2.2 Phenomena of the modified QED vacuum

As the Heisenberg-Euler action in some sense defines (a better approximation to) the true theory of classical electrodynamics, we can use it to study generic phenomena the go beyond standard classical phenomena.

2.2.1 Light propagation in a strong EM field

Starting from the weak-field Heisenberg-Euler action

$$\mathcal{L} = -\mathcal{F} + c_1 \mathcal{F}^2 + c_2 \mathcal{G}^2 \quad , \; c_1 = \frac{8}{45}\frac{\alpha^2}{m^4} \, , \; c_2 = \frac{14}{45}\frac{\alpha^2}{m^4} \, , \tag{2.42}$$

we can derive the corresponding equation of motion,

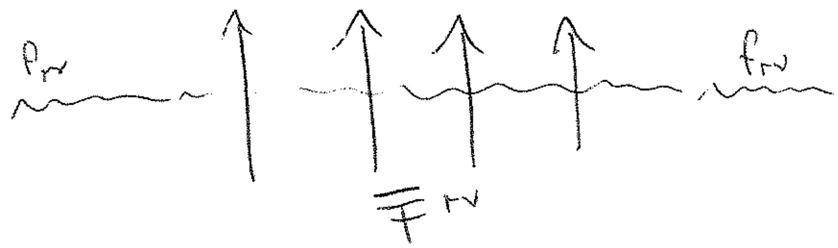
$$\underline{\underline{0 = -2\,\partial_\mu \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}}}} \quad , \quad \frac{\partial \mathcal{F}}{\partial F_{\alpha\beta}} = \frac{1}{2} F^{\alpha\beta} \, , \; \frac{\partial \mathcal{G}}{\partial F_{\alpha\beta}} = \frac{1}{2} \widetilde{F}^{\alpha\beta}$$

$$= \underline{\underline{\partial_\mu \left(F^{\mu\nu} - 2c_1 \mathcal{F} F^{\mu\nu} - 2c_2 \mathcal{G} \widetilde{F}^{\mu\nu} \right)}} \tag{2.43}$$

Let us consider a set-up of "pump-probe" type with a strong background field modifying the vacuum (pump) and a weak propagating field measuring resulting modifications (probe). This suggests to decompose

$$F^{\mu\nu} = \underset{\substack{\uparrow \\ \text{pump}}}{\bar{\bar{F}}^{\mu\nu}} + \underset{\substack{\uparrow \\ \text{probe}}}{f^{\mu\nu}} \tag{2.44}$$

with $f_{\mu\nu} \ll \bar{\bar{F}}_{\mu\nu}$ and $\bar{\bar{F}}_{\mu\nu} \simeq \text{const.}$



Linearizing the field equation with respect to $f^{\mu\nu}$ yields

$$0 = \partial_\mu f^{\mu\nu} - c_1 \bar{\bar{F}}_{\alpha\beta} \bar{\bar{F}}^{\mu\nu} \partial_\mu f^{\alpha\beta} - c_2 \tilde{\bar{\bar{F}}}_{\alpha\beta} \tilde{\bar{\bar{F}}}^{\mu\nu} \partial_\mu f^{\alpha\beta} \tag{2.45}$$

Let us drop the bar from now on : $\bar{\bar{F}} \to F$.

Next we assume $f^{\mu\nu}$ to represent a plane wave field which in Fourier space reads

$$f^{\mu\nu} \sim k^\mu a^\nu - k^\nu a^\mu \tag{2.46}$$

In Lorentz gauge, we also have $k_\mu a^\mu = 0$, such that (2.45) reads

$$0 = k^2 a^\nu - 2c_1 F_{\alpha\beta} F^{\mu\nu} k_\mu k^\alpha a^\beta - 2c_2 \tilde{F}_{\alpha\beta} \tilde{F}^{\mu\nu} k_\mu k^\alpha a^\beta . \tag{2.47}$$

In the classical limit where $c_{1,2} \to 0$, we rediscover

$k^2 a^\nu = 0$, implying that light propagates

"on the light cone", $k^2 = \vec{k}^2 - \omega^2 = 0$. (2.48)

Its (phase) velocity is then given by

$$v = \frac{\omega}{|\vec{k}|} = c = 1 \qquad (2.49)$$

The full equation (2.47) can be viewed as a matrix equation $0 = M^\nu{}_\beta a^\beta$ (Fresnel equation), and we are interested in non-trivial solutions.

Using as an ansatz:

$$a_\parallel^\mu \sim \tilde{F}^{\mu\nu} k_\nu \equiv (\tilde{F}k)^\mu \,, \quad a_\perp^\mu = (Fk)^\mu \,, \qquad (2.50)$$

we find

$$a_\parallel^\mu : \; 0 = \left(k^2 - 2c_2 (Fk)^2 + 2c_2 \mathcal{F} k^2\right)(\tilde{F}k)^\nu - \left(2c_1 \mathcal{G} k^2\right)(Fk^\nu)$$

$$a_\perp^\mu : \; 0 = \left(k^2 - 2c_1 (Fk)^2\right)(Fk)^\nu - \left(2c_2 \mathcal{G} k^2\right)(\tilde{F}k)^\nu \,, \qquad (2.51)$$

where we took advantage of the identities (2.6).

We observe that

$$k^2 = 0 + \mathcal{O}(c_{1,2})$$

is a consistent solution (in agreement with the classical limit. But from (2.51), we can also work out the leading-order correction $\sim c_{1,2}$. To this order, (2.51) is solved by the

<u>dispersion relations</u> (light cone conditions)

$$a_{\parallel}^{\mu}: \quad k^2 = 2c_2 (Fk)^2 , \tag{2.52}$$

$$a_{\perp}^{\mu}: \quad k^2 = 2c_1 (Fk)^2 .$$

To understand the geometry, let us more explicitly consider a purely magnetic field; then

$$(Fk)^2 = (F^{\mu\nu} k_\nu)^2 \hat{=} |\vec{k}|^2 B^2 \sin^2 \underbrace{\theta_B}_{\sphericalangle(\vec{B},\vec{k})} \tag{2.53}$$

This implies that

$a_{\perp}^{\mu} \sim Fk^{\mu}$ is polarized $\perp$ WRT the $(\vec{B},\vec{k})$ plane

$a_{\parallel}^{\mu} \sim \tilde{F}k^{\mu}$ " " in " $(\vec{B},\vec{k})$ plane.

A more intuitive quantity might be given in terms of the phase velocity

$$v = \frac{\omega}{|\vec{k}|} \quad , \quad k^\mu = (\omega, \vec{k}) \quad , \qquad (2.54)$$

yielding for the two modes:

$$v_\parallel = 1 - c_2 \frac{(Fk)^2}{|\vec{k}|^2} = 1 - \frac{14}{45} \frac{\alpha^2}{m^4} B^2 \sin^2\theta_B$$

$$v_\perp = 1 - c_1 \frac{(\tilde{F}k)^2}{|\vec{k}|^2} = 1 - \frac{8}{45} \frac{\alpha^2}{m^4} B^2 \sin^2\theta_B \qquad (2.55)$$

where, again, we worked to order $c_{1/2}$. Incidentally, the phase velocities are equal to the group velocities $v_{gr} = \frac{d\omega}{d|\vec{k}|}$, as the light cone conditions do not involve any nontrivial dependence on the frequency or the wave number. This is a direct consequence of the "soft-photon" approximation underlying the Heisenberg-Euler calculation.

In general, we expect the phase velocities to depend on the frequency $v = v(\omega)$, with (2.55) denoting the low-frequency limit $v_{(2.55)} = v(\omega \to 0)$.

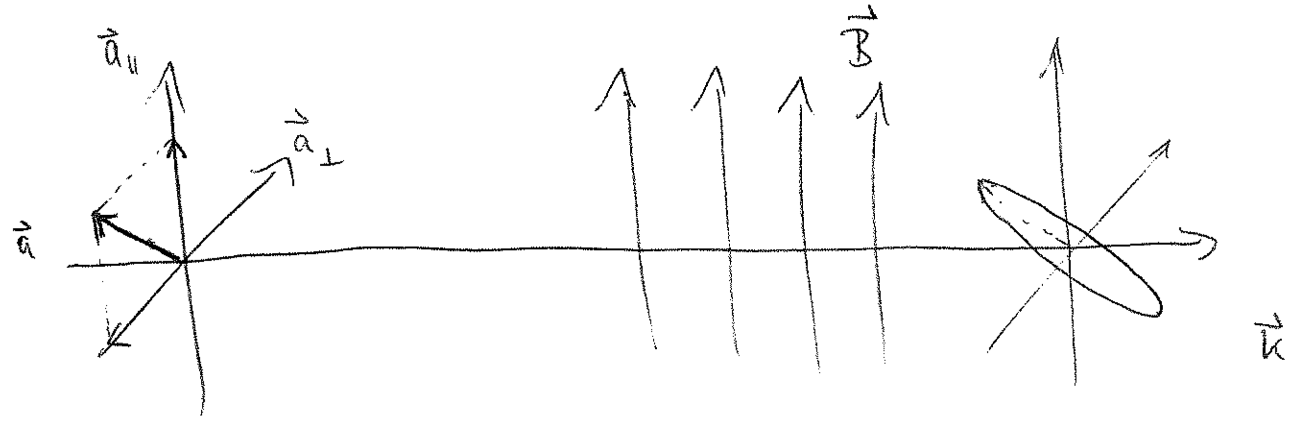
The most important phenomenological conclusion to be drawn from (2.55) is that the modified (magnetized) quantum vacuum is

<u>birefringent</u> : $$n_{\parallel,\perp} = \frac{1}{v_{\parallel,\perp}} \qquad (2.56)$$

The refractive indices depend on the polarization of the incoming photons WRT the magnetic field direction.

This "vacuum magnetic birefringence" has been discussed intensely in the literature, as it might give rise to the first (yet to be discovered) observable for quantum-induced nonlinear interactions of macroscopic electromagnetic fields.

A typical optical observable for birefringence is <u>ellipticity</u> which can be picked up by an originally linearly polarized beam after transversing a magnetic field:



The ellipticity is typically specified by an angle Φ characterizing the phase shift between the two polarization components:

optical path length

$$\Phi = 2\pi \, (n_\parallel - n_\perp) \, \frac{L}{\lambda} \tag{2.57}$$

probe beam wave length

$$\Rightarrow \Phi_{QED} = \frac{1}{15} \, \alpha \left(\frac{eB}{m^2} \right)^2 \frac{L}{\lambda} \left(1 + \underbrace{\frac{25}{4} \frac{\alpha}{\pi}}_{\text{2-loop contribution} \simeq 1\% \text{ effect}} \right) \tag{2.58}$$

Quite a number of different strategies have been discussed (or already used) for discovery experiments.

The most widely used strategy is to employ optical probe beams (i.e. $\lambda \simeq 500..1000\,nm$ fixed) and try to optimize $B^2 L$. This corresponds to using strong & large macroscopic magnets (typically dipoles or pulsed X-coils). This is being done in the PVLAS and BMV experiments (also in OSQAR, Q&A, ...). The limiting challenge in these cases so far is the precise measurement of the induced ellipticity in combination with the use of high-finesse optical cavities in order to increase L. A different strategy is based on the use of high-intensity lasers (i.e. $L \simeq (1..10)\,\mu m$ fixed), and try to optimize $\frac{B^2}{\lambda}$. Since B^2 is eventually also provided by the high-intensity laser, one aims at using short-wave length (e.g. X-ray) probe beams as could be provided by an XFEL. The advantage is that polarimetry for X-rays as a matter of principle is less affected by background noise; the disadvantage so far is that a high-intensity laser and a high-brilliance X-ray source have to be available at the same laboratory. Groups at FSU are working on this ...

Coming back to the underlying theory, let us emphasise again that the validity limits of eqs. (2.55) & (2.58) are dominated by those for the Heisenberg-Euler (+ weak-field) calculation, i.e., we require $\omega \ll m$ and $eB \ll m^2$. Both restrictions can be lifted by a study of the polarization tensor in a strong-field background (see, e.g. Tsai & Erber '75).

Finally, it is fascinating to see that the phase velocities are smaller than the vacuum velocity $v_{\parallel,\perp} < 1$. If this also holds for signal velocities this may point to a consistency between microcausality (built into QED) and macrocausality. Whether this is indeed the case also for other modified vacua (e.g. gravitational backgrounds) is an active field of research.

From (2.55), we can also deduce a polarization sum rule:

$$v = 1 - \frac{1}{2}(c_1+c_2)\frac{(F\hat{k})^2}{|\vec{k}|^2} = 1 - \frac{11}{45}\frac{\alpha^2}{m^4}B^2\sin^2\theta_B \qquad (2.59)$$

Averaging over propagation directions: $\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta_B\sin\theta_B \ldots$ yields

$$v = 1 - \frac{44}{135}\frac{\alpha^2}{m^4}\,\frac{1}{2}B^2$$

$$= 1 - \frac{44}{135}\frac{\alpha^2}{m^4}\,u \qquad (2.60)$$

where $u = \frac{1}{2}B^2$ denotes the energy density of the modified quantum vacuum.

As pointed out by Latorre, Pascual and Tarrach in 1995 Eq. (2.60) can serve as a "unified" formula that also holds for other forms of modifying the quantum vacuum.

For instance:

– thermal background

$$u = \frac{\pi^2}{15} T^4 \tag{2.61}$$

(explicitely calculated by Barton)

– Casimir (parallel-plates) background

$$u = -\frac{\pi^2}{720} \frac{1}{a^4} \tag{2.62}$$

" Scharnhorst effect "

– in curved space (suitably adjusting the coupling factors $\left(\frac{\alpha^2}{m^4} \to \frac{\alpha G_N}{m^2}\right)$)

A partial proof of the unified formula has been given by Dittrich, HG in 1998.

The issue of consistency between micro & macro-causality becomes particularly pressing for the case of the Casimir background as well as in curved spaces where u can be < 0. This has recently been discussed in detail by Shore & Hollowood in a series of works.

2.2.2 Photon splitting

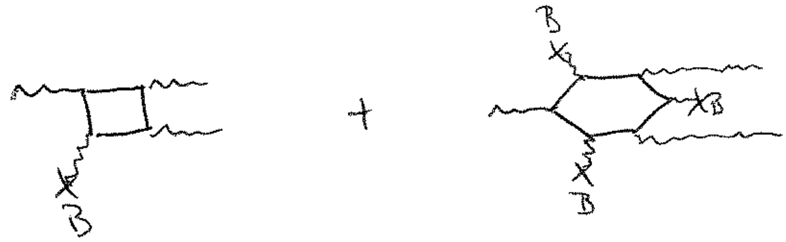
As the Heisenberg-Euler effective action is non-linear in the photon field, it can describe a variety of non-linear processes of the modified quantum vacuum, much in the same way as is familiar from nonlinear optics of dispersive media. In this language, the quantum vacuum can be viewed as a nonlinearly responding medium.

A paradigmatic phenomenon is "photon splitting"
(~ a $\chi^{(3)}$-process in nonlinear optics)

This process is forbidden in the trivial vacuum due to Furry's theorem

 $\sim \mathrm{tr}\ \mathrm{odd}\ \#\ \gamma\text{'s} = 0$

However, in a magnetized quantum vacuum, Furry's theorem can be circumvented by additional couplings to the B-field:



(2.63)

In the following, we will discuss photon splitting only rather qualitatively:
as has been shown by Adler in his famous work from 1971, the box diagram – though seemingly being the leading-order diagram – is kinematically suppressed compared to the hexagon diagram (this so-called Adler theorem holds for B = const.).

Schematically, the splitting matrix element is given by

$$\mathcal{M} \simeq f_1 f_2 f_3 \left. \frac{\partial^3 \mathcal{L}_{HE}}{\partial F \partial F \partial F} \right|_{\overline{F}} \qquad (2.64)$$

where f_i denote the propagating incoming & outgoing photons and $\overline{F}$ is, for instance, a strong magnetic background field. We have ignored here the Lorentz structure.

One further particularity of photon splitting is that the polarization algebra allows only the splitting process

$$\perp \Rightarrow \parallel + \parallel \qquad (2.65)$$

Heuristically, this can be understood from the fact that the $\perp$-mode is "faster" than the $\parallel$-mode. Other splitting channels are partly forbidden due to parity conservation.

This particularity gave rise to the speculation that photon splitting could be an effective mechanism to produce polarized soft photons in a strong field environment.

From the splitting matrix element the absorption coefficient for an incoming photon to undergo photon splitting can be computed by integrating over phase space

$$\kappa = \frac{1}{32\pi\,\omega^2} \int_0^\omega d\omega_1 \int_0^\omega d\omega_2 \; \underset{\substack{\uparrow \\ \text{energy conservation}}}{\delta(\omega-\omega_1-\omega_2)} \; |M|^2 . \qquad (2.66)$$

Using a soft photon effective action $\mathcal{L} = \mathcal{L}(\mathcal{F}, \mathcal{G}^2)$,
we find for photon splitting in a magnetic field:

$$K = \frac{\pi^2 \alpha^3}{15} \left(\frac{eB}{m^2}\right)^6 \left(\frac{\omega}{m}\right)^5 \sin^6 \theta_B \left(\frac{\partial^3 \mathcal{L}}{\partial \mathcal{F} \partial \mathcal{G}^2}\bigg|_B\right)^2 \frac{m^{16}}{e^{12}} \quad m \tag{2.67}$$

where the $\left(\frac{\omega}{m}\right)^5$-term arises for purely kinematic phase-space reasons.

Using the weak-field expansion of the Heisenberg-Euler action to 3rd order in the invariants, we find in the weak-field regime:

$$K \simeq 0.116\ \mathrm{cm}^{-1} \left(\frac{eB}{m^2}\right)^6 \sin^6 \theta_B \left(\frac{\omega}{m}\right)^5 \quad , \omega \ll m, B \ll \frac{m^2}{e} \tag{2.68}$$

From the full Heisenberg-Euler action, also the strong-field limit can be worked out, yielding

$$K \simeq 0.472\ \mathrm{cm}^{-1} \sin^6 \theta_B \left(\frac{\omega}{m}\right)^5 \quad , \omega \ll m, B \gg \frac{m^2}{e} \tag{2.69}$$

The inverse absorption length can be identified with the mean free path of a photon in a magnetic field to undergo photon splitting.

For instance, assuming that these formulas hold for

$$\omega/m \simeq 10^{-1} \quad , \text{ i.e. } \omega \simeq \mathcal{O}(10 \text{ keV}) \quad \text{in}$$

the X-ray regime, we find for $\theta_B = \frac{\pi}{2}$ a mean free path of

$$\frac{1}{\kappa} \simeq \mathcal{O}(1)\, 10^5 \text{ cm} \simeq \mathcal{O}(1) \text{ km} \tag{2.70}$$

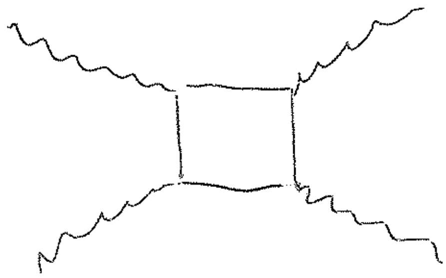
This is a length scale similar to the size of strong-magnetic field regions of highly-magnetized pulsars (radius $\mathcal{O}(10 \text{km})$). So-called magnetars are likely to develop field strength of up to $\frac{eB}{m^2} \simeq \mathcal{O}(10^3)$.

Hence photon splitting has been conjectured to be of relevance for the emission spectra and degree of polarization of photons from highly magnetized pulsars.

However, this has been under debate on and off during the past decades.

2.2.3 Light-by-light scattering

The possibility of light-light scattering was the original motivation of Euler's PhD thesis and has been published in 1936 for the first time. A comprehensive study was performed by Karplus & Neumann in 1951 during the flourishing years of QED



The full information about this scattering process in the soft-photon domain is encoded in the 4th order term of the Heisenberg-Euler action

$$\mathcal{L}_{HE}\Big|_{F^4} \simeq \frac{2}{45}\frac{\alpha^2}{m^4}\left(4\mathcal{F}^2 + 7\mathcal{G}^2\right) \qquad (2.71)$$

Again, we will just give a schematic summary here: the matrix element is schematically given by

$$\mathcal{M} \sim f_1 \dots f_4 \frac{\partial^4 \mathcal{L}}{\partial F \partial F \partial F \partial F} \qquad (2.72)$$

From which the differential cross-section can straightforwardly be computed, schematically:

$$\frac{d\sigma}{d\Omega} = \sum_{pol.} \int_{\substack{phase\\space}} |\mathcal{M}|^2 \tag{2.73}$$

The final result reads

$$\frac{d\sigma}{d\Omega} = \frac{139}{8100} \left(\frac{\alpha}{2\pi}\right)^2 r_0^2 \left(\frac{\omega}{m}\right)^6 \left(3 + \cos^2\theta\right)^2, \quad \omega \ll m$$

where $r_0 = \frac{\alpha}{m}$ is the "classical electron radius" and θ is the scattering angle in the center-of-mass frame. The strong $\left(\frac{\omega}{m}\right)^6$ suppression occurs again purely due to phase-space kinematics.

For an intuition, it is instructive to compare this cross-section to a standard QED low-energy cross-section for a tree-level process, e.g. Thomson scattering:

$$\frac{d\sigma}{d\Omega}(\omega \to 0) = \frac{1}{2} r_0^2 \left(1 + \cos^2\theta\right) \tag{2.74}$$

We observe that light-by-light scattering is suppressed by $\left(\frac{\alpha}{2\pi}\right)^2 \sim 10^{-6}$ as well as by $\left(\frac{\omega}{m}\right)^6$ which for optical photons is of the order of $\sim 5 \cdot 10^{-35}$.

Similar differences hold for the total cross section

$$\sigma_{\ell b \ell} = \frac{973}{10125} \frac{\alpha^2}{\pi} r_0^2 \left(\frac{\omega}{m}\right)^6 \quad , \quad \sigma_{\text{Thomson}} = \frac{8\pi}{3} r_0^2 \tag{2.75}$$

The light-by-light scattering cross-section as of today has not been measured directly sofar (say with lasers).

The most sensitive experiment with macroscopic EM field so far has been the PVLAS experiment.

From their non-observation of interactions, they deduce an upper bound for the cross-section:

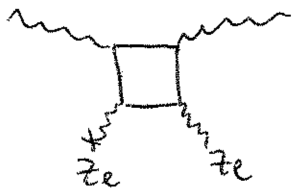
$$\sigma_{\ell b \ell} < 9.5 \cdot 10^{-59} \, cm^2 \quad @ \, 1064 \, nm \text{ (optical wavelength)}$$

(Zavattini et al I.J.M.P. A27 (2012) 1260017)

The QED prediction given above would correspond to

$$\sigma_{\ell b \ell} = 1.8 \cdot 10^{-65} \, cm^2 \quad @ \, 1064 \, nm \tag{2.76}$$

Still, light-by-light scattering has been experimentally explored in indirect or different experimental contexts. For instance, there exist measurements of Delbrück scattering of a photon off a high-Z nucleus



(2.77)

in the $\omega \gg m$ regime. This process is enhanced by the corresponding Z factors as well as by the high frequencies. These measurements do however not constitute a quantitative test of the Heisenberg-Euler action.

Light-by-light scattering also contributes as a subprocess to $g-2$ measurements. However, as all loop

momenta are integrated over, the $g-2$ measurements cannot test the Heisenberg-Euler limit quantitatively

(incidentally, the hadronic light-by-light contribution is still insufficiently understood for $g-2$ of the muon.)

Though the cross-section is small, it can play a dominant role for photon propagation over long distances. For instance, the propagation of high-energy photons through the universe is in principle limited due to scattering with thermal photons of the cosmic microwave background,
or with extragalactic background light
(In fact, there are currently hints for an anomalous transparency of the universe for TeV photons observed at HESS and MAGIC... ?)

So: a clean measurement of light-by-light scattering is urgently needed.