## 4.4 Georgi-Glashow Model in $D=4$

(one step further towards non-abelian gauge theories)

The Georgi-Glashow model is a Yang-Mills-Higgs system in $D=3$ with gauge group $G = SO(3)$.

The Lagrangian is

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2} (D_\mu \varphi)^a (D_\mu \varphi)^a + V(\phi),$$

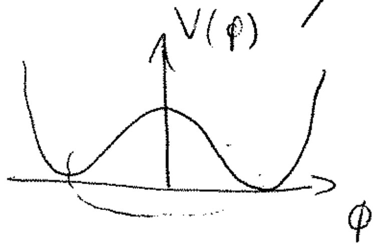
$$\text{where} \quad V(\varphi) = \frac{\lambda}{4} (\varphi^2 - v^2)^2, \tag{4.40}$$

$$\text{and} \quad D_\mu^{ab} = \partial_\mu + g f^{acb} A_\mu^c .$$

The scalar field transforms as a vector under $G$ (in adjoint representation)

$$\varphi' = U \varphi .$$

At weak coupling $g^2 \ll v$, the ground state of $\phi$ is determined by the potential minimum




$$\varphi^a \to \varphi_0^a , \quad \phi_0^2 = v^2 ,$$

$$\text{e.g.} \quad \varphi_0^a = v \, \delta^{a3} . \tag{4.41}$$

This expectation value provides masses for the gauge bosons by the Higgs mechanism:

$$\frac{1}{2}(D_\mu \phi)^2 \sim \ldots \frac{1}{2} g^2 v^2 \left( A^1_\mu A^1_\mu + A^2_\mu A^2_\mu \right)$$

In terms of complex fields, (4.42)

$$W^\pm_\mu = \frac{1}{\sqrt{2}} \left( A^1_\mu \pm i A^2_\mu \right),$$

the $W^\pm$ bosons describe particles with

mass $m_W = g v$, spin 1, charge $\pm g$

The remaining component of $A_\mu$ is a massless "photon": $A_\mu \equiv A^3_\mu$ with spin 1, charge 0.

Excitations on top of the condensate exist in terms of a Higgs scalar:

$$\phi^3(x) \to (v + H(x)), \text{ mass } m_H^2 = 2\lambda v^2, \text{ spin } 0, \text{ charge } 0$$

The expectation value $\Phi_0$ breaks the gauge group $SO(3)$; a residual symmetry remains:

the vacuum is invariant under rotations around the $\Phi_0^a$ axis in color space

$$G = SO(3) \quad \rightarrow \quad H = O(2) = U(1). \qquad (4.43)$$

The "photon" corresponds to the gauge boson of this $U(1)$.

For a semiclassical analysis, we are looking for stable solutions with finite action; the latter requires

$$\phi^a \rightarrow \Phi_0^a(x), \Phi_0^2 = v^2 \quad \text{for } |x| \rightarrow \infty \quad (S^2_\infty).$$
sufficiently fast

On the other hand, the space of all Higgs vacua is given by

$$(\Phi_0^1)^2 + (\Phi_0^2)^2 + (\Phi_0^3)^2 = v^2$$
$$\hat{=} \text{ Sphere } S^2 = G/H. \qquad (4.44)$$

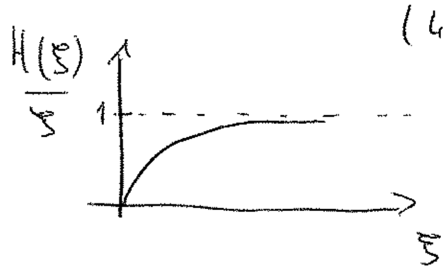
$\Rightarrow$ the "instantonic" finite-action solutions correspond to mappings $S^2_\infty \rightarrow S^2$, which are classified by the homotopy group $\Pi_2(S^2) = \mathbb{Z} \ni n$.

( solutions with different $n$ are not continuously deformable into each other ; they are separated by an infinite energy barrier ; thus they are stable )

The "most elementary" solution is the 't Hooft-Polyakov monopole

$$\phi^a = \frac{x^a}{g\, x^2}\, H(v g |x|) \qquad \text{(hedgehog)}$$

$$= v \frac{x^a}{|x|} \frac{H(v g|x|)}{v g |x|} \qquad (4.45)$$




$$A_i^a = -\varepsilon^a_{ij} \frac{x^j}{g|x|^2} \left(1 - K(v g |x|)\right) \quad , A_0^a = 0$$

(4.46)


where $H$ and $K$ are solutions of ordinary differential equations that follow from the EoM.

Also for this solution, one can define a $U(1)$ magnetic field

$$B_i = -\frac{1}{2}\frac{1}{v} \varepsilon_{ijk}\, \phi_0^a\, F^a_{jk} \; , \qquad (4.47)$$

giving rise to a magnetic flux of the form

$$g_M = \oint d\vec{f} \cdot \vec{B}_{\text{'t Hooft-Polyakov}} = \frac{4\pi}{g} \,, \tag{4.48}$$

describing a magnetic charge!

For more general solutions, one finds

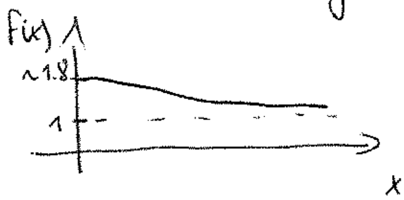
$$g_M = \frac{4\pi}{g}\, n \,, \quad n \in \Pi_2(S^2) \,.$$

The position of the monopole corresponds to a zero of the Higgs field.

There is no (obvious) singularity in the gauge potentials.

Plugging the 'tHP solution into the action gives

$$S_{\text{monopole}} = \frac{1}{2} \frac{m_W}{\alpha} f\Big(\underbrace{\frac{m_H}{m_W}}_{\sqrt{\frac{2\lambda}{g}}}\Big) \tag{4.49}$$




For weak coupling $\alpha \ll 1$, we find

$$S_{\text{monopole}} \gg 1 \,,$$

implying that the vacuum contains a dilute monopole gas / plasma.

( Only the long-range Coulombic interactions have to be considered; the more complicated near-field interactions are irrelevant.)

This suggests the effective action

$$\Gamma_{monop} = \frac{2\pi}{g^2} \sum_{a\neq b} \frac{q_a q_b}{|\vec{x}_a - \vec{x}_b|} + \frac{2\pi\, m_W}{g^2} f\left(\frac{m_H}{m_W}\right) \sum_a q_a^2 \qquad (4.50)$$

where $q_{a,b} = \pm 1$ denote the monopole charges in units of $g_M$, corresponding to the 'tHP and anti-'tHP monopoles (the higher solutions are suppressed by a larger action).

Comparison with compact $U(1)_3$ gives

$$M^2 \sim e^{-\frac{2\pi m_W}{g^2} f\left(\frac{m_H}{m_W}\right)}, \quad (\text{dual photon mass}) \qquad (4.51)$$

and the same line of argument as in $U(1)_3$ leads to confinement for static charges

$$\langle W(C)\rangle \sim e^{-\sigma\, \text{area}(S)}, \quad \sigma \sim g^2 M \qquad (4.52)$$