5 Gribov - Zwanziger Scenario (Baby version)

Magnetic monopoles, or more generally topological configurations (i.e., configurations that carry topological charges (such as a monopole charge characterized by a winding number) and exhibit defects in typical singular gauges) are conjectured to be "confiners", i.e., to be responsible for the low-energy behavior of YM theory and particularly for confinement.

Irrespective of their detailed nature (monopoles, vortices, etc...), their defect structure causes them to lie right near the Gribov horizon.

More generally, the configurations near the Gribov horizon can be expected to play a dominant role at low energies.

To see this, let us establish three simple properties of the first Gribov region:

The Gribov region Ω_I (region in config' space within the first Gribov horizon)

(i) contains the origin $A_\mu = 0$

(ii) is bounded in every direction

(iii) is convex

in the Landau gauge $\mathcal{F}^a[A] = \partial_\mu A^a_\mu$ (5.1)

Proof: (Zwanziger '82, '04)

The Faddeev-Popov operator in the Landau gauge is

$$M^{ab} := -\partial^2 \delta^{ab} - f^{acb} A^c_\mu \partial_\mu \equiv M_0^{ab} + M_1^{ab}[A] \qquad (5.2)$$

(on the space of transverse gauge potentials $A_\mu : \partial_\mu A_\mu = 0$)

(i) At the origin $A_\mu = 0$,

$$M^{ab}_{[A=0]} = M_0^{ab} = -\partial^2 \delta^{ab} \qquad (5.3)$$

which is strictly positive $\square$.

(ii) Note that $\operatorname{tr} M_1^{ab} = 0$, since $f^{acb} A^c$ is traceless;

$\Rightarrow$ there exists at least one negative eigenvalue of M_1 with eigen-vector φ_1 for any given A_μ

$E := \langle \varphi_1 | M_1[A] | \varphi_1 \rangle < 0$.

Moreover, $M_1[A]$ is linear in A_μ: $M_1[\lambda A_\mu] = \lambda M_1[A_\mu]$.

$$\Rightarrow \langle \varphi_1 | M[\lambda A] \varphi_1 \rangle = \langle \varphi_1 | M_0 | \varphi_1 \rangle - \lambda E \qquad (5.4)$$

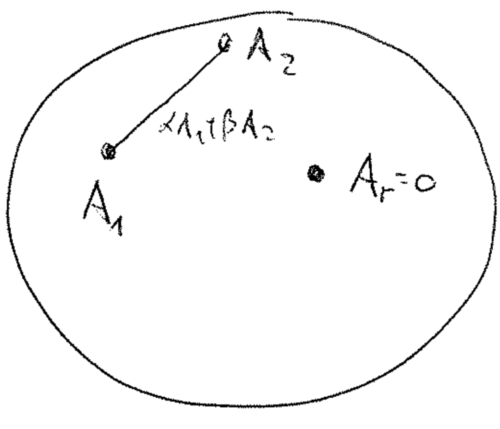
$\Rightarrow$ for λ sufficiently large, this becomes zero, meaning that $\Delta_{FP} = \det M[A]$ becomes zero, which characterizes the Gribov horizon.

(iii) Convexity means that for any $A_1, A_2 \in \Omega_I$, also $\alpha A_1 + \beta A_2$ with $\alpha + \beta = 1 \wedge \alpha, \beta \in [0,1]$ is in Ω_I

$$
\begin{aligned}
M[\alpha A_1 + \beta A_2] &= M_0 + M_1[\alpha A_1 + \beta A_2] \\
&= M_0 + \alpha M_1[A_1] + \beta M_1[A_2] \\
&= \alpha M_0 + \alpha M_1[A_1] + \beta M_0 + \beta M_1[A_2] \\
&= \alpha M[A_1] + \beta M[A_2]
\end{aligned}
\tag{5.5}
$$

$\Rightarrow$ since $M[A_1]$, $M[A_2]$ are positive, $M[\alpha A_1 + \beta A_2]$ is also positive

So, Ω_I can be depicted as



For small couplings, YM theory is well described by perturbation theory, i.e., by an expansion around $A_\mu = 0$.

However, in configuration space, these perturbative configurations are a set of very small measure. This is because in a very high-dimensional convex ~~space~~ and bounded space, most of the volume sits right under the surface (cf. volume of a high-dimensional ball).

Hence, we can expect that for larger couplings the dominant field configurations ~~[illegible]~~ sit right under the 1st. Gribov horizon; and topological configurations are an explicit example of these configs.

If this scenario holds, it has important consequences for the correlation functions.

Consider, e.g., the ghost correlator (2-point function)

$$\langle \bar{c}^a_{(-p)} c^b_{(p)} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{c} \mathcal{D}c \; \bar{c}^a(-p)\, c^b(p) \cdot e^{-S_{YM} - S_{gf} - \int \bar{c} M c} \qquad (5.6)$$

Perturbatively, we would have

$$\langle \bar{c}^a c^b \rangle \overset{\text{pert.}}{\simeq} \frac{1}{p^2} \delta^{ab} \tag{5.7}$$

(for small coupling).

Now, if at larger couplings the config's close to the Gribov horizon dominate, $\int \bar{c} M c$ is smaller on average than for the perturbative config's. Therefore, $\langle \bar{c}^a c^b \rangle$ is expected to diverge more strongly at low energies:

$$D_c(p) = \langle \bar{c}^a c^b \rangle_{(p)} = \frac{G(p)}{p^2} \delta^{ab} \quad ; \text{ where}$$

$$G\left(p^2/\Lambda^2_{QCD} \ll 1\right) \sim \left(p^2\right)^{-\kappa_c} \quad , \; \kappa_c > 0 \tag{5.8}$$

↑ ghost "dressing function"

↑ Zwanziger's horizon condition I

If gluons are confined in YM theory this should also have a consequence for the correlator which we parameterize as

$$D_{gl}(p) = \langle A^a_\mu A^b_\nu \rangle(p) = \frac{Z(p)}{p^2} \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \text{longitudinal} \tag{5.9}$$

↳ decouple in the Landau gauge

Now, one can show that if

$$\tilde{\mathcal{D}}_{gl}(x-x') = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} D_{gl}(p)$$

is smaller than 0 for some $(x-x')_\mu$, then (Ostowalder-Schrader positivity is violated and) the correlation function does not allow for a probabilistic interpretation, implying that the gluons would not belong to the physical Hilbert space of states.

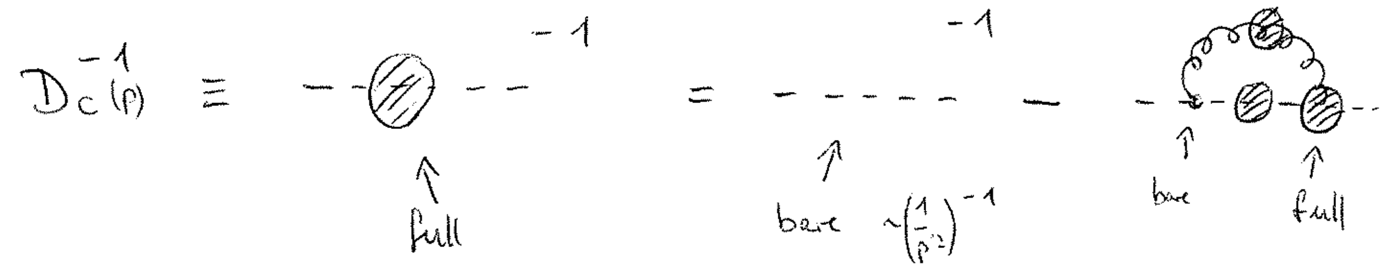
We observe that this positivity condition is necessarily violated if

$$0 = \lim_{p\to 0} \mathcal{D}_{gl}(p) \equiv \int d^4x\, \tilde{\mathcal{D}}_{gl}(x-x') \qquad (5.10)$$

Parameterizing $Z(p) \sim (p^2)^{-\kappa_A}$ for small p^2, (5.10) is satisfied if $\kappa_A < -1$

(Zwanziger's horizon condition II).

The "IR exponents" κ_A, κ_C are in fact related to each other. This can be deduced with the aid of nonperturbative functional methods such as Dyson-Schwinger equations. Expanding the generating functional in terms of correlation functions produces, e.g. the DSE for the ghost correlator:



Due to a theorem by Taylor ('81), the ghost-gluon vertex in Landau gauge is not (independently) renormalized, implying that the leading momentum structure is of the form of the bare vertex $\sim p_\mu$:

Counting IR momenta, this implies:

$$(p^2)^{1+\kappa_c} \simeq \underbrace{p^2}_{\text{subleading if } \kappa_c>0} + \int d^4p \; p \,(p^2)^{-1-\kappa_c} (p^2)^{-1-\kappa_A} \, p$$

$$\Rightarrow \quad 1+\kappa_C = \cancel{\#}\; 3 - 1 - \kappa_C - 1 - \kappa_A$$

$$\Rightarrow \quad \underline{\underline{-\kappa_A = 2\kappa_C}} \qquad (5.11)$$

This is compatible with Zwanziger's horizon conditions, if $\kappa_C > 0.5$

Solutions to (truncated) DSEs as well as (truncated) RG flow eqs. for the propagators give indeed (v. Smekal, Hauck, Alkofer 1997; Lerche, v. Smekal 2002; Fischer, Alkofer 2002)

$$\kappa_C \simeq 0.595\ldots \qquad (5.12)$$

This can be interpreted as a signature of confinement in the Landau gauge within the Gribov - Zwanziger scenario.

In absence of a mass scale in the IR, it is possible to prove that this solution is unique (Fischer, Pawlowski 2007).

However, ..., there is also a massive solution and the debate about the solutions is still ongoing...

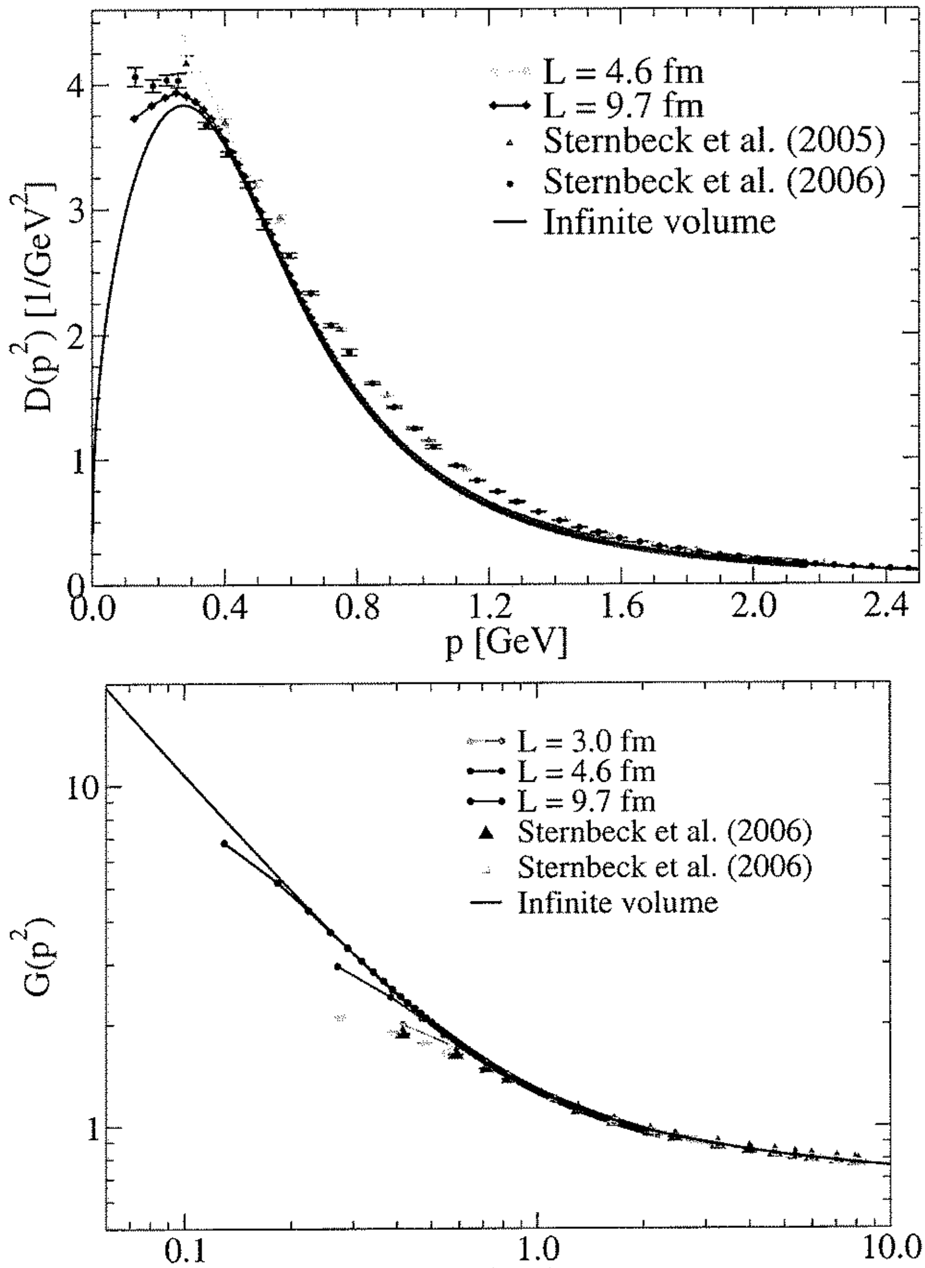


Fig. 8. DSE results for the gluon propagator and ghost dressing function on tori with different volumes compared to recent lattice calculations on similar manifolds. The lattice data are taken from Refs. [11,44].

Whereas both, the lattice and the DSE result at the smaller volume $V \approx (4.6\,\text{fm})^4$ seem to diverge, one starts to observe an infrared finite one, or perhaps even a slight infrared suppression, at the larger volumes $V \approx (9.7\,\text{fm})^4$. This indicates that the scaling behaviour of the lattice results with volume may be very similar to the ones of the DSE solution. If so, then our results

DSE solutions. One can show analytically that this is exactly the region where the omitted gluonic two-loop diagrams contribute significantly.