

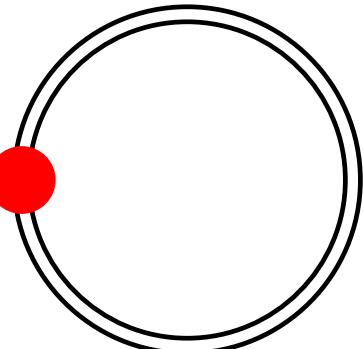
# Towards a renormalizable Standard Model without fundamental Higgs scalar

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- Renormalizability
- Towards the Standard Model
- Quest for **UV** completion

$$\partial_t \Gamma_k = \frac{1}{2} \partial_t R_k$$



& J. Jäckel, C. Wetterich: Phys.Rev.D69:105008,2004 (hep-ph/0312034)

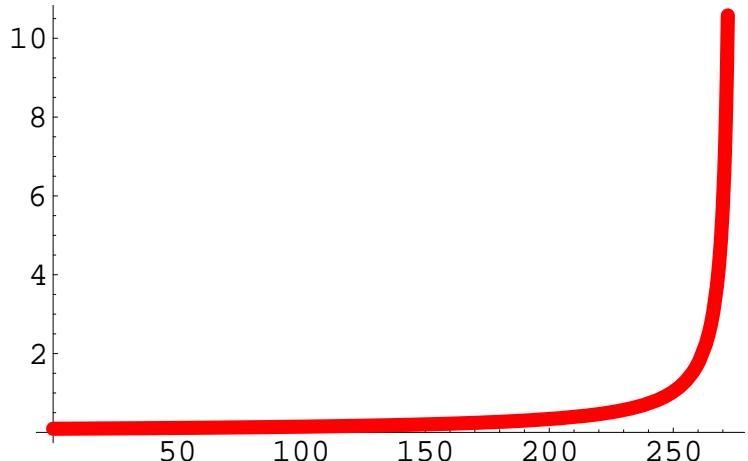
# Some shortcomings of the Standard Model

- ▷ abundance of parameters
  - ▷ origin of flavor physics
  - ▷ origin of neutrino physics
- 
- ▷ hierarchy problem (naturalness problem, finetuning problem):
$$\Lambda_{\text{UV}} \gg \Lambda_{\text{EW}} \quad (\gg \Lambda_{\text{QCD}})$$
  - ▷ triviality problem (Landau pole singularities) of U(1) and scalar Higgs sector

# Triviality problem

- ▷ QED: perturbation theory predicts its own failure (LANDAU'55)

$$\frac{1}{e_R^2} - \frac{1}{e_\Lambda^2} = \beta_0 \ln \frac{\Lambda}{m_R}, \quad \beta_0 = \frac{N_f}{6\pi^2}$$



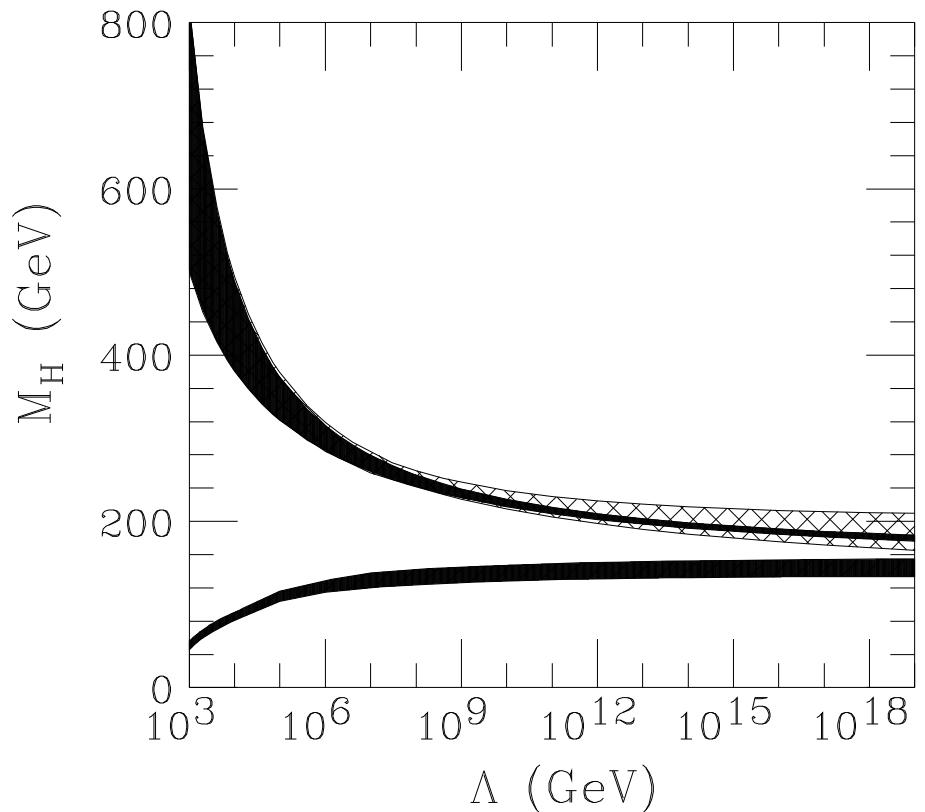
- ▷  $e_R^2$  and  $m_R$  fixed:  $\implies \Lambda_L \simeq m_R \exp\left(\frac{1}{\beta_0 e_R^2}\right) \simeq 10^{272} \text{GeV}$  (2 loop)  
**Landau pole singularity**

- ▷  $\lim (\Lambda/m_R) \rightarrow \infty: \implies e_R^2 \rightarrow 0$

**Triviality**

- ▷ Triviality problem:
  - scale of maximal UV extension

- ▷ triviality of the scalar Higgs sector:
- Higgs mass bounds in the Standard Model from Landau pole position
- ▷ Standard model  $\neq$  fundamental QFT(?)



(HAMBYE,RIESSELMANN'97)

# Hierarchy problem $\Lambda_{\text{UV}} \ggg \Lambda_{\text{EW}}$

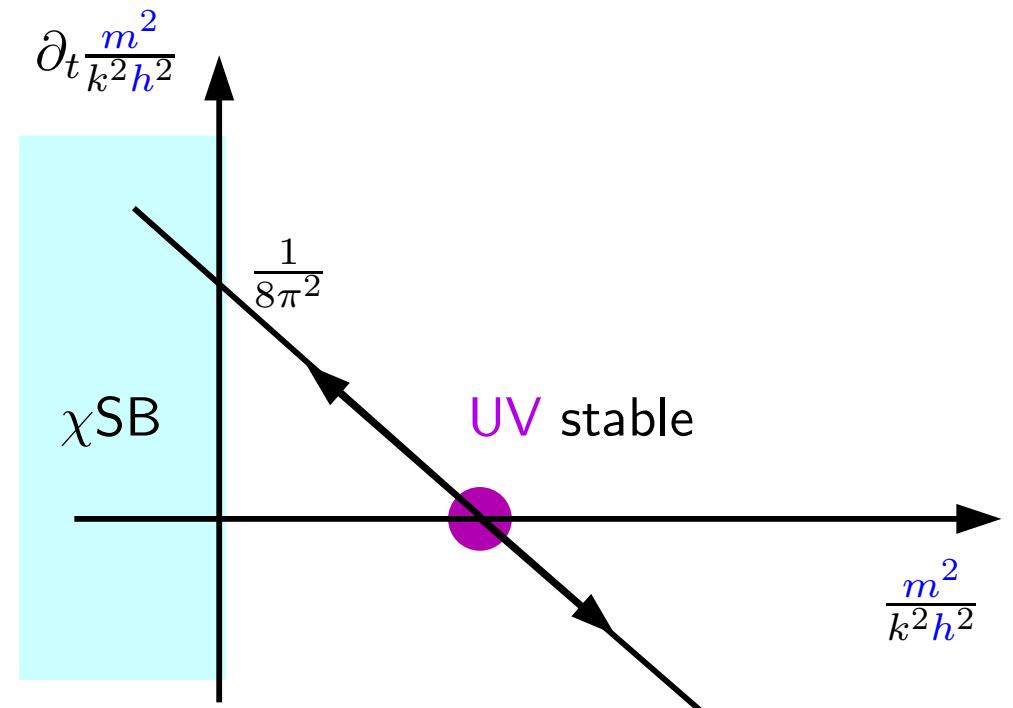
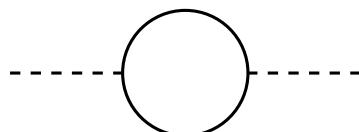
- ▷ renormalization of the scalar mass parameter (e.g.,  $\Lambda_{\text{UV}} = 10^{15} \text{ GeV}$ )

$$\underbrace{m_R^2}_{\sim 10^4 \text{ GeV}^2} \sim \underbrace{m_\Lambda^2}_{\sim 10^{30} \left(1 + \dots 10^{-26}\right) \text{ GeV}^2} - \underbrace{\delta m^2}_{\sim 10^{30} \text{ GeV}^2}$$

- ▷ RG viewpoint ( $\partial_t = k \frac{d}{dk}$ )

e.g., Yukawa theory:

$$\partial_t \frac{m^2}{k^2 h^2} = -2 \frac{m^2}{k^2 h^2} + \frac{1}{8\pi^2}$$



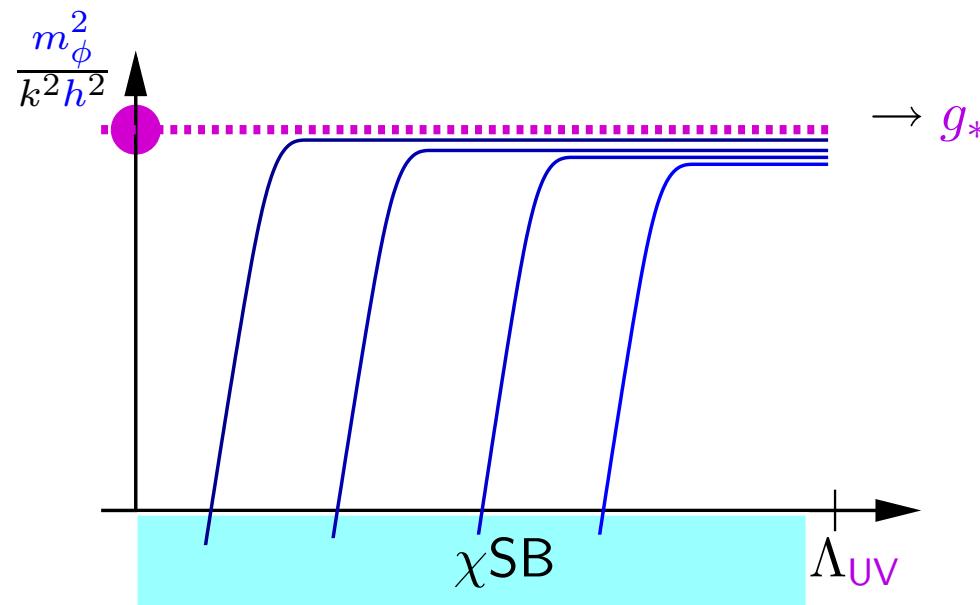
## $\simeq$ Finetuning problem

▷ Def.: “coupling”:  $g := \frac{m^2}{h^2 k^2}$ ,      “ $\beta$  function”:  $\partial_t g = \beta(g)$

▷ critical exponent  $\Theta$

$$\Theta = -\frac{\partial \beta(g_*)}{\partial g} = 2$$

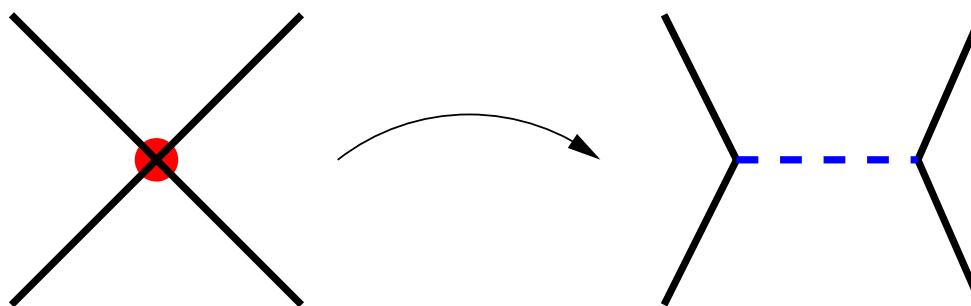
$\implies \Theta$ : measure for the required finetuning



# A composite Higgs particle . . . ?

- ▷ hierarchy and (part of the) triviality problem caused by fundamental Higgs scalar
- ▷ “top-quark condensation”:  $\phi \sim \bar{\psi}\psi$ 
  - (NAMBU'89; MIRANSKI, TABANASHI, YAMAWAKI'89)
  - (BARDEEN, HILL, LINDNER'91)
- (fermionic interactions  $\longleftrightarrow$  scalar Higgs sector) related by partial bosonization

(STRATONOVICH'57, HUBBARD'59)



$$2 \lambda_\sigma \bar{\psi}_L t_R \bar{t}_R \psi_L \rightarrow h (\bar{t}_R \psi_L \phi - \bar{\psi}_L t_R \phi^*) + m_\phi^2 \phi^* \phi$$

at  $\Lambda_{\text{UV}}$ :  $\lambda_\sigma = \frac{1}{2} \frac{h^2}{m_\phi^2} \Big|_{\Lambda_{\text{UV}}}$ , (compositeness condition)

$\implies$  reduction of parameters:  $m_H, m_t, v = f(\lambda_\sigma, \Lambda_{\text{UV}})$

. . . and it's shortcomings.

- ▷ fixing  $v \simeq 246\text{GeV}$  and  $\Lambda_{\text{UV}}$  predicts (BHL):

| $\Lambda_{\text{UV}}$ |               | $10^{19}$ | $10^{17}$ | $10^{15}$ | $10^{13}$ |
|-----------------------|---------------|-----------|-----------|-----------|-----------|
| $m_t/\text{GeV}$      | (quark loop)  | 143       | 153       | 165       | 180       |
| $m_t/\text{GeV}$      | (pert. RG)    | 218       | 223       | 229       | 237       |
| $m_t/\text{GeV}$      | (imp.lad.DSE) | 253       | 259       | 268       | 279       |
| $m_H/\text{GeV}$      | (quark loop)  | 290       | 309       | 333       | 364       |
| $m_H/\text{GeV}$      | (pert. RG)    | 239       | 246       | 256       | 268       |

⇒ simplest model ruled out ! (?)

- ▷ still a hierarchy problem:  $\lambda_\sigma \rightarrow \lambda_{\text{cr}}$  finetuning with exponent  $\Theta \simeq 2$
- ▷ perturbatively non-renormalizable:  $[\lambda_\sigma] = -2$

# Renormalizability

## ▷ Why?

- IR physics well separated from UV physics (cutoff  $\Lambda_{UV}$  independence)
- # of physical parameters  $< \infty$
- QFT should be predictive

⇒ realized in perturbative RG

## ▷ beyond perturbation theory?

⇒ Scenario of “Asymptotic Safety”

(WEINBERG'76)

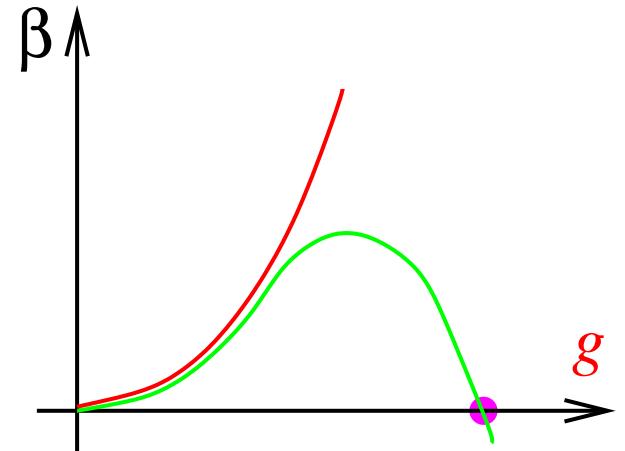
(GELL-MANN, LOW'54)

# Asymptotic Safety

- ▷ assumption: non-Gaußian UV stable fixed point  $g_*$

$$\partial_t g_i = \beta_i(g_1, g_2, \dots), \quad \beta_i(g_{*1}, g_{*2}, \dots) = 0$$

with at least one  $g_{*i} \neq 0$ .



- ▷ linearized fixed point regime:

$$\partial_t g_i = B_i^j (g_{*j} - g_j) + \dots, \quad B_i^j = \frac{\partial \beta_i}{\partial g_j} \quad (\text{stability matrix})$$

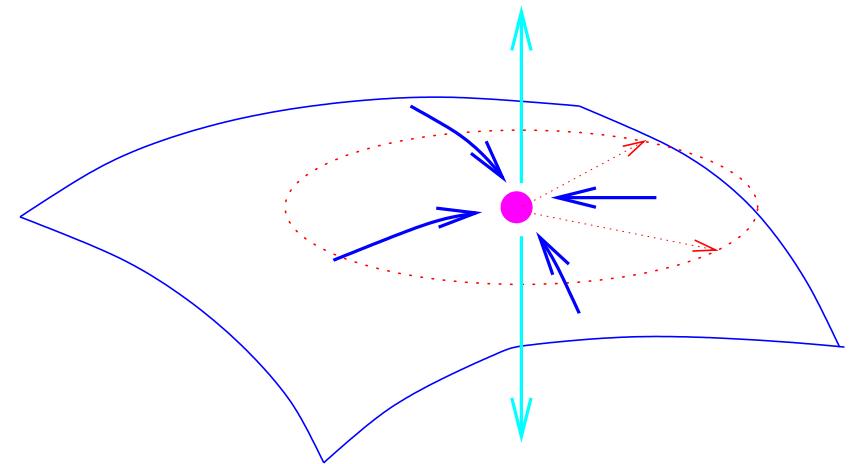
- ▷ general solution

$$g_i = g_{*i} + \sum_I C^I V_i^I \left( \frac{k_0}{k} \right)^{\Theta^I}, \quad B_i^j V_j^I = -\Theta^I V_i^I,$$

with initial conditions  $C^I = \text{const.}$

## asymptotic safety cont'd

- ▷  $\Theta^I < 0$ : UV repulsive  $\implies$  RG irrelevant
- ▷  $\Theta^I > 0$ : UV attractive  $\implies$  RG relevant
- ▷  $\Theta^I = 0$ : it depends . . .  $\implies$  RG marginal



- ▷ all RG relevant RG trajectories form the critical surface  $S$
- ▷ all relevant  $V_i^I$  span the tangent space to  $S$  at  $g_*$

$$\dim S = \Delta \hat{=} \# \text{ of relevant directions with } \Theta^I > 0$$

- ▷ Renormalization: for all irrelevant  $\Theta^I < 0$ , set  $C^I = 0$

$\implies$  limit  $k \rightarrow \Lambda_{\text{UV}} \rightarrow \infty$  can safely be taken

(cutoff independence ✓)

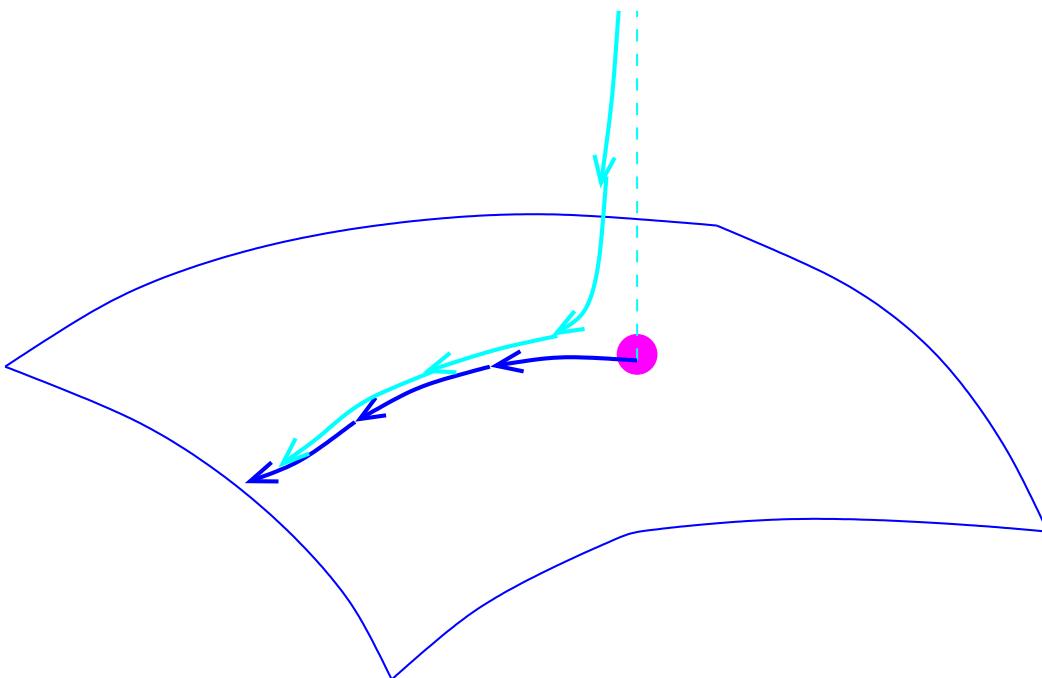
## asymptotic safety cont'd

- ▷ How many physical parameters?

$$\# \text{ of parameters} = \# \text{ of init.cond.} = \Delta = \dim \mathcal{S}$$

( $\# \text{ of physical parameters} < \infty$  iff  $\Delta < \infty$  ✓)

- ▷ Predictivity?



( universality & predictivity ✓)

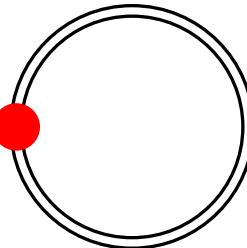
# Exact RG Flow Equation

IR:  $k \rightarrow 0$



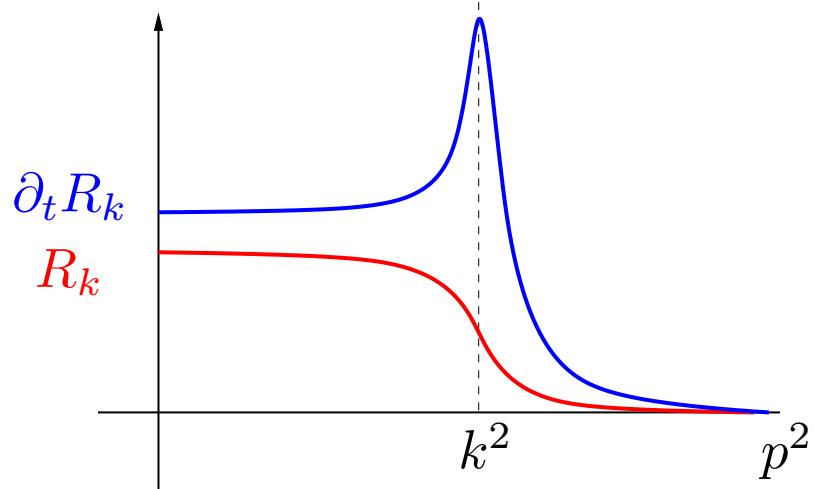
UV:  $k \rightarrow \Lambda$

$$\partial_t \Gamma_k = \frac{1}{2} S \text{Tr} \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1} =$$

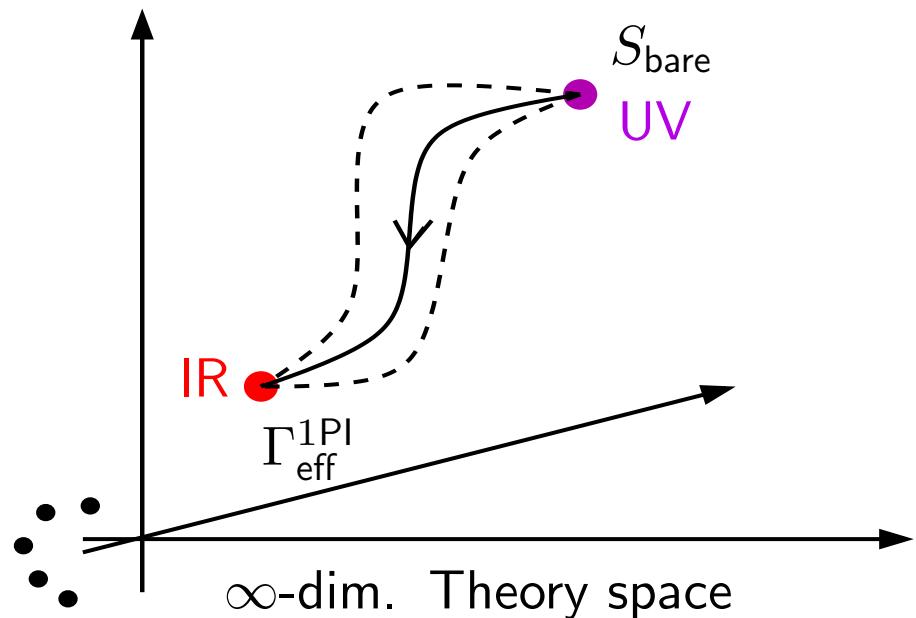


(WETTERICH'93)

▷ cutoff function  $R_k$



▷ RG trajectory:  $\Gamma_{k=\Lambda} = S_{\text{bare}} \rightarrow \Gamma_{k=0} = \Gamma_{\text{eff}}^{1\text{PI}}$



# An Example

- ▷ Nambu-Jona-Lasinio / Gross-Neveu in 3 dimensions,  $[\bar{\lambda}] = -1$ :

$$\Gamma_k = \int Z_\psi \bar{\psi} i\partial^\mu \psi + \frac{1}{2} \bar{\lambda} (\bar{\psi} \psi)^2 + \dots$$

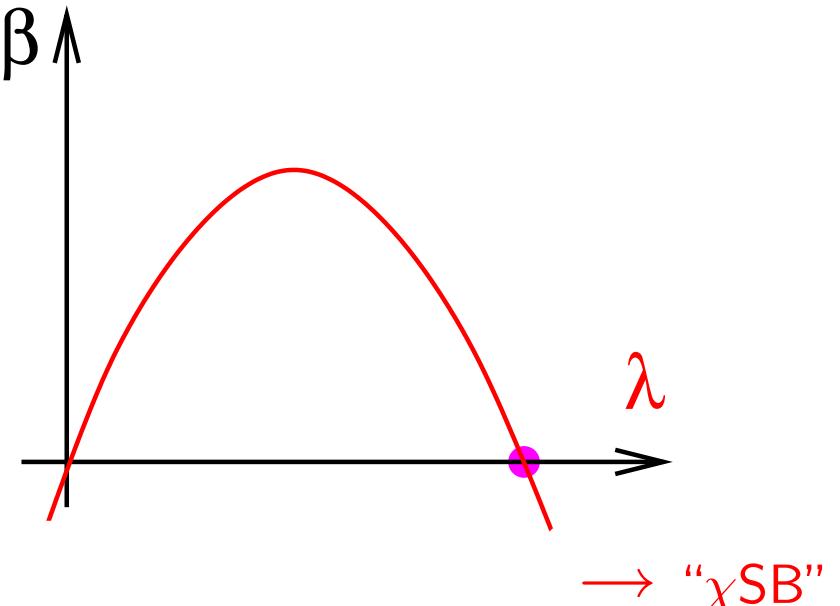
- ▷ flow of the dim'less coupling  $\lambda = k \bar{\lambda}$

$$\partial_t \lambda = \lambda - c \lambda^2$$

- ▷ UV fixed point  $\lambda_* = 1/c$

- ▷ critical exponent  $\Theta = 1$

⇒ asymptotically safe



(GAWEDZKI, KUPIAINEN'85; ROSENSTEIN, WARR, PARK'89; DE CALAN ET AL.'91)

# Towards the standard model . . .

- ▷  $\text{U}(1) \times \text{SU}(N_c)$  gauge symmetry + chiral  $\text{SU}(N_f)_L \times \text{SU}(N_f)_R$  flavor symmetry

$$\begin{aligned}\Gamma_k = & \int \bar{\psi} (\text{i} Z_\psi \not{\partial} + Z_1 \bar{g} \not{A} + Z_1^B \bar{e} \not{B}) \psi + \frac{Z_F}{4} F_z^{\mu\nu} F_{\mu\nu}^z + \frac{Z_B}{4} B^{\mu\nu} B_{\mu\nu} \\ & + \frac{1}{2} \left[ \bar{\lambda}_- (\text{V-A}) + \bar{\lambda}_+ (\text{V+A}) + \bar{\lambda}_\sigma (\text{S-P}) + \bar{\lambda}_{\text{VA}} [2(\text{V-A})^{\text{adj}} + (1/N_c)(\text{V-A})] \right]\end{aligned}$$

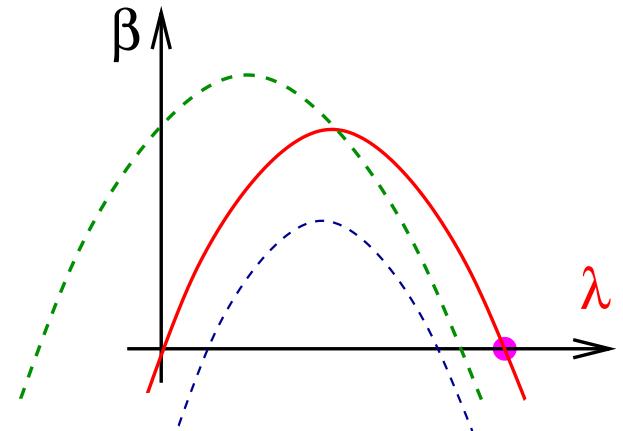
- ▷ four-fermion interactions

$$\begin{aligned}(\text{V-A}) &= (\bar{\psi} \gamma_\mu \psi)^2 + (\bar{\psi} \gamma_\mu \gamma_5 \psi)^2 \\ (\text{V+A}) &= (\bar{\psi} \gamma_\mu \psi)^2 - (\bar{\psi} \gamma_\mu \gamma_5 \psi)^2 \\ (\text{S-P}) &= (\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2 \equiv (\bar{\psi}_{\textcolor{blue}{i}}^a \psi_{\textcolor{blue}{i}}^b)^2 - (\bar{\psi}_{\textcolor{blue}{i}}^a \gamma_5 \psi_{\textcolor{blue}{i}}^b)^2 \\ (\text{V-A})^{\text{adj}} &= (\bar{\psi} \gamma_\mu \textcolor{blue}{T}^z \psi)^2 + (\bar{\psi} \gamma_\mu \gamma_5 \textcolor{blue}{T}^z \psi)^2\end{aligned}$$

- ▷ Standard Model:  $\mathcal{O}(10 \dots 100 \dots) \psi^4$  interactions

## $\lambda$ flow

▷ for instance  $\lambda_+$ :

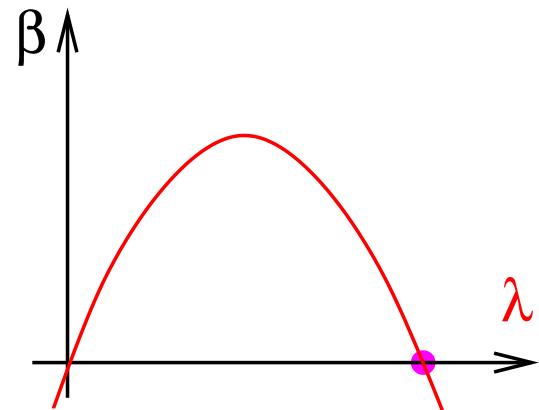


$$\begin{aligned}
 \partial_t \lambda_+ = & 2\lambda_+ - 4v_4 l_{1,1}^{(\text{FB}),4} \left[ \left( -\frac{3}{N_c} g^2 + 3e^2 \right) \lambda_+ \right] \\
 & - \frac{1}{8} v_4 l_{1,2}^{(\text{FB}),4} \left[ -\frac{12 + 3N_c^2}{N_c^2} g^4 - 48e^4 + \frac{48}{N_c} e^2 g^2 \right] \\
 & - 8v_4 l_1^{(\text{F}),4} \left\{ -3\lambda_+^2 - 2N_c N_f \lambda_- \lambda_+ - 2\lambda_+ (\lambda_- + (N_c + N_f) \lambda_{\text{VA}}) \right. \\
 & \quad \left. + N_f \lambda_- \lambda_\sigma + \lambda_{\text{VA}} \lambda_\sigma + \frac{1}{4} \lambda_\sigma^2 \right\}
 \end{aligned}$$

## Fermionic sector

- ▷ general structure for  $e^2, g^2 \rightarrow 0$

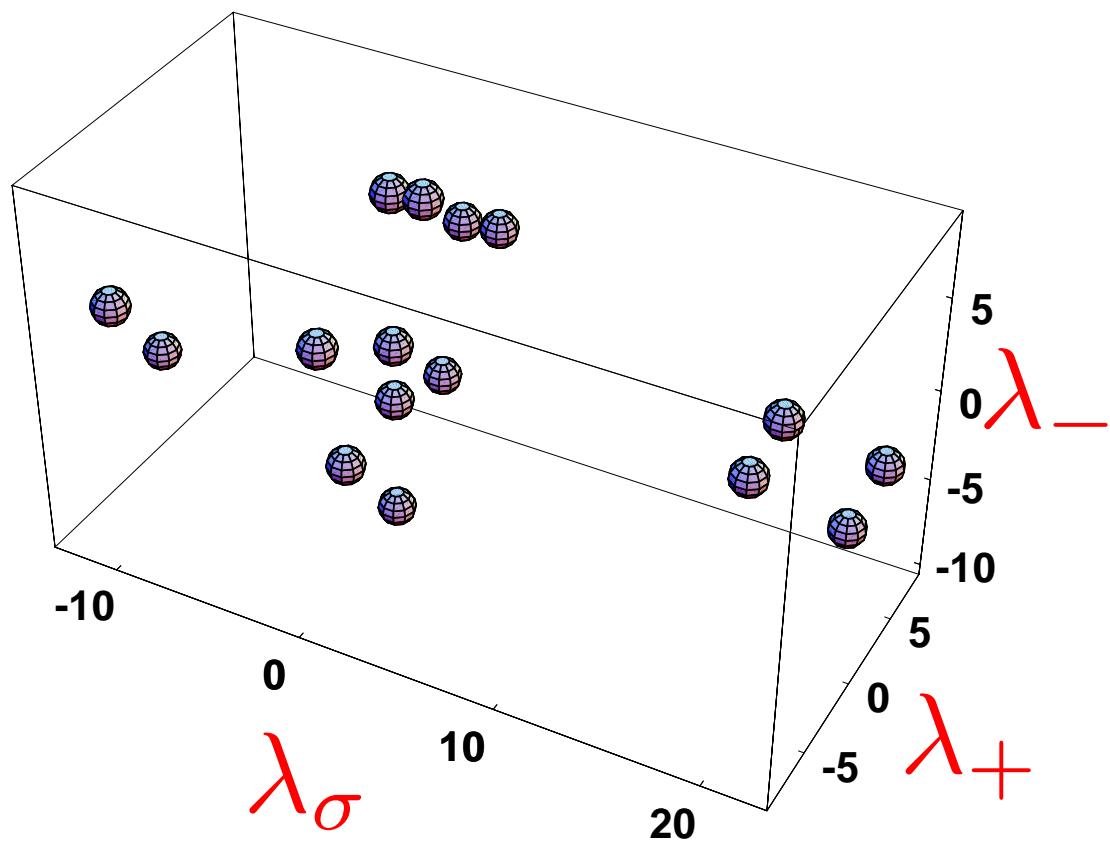
$$\partial_t \lambda_i = (d - 2)\lambda_i + \lambda_k A_i^{kl} \lambda_l$$



- ▷ 2 fixed points per  $\lambda$

$$\implies 2^4 = 16 \text{ fixed points}$$

- ▷ in general:  $2^n$  FP's  
for  $n = \#$  of  $\lambda$ 's



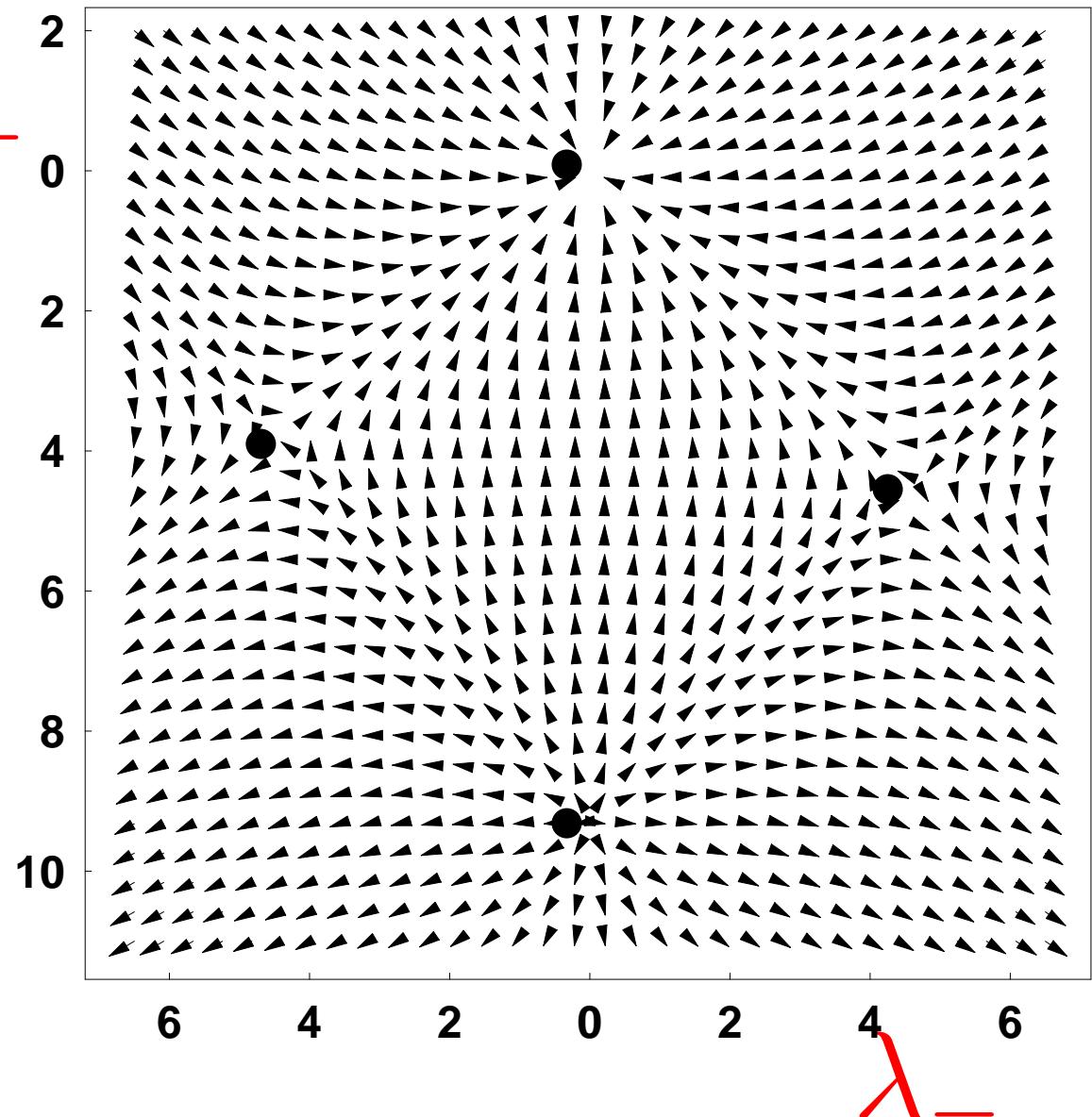
# How many physical parameters ?

- ▷ # of FP's with  
 $j$  relevant directions:

$$= \binom{4}{j}$$

- ➡ 4 FP's with  
only 1 parameter

$\lambda_+$



# Hierarchy problem ?

Yes!

- ▷ fixed point vector  $\lambda_{*i}$  is an eigenvector  $V_i$  of the **stability matrix**:

$$B_i^j \lambda_{*j} = -2 \lambda_{*i}, \quad \Theta_{\lambda_*} = 2 \leq \Theta_{\max}$$

⇒ Hierarchy problem is at least as bad

as in the Standard Model

Summary so far: no Higgs triviality, still U(1) triviality, (worse) finetuning

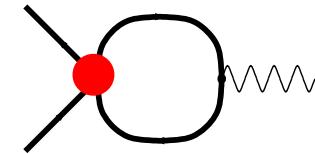
# Gauge interactions

Can we get rid of  $U(1)$  triviality with a  $\lambda$ -induced  $e_* > 0$  ?

Can the hierarchy problem be solved by a  $e_* > 0$  ?

- ▷  $\beta_{e^2}$  function from a modified Ward-Takahashi identity ( $c_i = \text{const.}$ )

$$\partial_t e^2 \equiv \beta_{e^2} = \eta_F e^2 + 2e^2 \frac{\sum_i c_i \partial_t \lambda_i}{1 + \sum_i c_i \lambda_i},$$



where, perturbatively,

$$\eta_F = \frac{1}{6\pi^2} N_f N_c$$

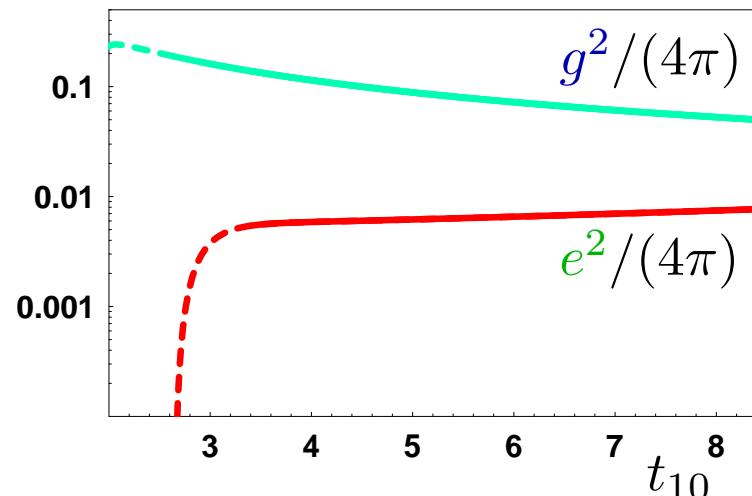
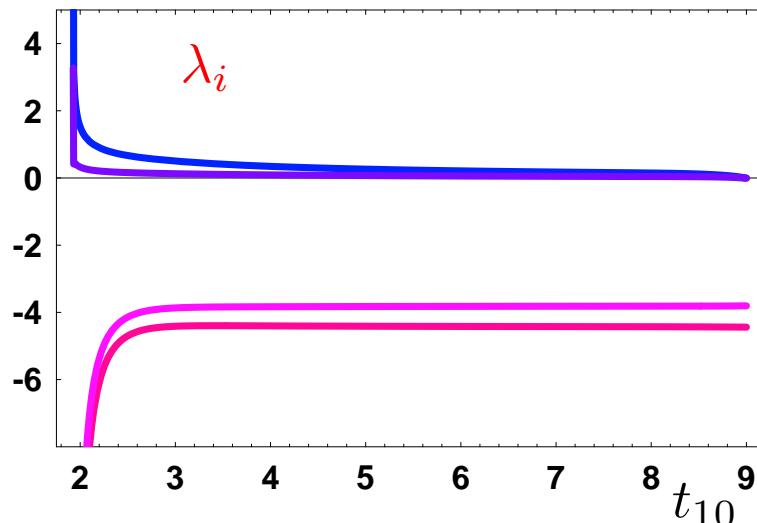
- ▷ at a global non-Gaussian fixed point:  $\partial_t \lambda_i \rightarrow 0$   
⇒ no non-Gaussian fixed point  $e_*$  from  $\lambda$  !!
- ▷ argument holds also for: “ $\psi^8$ ”, “ $\bar{\psi} A^n \psi$ ”, etc.

# Spontaneous symmetry breaking

▷ fermionic description:

$$\lambda \sim \frac{h^2}{m_\phi^2}$$

⇒ Symmetry breaking signaled by  $\lambda \rightarrow \infty$



▷ Symmetry-broken regime ⇒ partially bosonized description

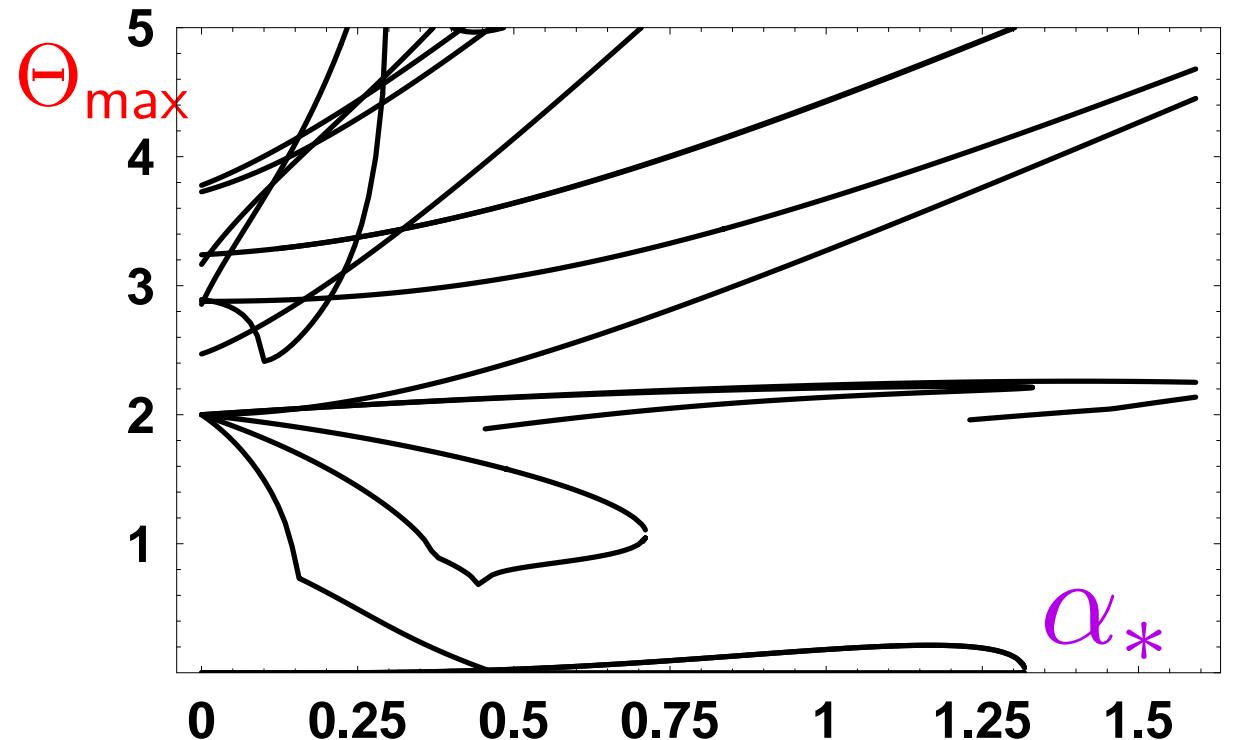
# Non-Gaußian gauge system

- ▷ Assumption: strong U(1) gauge dynamics induces a non-Gaußian fixed point

$$\eta_F(e_*) = 0, \quad \text{for } e_* > 0$$

- ▷ Consequences: critical exponents

⇒ an asymptotically safe U(1) sector potentially solves the hierarchy problem



# Conclusions

- (Toy-)Standard Model without fundamental Higgs scalar  
but with fermionic self-interactions can be asymptotically safe
- Plethora of fixed points with SM symmetries (  $\Rightarrow$  universality classes)
- # of physical parameters can be less than in the SM
- U(1) triviality and hierarchy problems remain

# Outlook

- +  $SU(2)_L$
- strong U(1) ?
- Fermionic theories with large anomalous dimensions ?