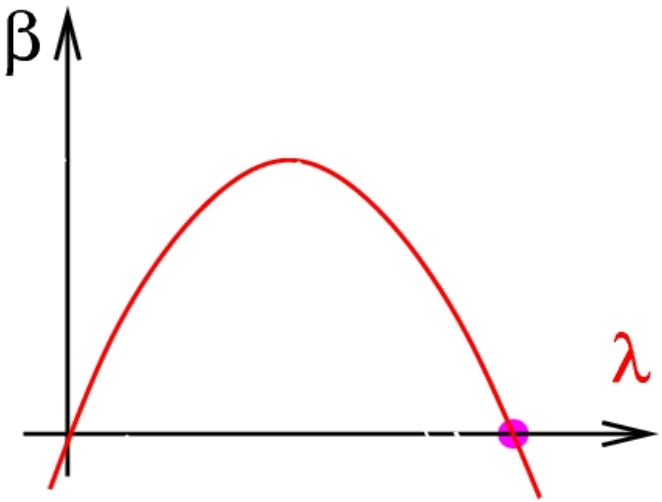
▷ gradual approach to criticality

$$\frac{T_{\rm cr}(m) - T_{\rm cr}(0)}{T_{\rm cr}(0)} \simeq 0.0$$

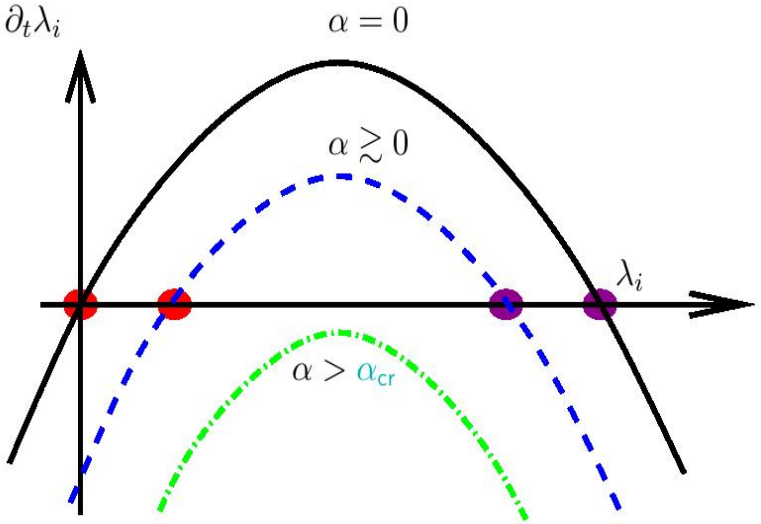
for $m \lesssim 10$eV, $m_\pi \lesssim 200$MeV

Quark Mass Dependence

▷ NJL, quark-meson model:



▷ QCD RG flow (BRAUN'06)



▷ critical initial conditions

$$\frac{T_{cr}(m) - T_{cr}(0)}{T_{cr}(0)} \simeq 0.3$$

for $m \lesssim 10$eV, $m_\pi \lesssim 200$MeV

▷ gradual approach to criticality

$$\frac{T_{cr}(m) - T_{cr}(0)}{T_{cr}(0)} \simeq 0.0$$

for $m \lesssim 10$eV, $m_\pi \lesssim 200$MeV

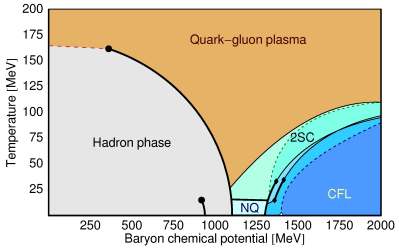
▷ lattice gauge theory (KARSCH,LAERMANN,PEIKERT'01)

$$\frac{T_{cr}(m) - T_{cr}(0)}{T_{cr}(0)} \simeq 0.04$$

Conclusions

- low-energy QCD models:
  - check relevant DoFs
  - chiral phase boundary
  - “first guide” to full QCD
- functional methods $\longrightarrow$ full QCD calculations
  - RG $\longleftrightarrow$ critical phenomena

$\Longrightarrow$ finite μ



$\Longrightarrow$ Polyakov loop (Braun,HG,Pirner’05; Braun,HG,Pawlowski’XX)