3.3 QCD effective 1-loop action for covariant-constant fields

In analogy to the Heisenberg-Euler action, let us try to compute the above-given determinants for covariant constant fields, satisfying

$$D_\mu F^{\mu\nu} = 0 \qquad (3.43)$$

Though it is not necessary, it makes the calculation transparent if we choose $F^a_{\mu\nu}$ to describe a pseudo-abelian magnetic field:

$$F^a_{\mu\nu} = n^a F_{\mu\nu} \quad , \quad n^a n^a = 1 \quad \text{(unit vector in adjoint color space)}$$

$$F_{12} = B = -F_{21} = \text{const}. \qquad (3.44)$$

Furthermore, it is possible to show that $\Gamma^{1\text{-loop}}$ is independent of the gauge parameter α for fields satisfying (3.43). This allows us to choose Feynman gauge $\alpha = 1$, which simplifies the gluon determinant to

$$\Gamma^{1\text{-loop}}_{YM}[A] = \frac{1}{2} \ln\det\left(-D^2 \delta_{\mu\nu} + 2g f^{abc} F^c_{\mu\nu}\right) - \ln\det\left(-D^2\right) \qquad (3.45)$$

Color space diagonalization can formally simply be done by noting that the only color structure

$$-i f^{abc} n^c \equiv (T^c)^{ab} n^c \tag{3.46}$$

is hermitean, yielding real eigenvalues

$$\{\text{spectrum of } (T^c n^c)\} =: \nu_\ell \quad , \ell = 1, \dots, N_c^2 - 1 \tag{3.47}$$

Furthermore, we need the spectrum of the covariant Laplacian (which is known from QM):

$$\{\text{spectr. } (-D^2)\} = p_0^2 + p_z^2 + g|\nu_\ell B|(2n+1), \quad n = 0, 1, \dots \tag{3.48}$$

and the spectrum of the gluon-spin-field coupling

$$2ig F_{\mu\nu}(T^c n^c) \sim 2ig\, \nu_\ell \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & -B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \{\text{spect. } 2igF_{\mu\nu}(T^c n^c)\} = \begin{cases} \left.\begin{matrix} +2g|\nu_\ell B| \\ -2g|\nu_\ell B| \end{matrix}\right\} \text{transversal to } B \text{ field} \\ 0 \quad \text{multiplicity } 2 \end{cases} \tag{3.49}$$

Hence, we find

$$\Gamma^{1\text{-loop}}_{YM}[A] = \sum_{\ell=1}^{N_c^2-1} \Big\{ \frac{1}{2}\mathrm{Tr}\ln\left(-D^2 + 2g|\nu_\ell B|\right) + \frac{1}{2}\mathrm{Tr}\ln\left(-D^2 - 2g|\nu_\ell B|\right) + \cancel{2\cdot\frac{1}{2}\mathrm{Tr}\ln(-D^2)} - \cancel{\mathrm{Tr}\ln(-D^2)} \Big\} \tag{3.50}$$

We observe that the Faddeev-Popov determinant exactly cancels the "unphysical" longitudinal and timelike gluon contribution.

We are left with the fluctuation spectra of the transversal gluons:

$$\{\text{spect.}\ -D^2 + 2g|\nu_e B|\} = p_0^2 + p_z^2 + g|\nu_e B|(2n+3)$$
$$n = 0, 1, \ldots,$$
$$\{\text{spect.}\ -D^2 - 2g|\nu_e B|\} = p_0^2 + p_z^2 + g|\nu_e B|(2n-1) \qquad (3.51)$$

For the second line at $n=0$ we observe a region $(p_0^2 + p_z^2) < g|\nu_e B|$, where the eigenvalues are negative. This part is called the Nielsen-Olesen unstable mode. It is also sometimes called a "tachyonic" mode as the dispersion looks like that of a negative mass^2 state.

The observation of this unstable mode has led to a long discussion in the literature. Its existence shows that their are long wavelength modes with $p_0^2 + p_z^2$ small that do not "cost" any action (for suitable gluon-spin-field orientation. Instead, their is a "downhill" direction which naively seems unbounded.

This can be interpreted as follows: the covariant constant magnetic field is not a minimum of the action but a saddle point. Thus the cov. const. B field itself is unstable.

The true vacuum state must entail a stabilizing mechanism. One possibility is given by letting the B-field regions to be of finite extent only. For instance a finite extent L_z in z-direction leads to an IR cutoff for $p_z \sim \frac{\pi}{L_z}$. This might be similar to domain-like structures in ferromagnets.

This picture together with the necessity of magnetic flux conservation lead to the picture of a "spaghetti vacuum" (also called Copenhagen vacuum).

However, this vacuum is technically difficult to deal with (moreover, the relevance of perturbation theory for arriving at this conclusion is unclear.).

Hence, we will still deal with $F =$ cov. const. in the following, paying particular attention to the unstable mode.

Let us technically still proceed naively, performing the log-det calculation in the propertime formalism:

$$\Gamma_{YM}^{1\text{-loop}} = -\frac{1}{2}\sum_{\ell=1}^{N_c^2-1}\int_{1/\Lambda^2}^{\infty}\frac{dT}{T}\,\mathrm{Tr}\left(e^{-(-\partial^2+2g|\nu_\ell B|)T} + e^{-(-\partial^2-2g|\nu_\ell B|)T}\right) + \text{const}$$

$$= -\frac{1}{2}\sum_{\ell}\int_{1/\Lambda^2}^{\infty}\frac{dT}{T}\int\frac{dp_0}{2\pi/L}\int\frac{dp_z}{2\pi/L}\sum_{n=0}^{\infty}\frac{(g|\nu_\ell B|)}{2\pi/L^2}\,e^{-(p_0^2+p_z^2)T}\left[e^{-g|\nu_\ell B|T(2n+3)} + e^{-g|\nu_\ell B|T(2n-1)}\right]$$

$$+\text{const}$$

$$= -\frac{\Omega}{2}\sum_{\ell}\frac{g|\nu_\ell B|}{2\pi}\int_{1/\Lambda^2}^{\infty}\frac{dT}{T}\frac{1}{(4\pi T)}\left[e^{-3g|\nu_\ell B|T}\sum_{n=0}^{\infty}e^{-(2g|\nu_\ell B|T)n}\right.$$

$$\left. + e^{g|\nu_\ell B|T}\underbrace{\sum_{n=0}e^{-(2g|\nu_\ell B|T)n}}_{=\frac{e^{g|\nu_\ell B|T}}{2\sinh g|\nu_\ell B|T}}\right] + \text{const}$$

$$= -\frac{\Omega}{16\pi^2}\sum_{\ell} g|\nu_\ell B|\int\frac{dT}{T^2}\;\frac{\cosh 2g|\nu_\ell B|T}{\sinh g|\nu_\ell B|T} \qquad + \text{const}$$

$$\overset{\cosh 2x = 2\sinh^2 x + 1}{=} -\frac{\Omega}{16\pi^2}\sum_{\ell=1}^{N_c^2-1}\int_{1/\Lambda^2}^{\infty}\frac{dT}{T^3}\left\{\frac{g|\nu_\ell B|T}{\sinh g|\nu_\ell B|T} + 2g|\nu_\ell B|T\sinh g|\nu_\ell B|T\right. \\ \left. + \text{const}\right\} \qquad (3.52)$$

While the first term is identical to a bosonic fluctuation contribution (cf. scalar QED), the second term contains the unstable mode's contribution. In any case, Eq. (3.52) represents the unrenormalized

Yang - Mills effective 1-loop action.

This expression carries several divergencies: of course, we find the expected UV divergencies for $T \to \frac{1}{\Lambda^2} \to 0$, giving rise to charge and field strength renormalization. The corresponding β function coefficient can be read off from the term $\sim B^2$ of the integrand

$$\frac{1}{T^3}\left\{\frac{g|V_cB|T}{\sinh g|V_cB|T} + 2g|V_cB|T \sinh g|V_cB|T\right\}$$

$$= \underset{\substack{\uparrow\\ \text{zero point}\\ \text{energy. Subtracted}\\ \text{by defining } \Gamma[B=0]=0}}{\frac{1}{T^3}} + \Big(\underset{\substack{\nearrow\\ \text{paramagnetic}\\ \text{spin-field coupling, also containing the unstable mode}}}{2} - \underset{\substack{\uparrow\\ \text{diamagnetic}\\ \text{KG coupling}}}{\frac{1}{6}}\Big)(g|V_cB|)^2 \underset{\substack{\nwarrow\\ \text{log-divergence}}}{\frac{1}{T}} + \mathcal{O}(T)$$

$$= \frac{1}{T^3} + \frac{11}{6}(g|V_cB|)^2\frac{1}{T} + \mathcal{O}(T) \tag{3.53}$$

As we will see below, the prefactor $\sim \frac{11}{6}$ corresponds to the famous Yang-Mills β-function coefficient giving rise to the celebrated property of asymptotic freedom in QCD. Both, the sign and the absolute value depend crucially on the presence of the paramagnetic spin-field coupling, which also gives rise to the unstable mode. Hence, simply removing the unstable mode from the theory is not a legitimate option.

This unstable mode gives rise to another divergence occurring at the upper bound of the propertime integrand, $T \to \infty$, corresponding to an IR divergence in accordance with our expectations.

In the literature, several suggestions have been put forward to deal with this problem. For instance, in a Minkowskian formulation, the natural contour integration would have been along the imaginary T axis:

$$\int^{\infty} dT \to \int^{\pm i\infty} dT$$

Whereas the diamagnetic term would still have yielded a real contribution to the integral, the paramagnetic would have also contributed an imaginary part $\mathrm{Im}\,\Gamma[B] = \mp i \frac{N_c}{16\pi} (gB)^2$.

This imaginary part can be interpreted as a decay rate of the covariant constant B field.

Here, we proceed in a different manner. In order to disentangle the UV from the IR divergences (which have a very different meaning), we introduce an IR regulator in the form of an artificial gluon mass term. However, this "gluon mass" μ does not break gauge invariance as it is introduced in the propertime framework:

$$\Gamma = \int d^4x\, \mathcal{L} = \Omega \mathcal{L}$$

$$\Rightarrow \mathcal{L} = \underbrace{\mathcal{L}^0}_{=\frac{1}{2}B^2} + \mathcal{L}^1 \Rightarrow \frac{1}{2}B^2$$

$$- \frac{1}{16\pi^2} \sum_{\ell=1}^{N_c^2-1} \int_{1/\Lambda^2}^{\infty} \frac{dT}{T^3} e^{-\mu^2 T} \left\{ \frac{g|\nu_\ell B|T}{\sinh g|\nu_\ell B|T} + 2g|\nu_\ell B|T \sinh g|\nu_\ell B|T - 1 - \frac{11}{6} g|\nu_\ell B|T^2 \right\}$$

$$- \frac{11}{3\cdot 32\pi^2} \sum_{\ell=1}^{N_c^2-1} (g|\nu_\ell B|)^2 \underbrace{\int_{1/\Lambda^2}^{\infty} \frac{dT}{T}\, e^{-\mu^2 T}}_{= \ln \frac{\Lambda^2}{\mu^2} + \text{const} + \mathcal{O}(\mu^2/\Lambda^2)}$$

$$\sum_\ell |\nu_\ell|^2 = n^a n^b t_r T^a T^b = N_c$$

$$= \frac{1}{2}\left(1 - \frac{11 N_c}{3\cdot 16\pi^2} g^2 \ln \frac{\Lambda^2}{\mu^2}\right) B^2$$

$$- \frac{1}{16\pi^2} \sum_{\ell=1}^{N_c^2-1} \int_{1/\Lambda^2 \to 0}^{\infty} \frac{dT}{T^3} \left\{ \frac{g|\nu_\ell B|T}{\sinh g|\nu_\ell B|T} + 2 g|\nu_\ell B|T \sinh g|\nu_\ell B|T - 1 - \frac{11}{6} g|\nu_\ell B|T^2 \right\} e^{-\mu^2 T}$$

$$=: \frac{1}{2} B_R^2 + \mathcal{L}_R^1 (g_R B_R, \mu) \qquad (3.54)$$

This defines the renormalized field strength and coupling

$$B_R^2 = B^2 Z_F^{-1}, \quad g_R^2 = g^2 Z_F$$

$$Z_F^{-1} = 1 - \frac{11 N_c g^2}{3\cdot 16\pi^2} \ln \frac{\Lambda^2}{\mu^2} + \text{const} \qquad (3.55)$$

with μ playing the role of the renormalization scale. The corresponding β function is

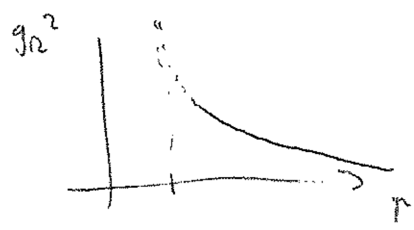
$$\beta_{g^2} = \mu \frac{\partial}{\partial\mu} g_R^2(\mu) = -\frac{11\,N_c}{3} \frac{g_R^4}{8\pi^2} + \mathcal{O}(g_R^6), \tag{3.56}$$

predicting an asymptotically free running coupling in the UV ($\mu \to \infty$)

$$g_R^2(\mu^2) = \frac{g_R^2(\mu_0)}{1 + \frac{11 N_c}{3} \frac{g_R^2(\mu_0)}{16\pi^2} \ln \frac{\mu^2}{\mu_0^2}} \tag{3.57}$$

Since $g_R^2 \to 0$ at high scales, any artificially introduces UV cutoff Λ can safely be removed. Hence $SU(N_c)$ gauge theories can be valid fundamental QFTs on all scales.

The running of the coupling exhibits a Landau pole singularity in the IR, which is considered to be an artefact of perturbation theory.



Phenomenologically, the standard renormalization point of QCD is chosen to be the Z mass pole,

$$\frac{g_R^2(\mu = M_Z^2)}{4\pi} \simeq 0.117 \tag{3.58}$$

Let us return to

$$\mathcal{L}_R^1(g_R B_R, \mu) = -\frac{1}{16\pi^2}\sum_{l=1}^{\frac{N_c^2-1}{2}}\int_{1/\Lambda^2\to 0}^{\infty}\frac{dT}{T^3}\, e^{-\mu^2 T}\left\{\frac{g_R|\nu_l B_R|T}{\sinh g_R|\nu_l B_R|T} + 2 g_R|\nu_l B_R|T \sinh g_R|\nu_l B_R|T - 1 - \frac{11}{6}(g_R|\nu_l B_R|T)^2\right\} \tag{3.59}$$

Now we wish to investigate the vacuum requiring us to get rid of the IR scale μ (without getting back the problem of the unstable mode). In fact, the integral (3.59) can be evaluated in terms of special functions (derivative of Hurwitz ζ functions). This result can analytically be continued to small values of μ (giving us back an imaginary part for $\mu^2 < g_R|\nu_l|B_R$). However, the real part acquires a simple form in this limit ($g_R B_R \to gB$)

$$\mathcal{L}_R^1(gB) = \frac{1}{4} b_0 (gB)^2 \ln\frac{gB}{\mu^2} + \mathcal{O}((gB)^2) + \operatorname{Im}\mathcal{L}_R^1 \tag{3.60}$$

with $b_0 = \frac{11}{3}\frac{N_c}{8\pi^2}$.

Hence, the total effective Lagrangian reads

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}B^2 + \frac{1}{4} b_0 (gB)^2 \ln\frac{gB}{\mu^2} + \mathcal{O}((gB)^2)\ldots\\ &=: \frac{1}{4} b_0 \left(g^2 \tfrac{1}{2}B^2\right)\ln\frac{2g^2\frac{1}{2}B^2}{e\kappa^2} + \ldots\\ &= \frac{1}{4} b_0 (g^2\mathcal{F})\ln\frac{2g^2\mathcal{F}}{e\kappa^2} + \ldots\end{aligned} \tag{3.61}$$

where we have expressed the magnetic field in terms of the invariant $\mathcal{F} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$ and introduced

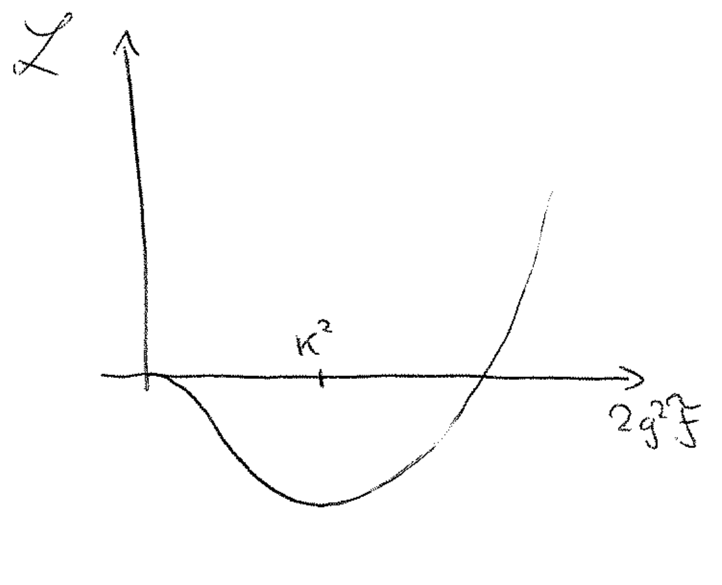
$$K^2 = \frac{\mu^4}{e} e^{-\frac{4}{b_0 g^2}} . \qquad (3.62)$$

Now, K is in fact RG invariant, since

$$\mu \frac{\partial}{\partial \mu} K = \mu \frac{\partial}{\partial \mu} \left(\frac{\mu^2}{\sqrt{e}} e^{-\frac{2}{b_0 g^2}} \right) = 2 \frac{\mu^2}{\sqrt{e}} e^{-\frac{2}{b_0 g^2}} + \frac{\mu^2}{\sqrt{e}} e^{-\frac{2}{b_0 g^2}} \cdot \left(-\frac{2}{b_0}\right) \underbrace{\mu \frac{\partial}{\partial \mu} g^{-2}}_{-\frac{1}{g^4}\beta_{g^2}}$$

$$= 2 \frac{\mu^2}{\sqrt{e}} e^{-\frac{2}{b_0 g^2}} \left(1 - \frac{1}{b_0} \left(-\frac{1}{g^4} \right) (-b_0 g^4) \right) = 0 \qquad (3.63)$$

Hence the Lagrangian (3.61) is RG invariant $\mu \frac{\partial}{\partial \mu} \mathcal{L} = 0$.
We conclude the Yang-Mills theory dynamically generates a scale K (often called $\simeq \Lambda^2_{QCD}$) which has to be determined from experiment. Plotting $\mathcal{L}$ as a function of $2g^2\mathcal{F}$, we get



We observe that $\mathcal{L}$ acquires a non-trivial minimum at $2g^2\mathfrak{F}_{min} = (gB_{min})^2 = \kappa^2$ (3.64)

From this, we are tempted to conclude that the Yang-Mills (as well as the QCD) vacuum prefers a non-vanishing gluon field strength, a so-called "gluon <u>condensate</u>".

In fact, such a concept is also successfully used in other approaches to low-energy QCD such as the operator product expansion.

However, this conclusion has to be examined critically:

- criticism 1: the problem of the unstable mode has not been solved but rather circumvented by analytic continuation.
- criticism 2: a covariant-constant field singles out a preferred direction in spacetime (magnetic field direction). This vacuum breaks Lorentz invariance, but no such breaking is observed in experiments

These two points might be resolved by a more adequate background field (à la spaghetti vacuum ...)

However, there is a more severe point of criticism:

- criticism 3: the 1-loop calculation can only be justified at small coupling, say $\frac{g^2}{4\pi} \ll 1$.

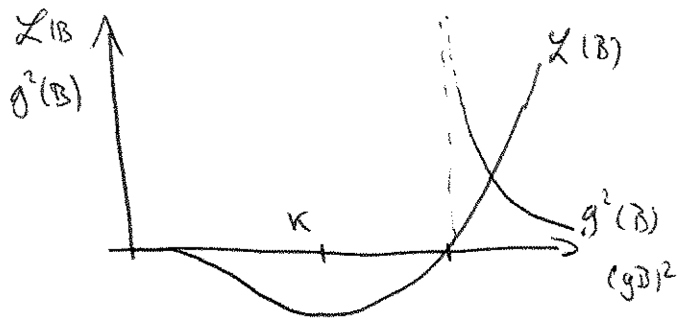
From the β-function and the running coupling (3.56 & 3.57), we know that the coupling grows strong at low-energy scales. Now, the gluon condensate itself provides a scale, such that we can try to formulate the response of the coupling strength to a given field strength in terms of

$$g^2(B) = \frac{g_0^2}{1+\frac{1}{2}b_0 g_0^2 \ln \frac{gB}{\mu_0^2}} \quad , \quad g_0^2 = g^2(\mu_0) \quad . \tag{3.65}$$

Using the invariant scale $\kappa = \mu_0^2 \, e^{-\frac{2}{b_0 g_0^2} - \frac{1}{2}}$, we can rewrite (3.65) as

$$g^2(B) = \frac{g_0^2}{\frac{1}{2} b_0 g_0^2 \left(\ln \frac{gB}{\kappa} - \frac{1}{2}\right)} \quad . \tag{3.66}$$

Plotting both $\mathcal{L}(B)$ and $g^2(B)$,



we observe that the coupling runs into the IR Landau-pole singularity. In other words, perturbation theory breaks down before the minimum of the action is reached. Hence, the one-loop approximation breaks down in the physically interesting domain.

There are, of course, further points of criticism which a complete study of the QCD vacuum has to address:

- criticism 3: $\mathcal{L}(\mathcal{F})$ in our calculation just includes the dependence on one invariant $\mathcal{F}$. However, as discussed in the QED chapter, there are much more invariants in YM theory even for covariantly constant fields.

- criticism 4: even if we solved the unstable mode problem, there is a priori no ordering criterion that would justify an expansion in terms of slowly-varying fields. For instance even for the lowest lying states states above the vacuum – the glue balls – we expect essential contributions from higher momentum components.

So far, we have performed explicit computations only for Yang-Mills theory. If we include the quark degrees of freedom, we obtain further contributions of Heisenberg-Euler type $\sim N_f \cdot \ln \det(i\not{D} + m)$.

Neglecting the quark masses for light quarks, the only modification to the above discussion is a contribution to the running of the coupling in terms of a modified β function:

$$b_0 \to \frac{1}{8\pi^2}\left(\frac{11 N_c}{3} - \frac{2}{3} N_f\right) \qquad (3.67)$$

We observe that, e.g., $SU(N_c = 3)$ QCD remains asymptotically free for $N_f < \frac{33}{2} = 16.5$.

3.4 The Leading-Log model

(Pagels & Tomboulis '78, Adler '82, Adler & Piran '84, Lehmann, Wu '84, ...)

We conclude this chapter with one of the simplest models of confinement. This model is based on the completely unjustified assumption that a full non-perturbative calculation would clarify all points of criticism mentioned above, but finally still lead to a full quantum action of the type

$$\mathcal{L} = -\frac{1}{4} b_0 \mathcal{F} \ln \frac{2|\mathcal{F}|}{e\kappa^2} - A_\mu^a J_\mu^a \qquad (3.68)$$

(here, we rotated back to Minkowski space, $\mathcal{F} = \frac{1}{2}(B^2 - E^2)$, $\mathcal{L}_E \Rightarrow -\mathcal{L}_M$, and rescaled the field $g^2\mathcal{F} \Rightarrow \mathcal{F}$.)

This defines a non-linear model which we apply to the case of a meson-type source (with static quarks)

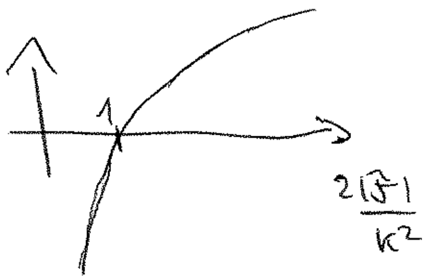
$$J^{a0} = Q\,\hat{m}^a \left(\delta^{(3)}(\vec{x}-\vec{x}_1) - \delta^{(3)}(\vec{x}-\vec{x}_2)\right) \text{ with} \qquad (3.69)$$

$R = |\vec{x}_1 - \vec{x}_2|$. The Euler-Lagrange equation can be written in the form

$$J^{a\nu} = \partial_\mu \left[\left(\frac{\partial \mathcal{L}}{\partial \mathcal{F}} \right) F^{a\mu\nu} \right] =: \partial_\mu \left(\varepsilon(\mathcal{F}) F^{a\mu\nu} \right) \qquad (3.70)$$

with the dielectricity "constant"

$$\varepsilon(\mathcal{F}) = \frac{1}{4} b_0 \ln \frac{2|\mathcal{F}|}{\kappa^2}$$



As the source is pseudo-abelian source, we expect to find pseudo-abelian solutions, $F^a_{\mu\nu} = \hat{n}^a F_{\mu\nu}$.

Writting (3.70) in non-covariant form

$$\vec{\nabla} \cdot \vec{D} = J^0 \quad , \quad \vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \times \vec{H} = 0 \quad , \quad \underbrace{\vec{\nabla} \cdot \vec{B} = 0}_{\partial_\mu \tilde{F}^{\mu\nu} = 0} \qquad (3.71)$$

The vacuum of this model is characterized by the non-linear constituent equations

$$\vec{D} = \varepsilon \vec{E} \quad , \quad \vec{H} = \varepsilon \vec{B} \qquad (3.72)$$

Hence the QCD-Vacuum is fully mapped onto a problem of nonlinear electrodynamics.

Since $\vec{\nabla} \times \vec{H} = 0$, we can write

$$0 = \int d^3x\ \vec{A} \cdot (\vec{\nabla} \times (\varepsilon \vec{B})) = \int d^3x\ \varepsilon \vec{B}^2 - \oint d\vec{f} \cdot \varepsilon (\vec{A} \times \vec{B}) \qquad (3.73)$$

Assuming that the surface term at infinity vanishes, we conclude

$$\varepsilon \vec{B}^2 = 0 \quad . \qquad (3.74)$$

This can be satisfied in the following manner:

(I) $\vec{B} = 0$, $\vec{E}^2 > \kappa^2$ (as we expect for the field near the sources)

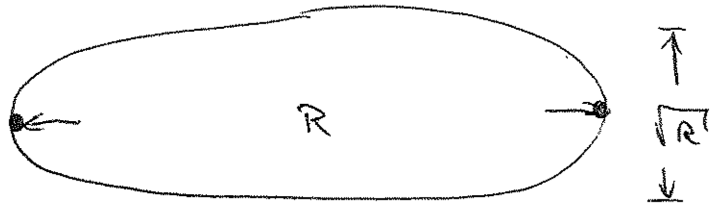
(II) $\varepsilon = 0$, $2\mathcal{F} = \kappa^2$ (associated with the vacuum solution) (3.75)

The detailed analysis of the partial differential equation (3.70) is rather extensive. It turns out to be a quasilinear PDE of second order which is elliptic in region (I) and parabolic in (II). Main results of this analysis show:

– region (I) and region (II) merge at a

finite distance from the $Q, \bar{Q}$ charges.
The transition is continuous and differentiable.

- the boundary is a free characteristic (PDE language) and scales for large R as

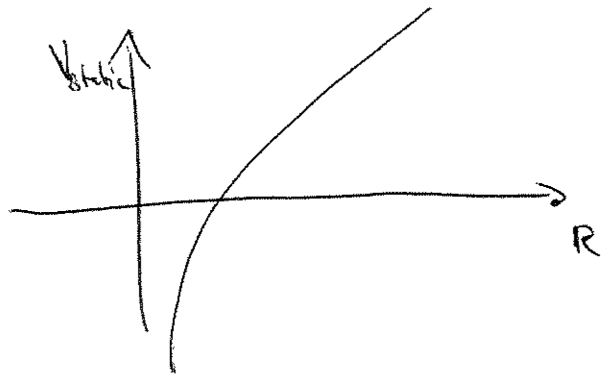


- inside and outside solutions are causally disconnected (charge distributions inside do not exert any influence on the fields outside.)

- the solution provides for statements on the static potential (integral of the energy density of the inside solution)

$$V_{static} = \begin{cases} K Q R + \frac{2}{3} Q^{3/2} \sqrt{\frac{2}{\pi b_0}} \sqrt{K} \ln(K^{1/2} R) + \mathcal{O}(1) & \text{for } \sqrt{K} R \gg 1 \\ -\frac{Q^2}{4\pi R} \frac{1}{b_0 \left(\ln \frac{1}{\sqrt{K} R} + \text{const.}\right)} + \mathcal{O}\left(\frac{\ln \ln}{\ln}, \frac{1}{\ln^3}\right) & \text{for } \sqrt{K} R \ll 1 \end{cases} \qquad (3.76)$$

Hence, the model predicts a log-modified Coulomb potential in agreement with perturbation theory in the limit of small separations.
And a linearly rising confining potential at large distances



We conclude that the model qualitatively agrees with experimental and lattice results. The quantitative analysis of Adler & Piran even indicated reasonable quantitative agreement.

This simple model has successively inspired more elaborate model that can also deal with baryonic states. This class of models is typically called

"models of dielectric confinement"