2.5 Leading-Log model of Confinement
(= Savvidy model)

(Pagels & Tomboulis '78; Adler '82, Adler & Piran '84; Lehmann, Wu '84 ...)

Despite the obvious deficiencies of the one-loop calculation, consider $\Gamma^{1\text{-loop}}[A]$ as the simplest example of a possible complete effective action of QCD. The true action will, of course, depend on many more invariant operators of more complicated color and Lorentz structures. But already the present simple approximation features a nontrivial aspect: the "gluon condensate".

Therefore it is worthwhile to study the resulting Quantum Equations of Motion; for this, we need to go over to Minkowski space:

$$\mathcal{L}_E = -\mathcal{L}_M \,, \quad \mathcal{F} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \qquad (2.91)$$

We furthermore rescale the coupling into the field strength for convenience,

$$g^2 \mathcal{F} \to \mathcal{F} \qquad (2.92)$$

The Minkowski space Lagrangian then reads

$$\mathcal{L} = -\frac{1}{4} \, b_0 \, \mathcal{F} \ln \frac{2|\mathcal{F}|}{e \kappa^2} - A^a_\mu J^{a\mu}, \qquad (2.93)$$

where we included a source term which we choose to be provided by a static quark anti-quark pair at a distance R:

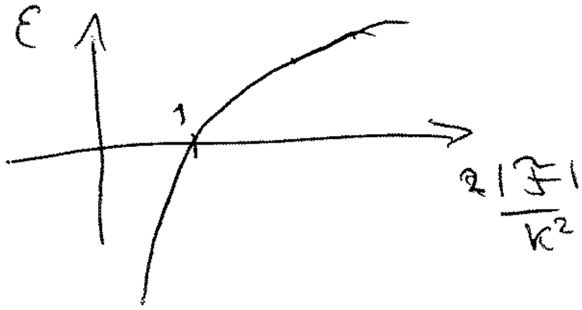
$$J^{a0} = Q \, n^a \left[\delta^{(3)}(\vec{x} - \vec{x}_1) - \delta^{(3)}(\vec{x} - \vec{x}_2) \right], \; |\vec{x}_1 - \vec{x}_2| = R \qquad (2.94)$$

The Quantum EoM read:

$$+J^{a\nu} = -\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A^a_\nu)} = \partial_\mu \left(-\frac{\partial \mathcal{L}(\mathcal{F})}{\partial \mathcal{F}} F^{a\mu\nu} \right) \equiv \partial_\mu \left(\varepsilon(\mathcal{F}) F^{a\mu\nu} \right) \qquad (2.95)$$

where we introduced the vacuum dielectricity constant

$$\varepsilon(\mathcal{F}) = -\frac{\partial \mathcal{L}}{\partial \mathcal{F}} = \frac{1}{4} b_0 \ln \frac{2|\mathcal{F}|}{\kappa^2}. \qquad (2.96)$$



The source-free QEoM can be satisfied by

$$\begin{aligned} &(a) \quad F_{\mu\nu} = 0 && \text{(unstable)} \\ &(b) \quad 2|\mathcal{F}| = \kappa^2 \,,\; \varepsilon = 0 && \text{(stable)} \end{aligned} \quad (2.97)$$

For pseudo-abelian sources, there is a pseudo-abelian solution, $F^a_{\mu\nu} = n^a F_{\mu\nu}$, which has to satisfy (in non-covariant notation):

$$\begin{aligned} \vec{\nabla}\cdot\vec{D} &= J^0 \,, & \vec{\nabla}\times\vec{E} &= 0 \\ \vec{\nabla}\times\vec{H} &= 0 \,, & \vec{\nabla}\cdot\vec{B} &= 0 \end{aligned} \quad (2.98)$$

supplemented by the material equations

$$\vec{D} = \varepsilon\,\vec{E} \,,\quad \vec{H} = \varepsilon\vec{B} \,. \quad (2.99)$$

To summarize, we have mapped the QCD vacuum onto nonlinear electrodynamics.

From $\vec{\nabla}\times\vec{H} = 0$, it follows that

$$0 = \int d^3x \; \vec{A}\cdot(\vec{\nabla}\times(\varepsilon\vec{B})) = \int d^3x \; \varepsilon\vec{B}^2 - \oint d\vec{f}\cdot\varepsilon(\vec{A}\times\vec{B}) \,, \quad (2.100)$$

where the last term vanishes for a solution that approaches (2.97)(b) at infinity.

For a solution with $\varepsilon \geq 0$, we obtain

$$\varepsilon \vec{B} = 0 \tag{2.101}$$

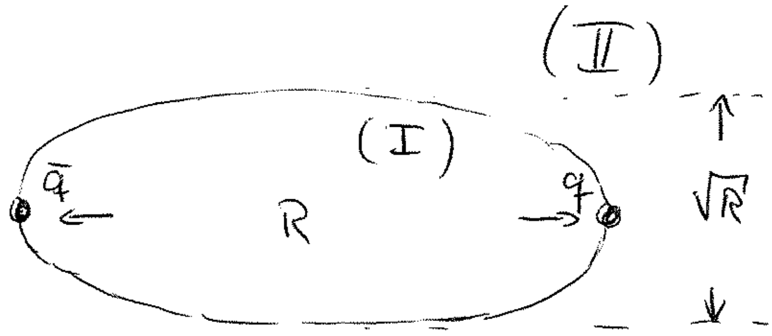
$$\Rightarrow \begin{cases} (\mathrm{I}) & \vec{B} = 0 \ , \ \vec{E}^2 > \kappa^2 \\ (\mathrm{II}) & \varepsilon = 0 \ , \ 2\mathcal{F} = \kappa^2 \end{cases}$$

Region (I) is expected to be observed near the sources where the electric field should show some resemblance with the Coulomb field.
Region (II) is the vacuum solution.

In fact, the QEoM can be solved in an analytic expansion using a $\vec{D}$-flux formulation, resulting in a quasi-linear PDeq. for the flux potential.

We will not review this (interesting) technical step here, but just list a few of the final results:

The static quarks are surrounded by a "bag" of region (I) of ellipsoidal shape. Outside the bag, region (II) extends to infinity :



The thickness of the bag scales with $R^{1/2}$.

Regions (I) and (II) are causally disconnected:

the source distribution inside cannot exert any influence on the field configuration outside

(the PDeq. turns from elliptic (I) to parabolic (II) on the boundary ; there , the normal second derivative vanishes , ∂_m^2 , from the PDeq) .

The static quark - anti-quark potential for long and short distances R reads

$$V_{static} = -\int d^3x\, \mathcal{L}\left(A|_{\text{QEoM}}\right)$$

$$= \begin{cases} \kappa Q R + \frac{2}{3} Q^{3/2} \sqrt{\frac{2}{\pi b_0}}\,\sqrt{\kappa}\, \ln(\sqrt{\kappa} R) + \mathcal{O}(1) & ; \sqrt{\kappa}R \gg 1 \\ -\frac{Q^2}{4\pi R}\,\frac{1}{b_0\left(\ln\frac{1}{\sqrt{\kappa}R} + \text{const}\right)} + \mathcal{O}\left(\frac{\ln \ln}{\ln}, \frac{1}{\ln^3}\right) & , \sqrt{\kappa}R \ll 1 \end{cases}$$

(2.102)

We observe a linear confinement for large R and a log-modified Coulomb potential for small R.

For heavy-quarkonium spectroscopy, the model shows reasonable agreement with experimental data for $\sqrt{\kappa} \simeq 229$ MeV. However, the string tension comes out somewhat too small ($Q = \sqrt{(4/3)}$ for $SU(3)$)

$$\Rightarrow \sqrt{\sigma} = \sqrt{Q\kappa} \simeq 246 \text{ MeV}$$

which should be compared to $\sqrt{\sigma}\,|_{Exp} \simeq 440$ MeV.

The predictions for the string thickness ("string roughening") $\sim R^{1/2}$ and the -leading correction to linear confinement $\sim \ln R$ should be compared with, e.g., the bosonic string model, where the roughening goes like $\ln R$ (not yet measured precisely on the lattice), and the static potential is

$$V_{BS}(R) = \sigma R - \underbrace{\frac{\pi(D-2)}{24}}_{\text{universal "Lüscher term"}} \frac{1}{R} \qquad \sqrt{\sigma} R \gg 1 \qquad (2.103)$$

The latter has been confirmed on the lattice.

To summarize, the leading-log model is a first and simple confinement model. The mechanism for confinement arises in this model from the dielectric properties of the quantum vacuum.

Due to its perturbative origin the model is, however, not well founded and various qualitative details are in contradiction with other methods.

Nevertheless, the possibility remains that a nonperturbative computation results in an effective action (of more complicated structure) that supports a dielectric confinement mechanism of the type described here.