4 ~~3~~ Confinement & Monopoles

4.~~3~~.1 Prerequisites

A class of popular models of confinement is based on the idea of a dual Meissner effect.

To understand this duality hypothesis, let us first sketch the Meissner effect which is a characteistic feature of superconductivity.

Type-I superconductivity can be described by a condensation of Cooper pairs which are bosonic electron composites which occupy the same quantum state. This condensate is associated with a macroscopic wave function

$$\phi(\vec{x},t) = \sqrt{|\underbrace{q}_{=-2e\text{, Cooper pair charge}}| \underbrace{N}_{\#\text{ Cooper pairs}}} \cdot \underbrace{\chi(\vec{x},t)}_{\text{normalized wave function of one Cooper pair}} \qquad (3.1)$$

The macroscopic charge density is

$$\rho(\vec{x},t) = -|\phi|^2_{(\vec{x},t)} = q\,N\,|\chi|^2_{(\vec{x},t)} \qquad (3.2)$$

The conservation of the number of Cooper pairs implies a continuity equation,

$$\frac{\partial \varrho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0 . \qquad (3.3)$$

From (3.2) together with Schrödingers Eq., $i\hbar\dot{\phi} = H\phi$, $H = -\frac{\vec{D}^2}{2m}$, $\vec{D} = \vec{\nabla} - \frac{iq}{\hbar}\vec{A}$, we obtain the Cooper current

$$\vec{j} = \frac{i\hbar}{2m}\left(\phi(\vec{D}\phi)^* - \phi^*\vec{D}\phi\right) . \qquad (3.4)$$

For a homogeneous superconductor with $\varrho(\vec{x},t) \simeq \varrho = \text{const}$ and

$$\phi(\vec{x},t) = \sqrt{|\varrho|}\, e^{i\varphi(\vec{x},t)} , \qquad (3.5)$$

the current simplifies to

$$\vec{j} = \frac{\varrho\hbar}{m}\left(\vec{\nabla}\varphi(\vec{x},t) - \frac{q\vec{A}}{\hbar}\right) . \qquad (3.6)$$

The interaction between the Cooper current and a magnetic field, obeying the Maxwell's equations

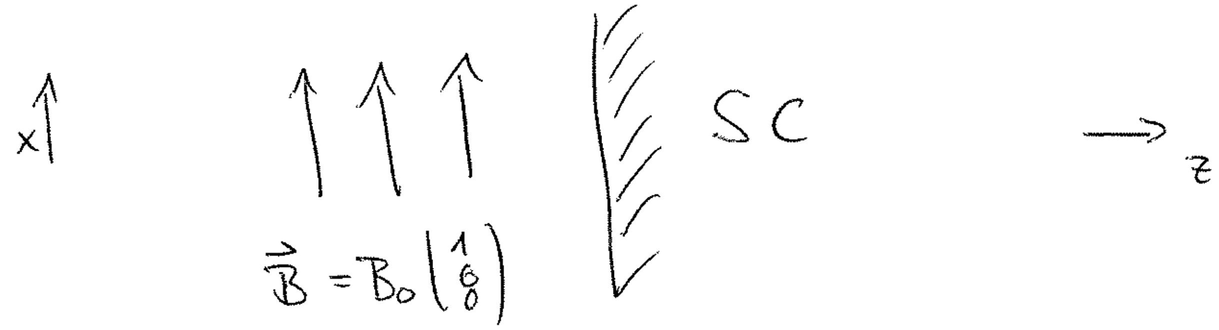
$$\vec{\nabla}\cdot\vec{B} = 0 \quad , \quad \vec{\nabla}\times\vec{B} = \frac{\vec{j}}{\varepsilon_0 c^2} , \qquad (3.7)$$

implies

$$-\vec{\nabla}^2\vec{B} = \vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \frac{1}{\varepsilon_0 c^2}\vec{\nabla}\times\vec{j} = -\frac{\varrho q}{\varepsilon_0 c^2}\vec{\nabla}\times\vec{A} = -\frac{\varrho q}{\varepsilon_0 c^2}\vec{B}$$

$$\Rightarrow \underline{\underline{\vec{\nabla}^2\vec{B} = \frac{1}{\lambda_L^2}\vec{B}}} \qquad (3.8)$$

where λ_L denotes the London penetration depth. Consider, e.g., a magnetic field close to a superconductor,

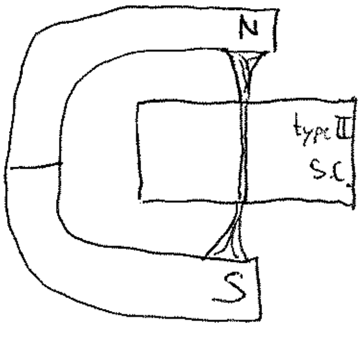


then the field inside the S.C. has to obey (38), yielding

$$\vec{B}(\vec{z}) = B_0 \left(e^{-z/\lambda_L}, 0, 0 \right) , \quad z > 0 . \tag{3.9}$$

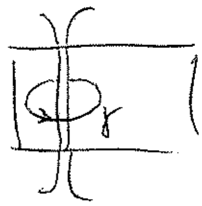
The magnetic field vanishes exponentially inside the S.C. : <u>Meissner effect</u>

For a type II superconductor, the magnetic field can penetrate into th S.C. in terms of thin magnetic flux lines (Abrikosov vortices)



(Shubnikov phase)

The flux through the vortex yields

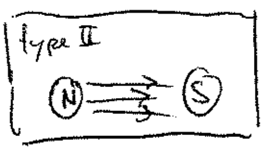


$$\Phi = \oint_\gamma d\vec{x}\cdot\vec{A} = \frac{\hbar}{q}\oint_\gamma d\vec{x}\cdot\vec{\nabla}\varphi = \underbrace{\frac{2\pi\hbar}{q}}_{\text{flux quantum } \Phi_0} n \,, \qquad (3.10)$$

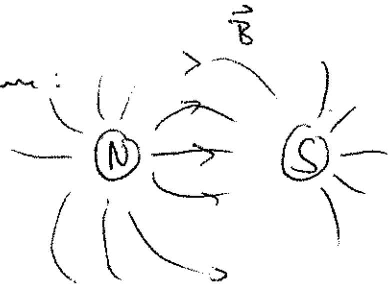
$$\vec{\nabla}\times\vec{B} = 0 = \vec{j} = \frac{q\hbar}{m}\left(\vec{\nabla}\varphi - \frac{q}{\hbar}\vec{A}\right)$$

Since the single-valuedness of the wave function fixes the phase $\varphi(\vec{x},t)$ only up to a discontinuity of $2\pi n$, $n\in\mathbb{Z}$ $(=\pi_1(U(1)))$. This results in flux quantization.

Now, imagine that we have two magnetic monopoles of opposite charge Ⓝ and Ⓢ at our disposal. If we bring these monopoles into a type-II S.C., the Meissner effect enforces a string-like flux distribution



vacuum:



The static potential would then be linear

$V^{NS}_{s.c.}(R) \sim R$ in contrast to the vacuum potential

$V^{NS}_{vac}(R) \sim \frac{1}{R}$.

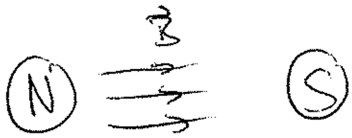
(NB: incidentally, if monopoles existed, the flux distribution of type I S.C. would also be string-like for two static monopoles.)

This gedankenexperiment gives rise to a confinement picture based on a hypothetical dual Meissner effect ('t Hooft, Mandelstam '76):

S.C.

magn. Meissner effect.

magn. flux quantization



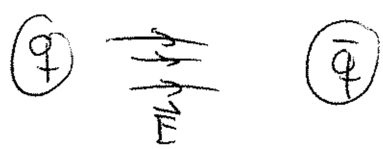
condensation of electric charges, "Cooper pairs"

QCD vacuum

electric. Meissner effect

electric flux quant.

(o.k.: quark charges are quantized)



condensation of magnetic charges, monopole pairs ?

The obvious question is: are there field configurations in YM theories with a monopole-like charge content?

(to be answered later...)

4.2 Magnetic Monopoles in abelian gauge theory

The source-free Maxwell's equations exhibit a duality symmetry $\vec{E} \to \vec{B}$, $\vec{B} \to -\vec{E}$.
Promoting this symmetry to hold also in the presence of sources requires the existence of magnetic charges and currents. E.g. electro-magneto statics is described by

$$\vec{\nabla} \cdot \vec{E} = \rho \quad , \quad \vec{\nabla} \cdot \vec{B} = \rho_M \qquad (4.11)$$

with the magnetic field of a magnetic monopole given by

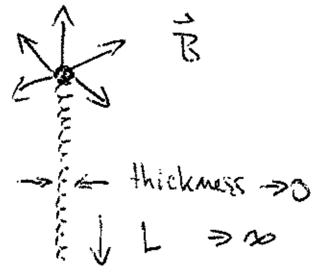
$$\vec{B} = \frac{g}{4\pi} \frac{\hat{r}}{r^2} \qquad (4.12)$$

Representing this field by a gauge potential requires singular structures,

$$g = \int_V \rho_M \, d^3x = \int_V d^3x \, \vec{\nabla} \cdot \vec{B} \stackrel{?}{=} \int_V d^3x \, \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) \qquad (4.13)$$

If $\vec{A}$ is non-singular but regular, the last term $= 0$.

The monopole gauge potential can be constructed from that of an idealized infinitely long solenoid



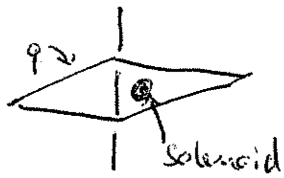
$$\vec{A} = \frac{g}{4\pi} \frac{1-\cos\theta}{r\sin\theta}\,\hat{\phi} \equiv \frac{g}{4\pi}(1-\cos\theta)\,\vec{\nabla}\varphi \tag{4.14}$$

$$\Rightarrow \quad \vec{B} = \vec{\nabla}\times\vec{A} = \underbrace{\frac{g}{4\pi r^2}\,\hat{r}}_{\text{Monopole}} + g\,\underbrace{\delta^{(2)}(\vec{x}_\perp)\,\Theta(-z)}_{\text{Solenoid}}\,\vec{e}_z \tag{4.15}$$

Can ~~the~~ the solenoid be detected in the limit thickness $\to 0$, $L\to\infty$?

classically : no

quantum mechanically : (yes) by Aharonov-Bohm effect



But the A.B. effect is invisible for a solenoid flux

$$\left.\begin{array}{l} \Phi = \dfrac{2\pi\hbar}{q}\,n \\ \parallel \\ \Phi_{\text{solenoid}} = g \end{array}\right\} \Rightarrow \quad \underline{\underline{\frac{q\cdot g}{2\pi\hbar} = n}}\,, \quad n\in\mathbb{Z} \tag{4.16}$$

(since then the interference pattern is shifted by 2π)

Dirac quantization condition

(stating that the existence of a single monopole in the universe requires electric charges to be quantized.)