Lattice Quantum Gravity

Jack Laiho
(in collaboration with Daniel Coumbe)
University of Glasgow

Jena
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Quantum Gravity

Quantizing gravity is one of the outstanding problems in theoretical physics.

- Straightforward implementation as a perturbative quantum field theory is not renormalizable
- Can still be formulated as an effective field theory at low energies
- Explicitly confirmed that a counter-term is necessary at 2-loop order for pure gravity [Goroff + Sagnotti, NPB266, 709, 1986] and 1-loop for gravity+matter [t'Hooft+Veltman]
- New couplings at each order in perturbation theory leads to a loss of predictive power.

Approach to Quantum Gravity

Weinberg proposed idea that gravity might be Asymptotically Safe in 1976 [Erice Subnucl. Phys. 1976:1]. This scenario would entail:

- Gravity is effectively renormalizable when formulated non-perturbatively.
- Renormalization group flows of couplings have a non-trivial fixed point, with a finite dimensional ultraviolet critical surface of trajectories attracted to the fixed point at short distances.
- In a Euclidean lattice formulation the fixed point would show up as a second order critical point, the approach to which would define a continuum limit.

History

- Within the last decade renormalization group studies have been done that suggest that pure gravity has a non-trivial fixed point with a small number of adjustable parameters (there is evidence that there are only three). [Lauscher + Reuter Phys. Rev. D65, 025013 (2002), Litim, PRL 92, 201301 (2004), Codello, et al. Ann. Phys. 324, 414 (2009) arXiv:0805.2909; Benedetti, et al, Mod. Phys. Lett. A24, 2233 (2009) arXiv:0901.2984].
- These calculations use truncations of the complete effective action. Errors can only be estimated by comparing various truncations, but systematic errors are hard to estimate.
- A lattice formulation would serve as a powerful cross-check. There have been several attempts...

Lattice gravity

- A number of different approaches to lattice gravity were introduced in the 1990's.
- Euclidean dynamical triangulations (EDT) was among the most popular formulations. [Ambjorn, Carfora, and Marzuoli, The geometry of dynamical triangulations, Springer, Berlin, 1997] Lattice geometries are approximated by triangles with fixed edge lengths. The dynamics is contained in the connectivity of the triangles, which can be added or deleted.
- In lattice gravity, the lattice itself is a dynamical entity, which evolves in Monte Carlo time. The dimension of the building blocks can be fixed, but the effective fractal dimension must be calculated from simulations.
- The EDT formulation was shown to have two phases, a "crumpled" phase with infinite Hausdorff dimension and a branched polymer phase, with Hausdorff dimension 2. The critical point separating them was shown to be first order, so that new continuum physics is not expected. [Bialas et al, Nucl. Phys. B472, 293 (1996), hep-lat/9601024; de Bakker, Phys. Lett. B389, 238 (1996), hep-lat/9603024]

Causal Dynamical Triangulations

In the late 90's, Ambjorn and Loll introduced Causal Dynamical Triangulations (CDT) [NPB 536, 407 (1998), hep-th/9805108] . They introduced a causality condition, where only geometries that admit a time foliation are included in the path integral.

- Simulations from 2004-2005 show a good semi-classical limit, with (Euclidean) de Sitter space as a solution. [Ambjorn, et. al., PRD 78, 063544 (2008), arXiv:0807.4481.]
- Striking result is a running effective dimension
- Effective (spectral) dimension runs from ~ 2 at short distances to ~ 4 at long distances. [Ambjorn, et. al., PRL 95, 171301 (2005), hep-th/0505113.]

Point of departure

Revisiting the EDT simulations for a few reasons [JL + Coumbe, PRL 107, 161301 (2011)]:

- If fixing the foliation in CDT is a gauge condition that does not remove physical degrees of freedom, should be possible to simulate using a covariant formulation.
- CDT has three tunable parameters: G_N, Λ and Δ, which is an asymmetry parameter between space-like and time-like links, so that CDT lattices are anisotropic in the time direction. Taking the RG studies seriously, maybe UV critical surface is 3 dimensional.
- Perhaps good results of CDT are a result of having 3 parameters, not the causality condition?
- Revisiting EDT simulations with a third parameter in the action.

Einstein Hilbert Action

Continuum path-integral:

$$Z = \int \mathcal{D}g \; e^{iS[g]}, \qquad (1)$$

$$S[g_{\mu\nu}] = \frac{k}{2} \int d^d x \sqrt{-\text{det}\; g}(R - 2\Lambda), \qquad (2)$$

where $k = 1/(8\pi G_N)$.

Discrete action

Discrete Euclidean (Regge) action is

$$S_E = k \sum 2V_2\delta - \lambda \sum V_4, \tag{3}$$

where $\delta = 2\pi - \sum \theta$ is the deficit angle around a triangular face, V_i is the volume of an i-simplex, and $\lambda = k\Lambda$. Can show that

$$S_E = -\frac{\sqrt{3}}{2}\pi k N_2 + N_4 \left(\frac{5\sqrt{3}}{2} k \arccos \frac{1}{4} + \frac{\sqrt{5}}{96}\lambda \right) \tag{4}$$

where N_i is the total number of i-simplices in the lattice. Conveniently written as

$$S_E = -\kappa_2 N_2 + \kappa_4 N_4. \tag{5}$$

Measure term

Adding a measure term was investigated somewhat in the 90's, but not in great detail [Brugmann and Marinari, PRL 70, 1908 (1993), hep-lat/9210002; Bilke, et. al., PLB 432, 279 (1998), hep-lat/9804011.]. In the continuum:

$$Z = \int \mathcal{D}g \, \sqrt{-\text{det } g}^{\beta} \; e^{iS[g]}, \tag{6}$$

Going to the discretized theory, we have

$$\sqrt{-\text{det } g}^{\beta} \rightarrow \prod_{j=1}^{N_2} o(t_j)^{\beta}, \tag{7}$$

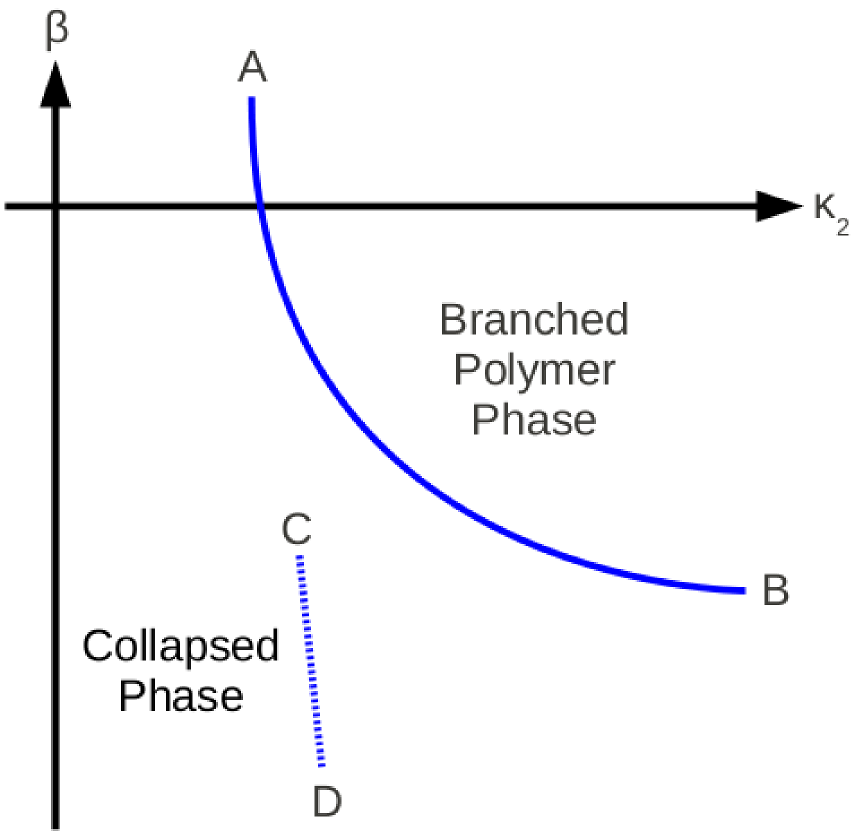
where $o(t_j)$ is the order of triangle t_j, i.e. the number of 4-simplices to which a triangle belongs. Can incorporate this term in the action by taking exponential of the log. β is our third parameter.

Simulations

Methods for doing these simulations were introduced in the 90's. I wrote new MC code, and my student and I performed extensive tests against the literature.

- The Metropolis Algorithm is implemented using a set of local update moves.
- Lists of simplices and sub-simplices are stored in arrays. Pointers to nearest neighbors of any given four-simplex and pointers to the sub-simplices of any given four-simplex are also stored to speed up the local moves.
- We relax the combinatorial manifold constraints on the space of triangulations used in the path integral. This leads to significantly reduced finite-size effects (for both exactly solvable 2-d models and comparing 4-d results for the unphysical phases). [Bilke and Thorleifsson, PRD 59, 124008 (1999), hep-lat/9810049.]

Phase diagram



Spectral Dimension

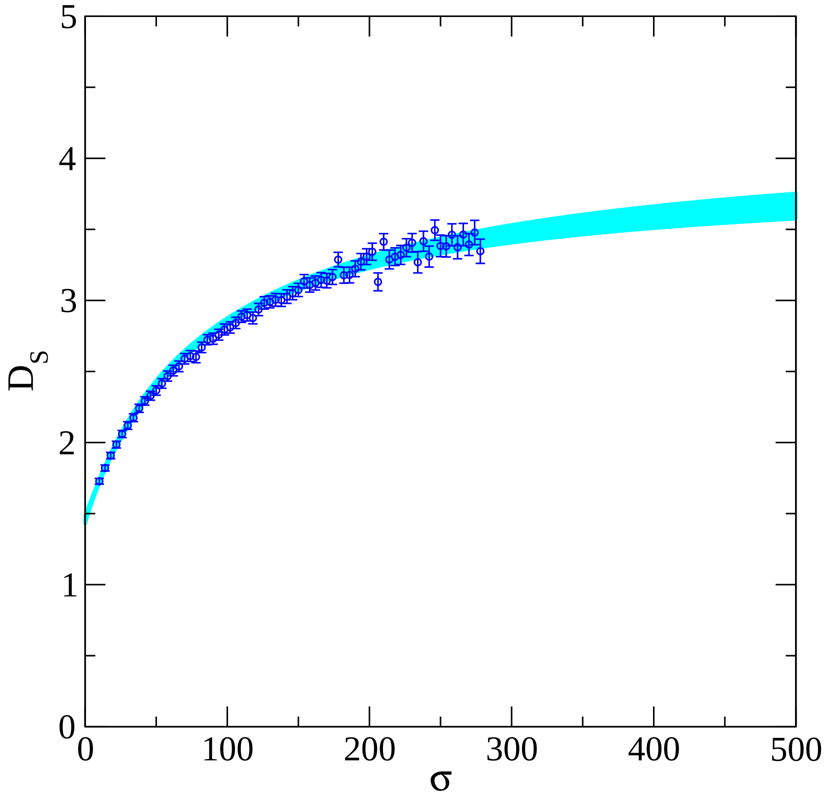
Spectral dimension is defined by a diffusion process

$$D_S(\sigma) = -2\frac{d\log P(\sigma)}{d\log\sigma}, \tag{8}$$

where σ is the diffusion time step on the lattice, and $P(\sigma)$ is the return probability, i.e. the probability of being back where you started in a random walk after σ steps.

Spectral Dimension

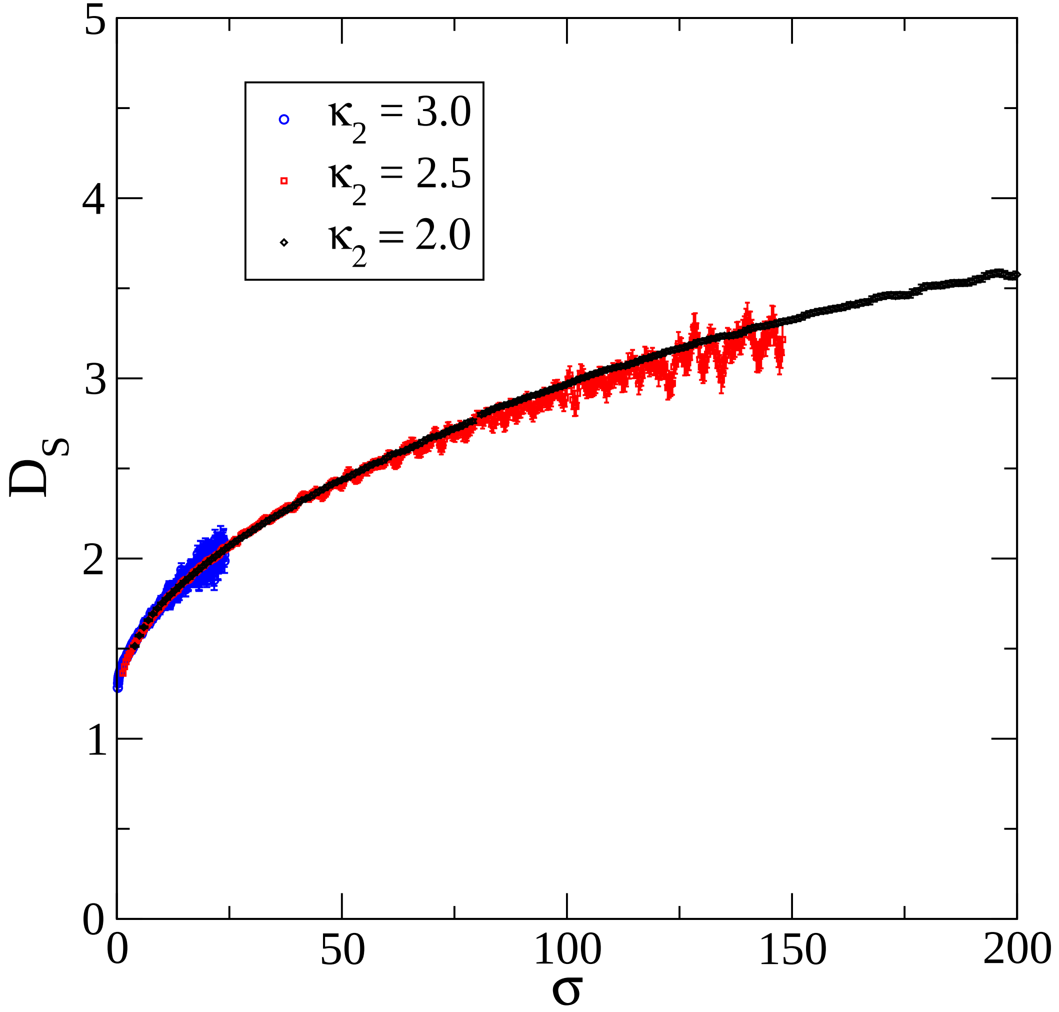
χ^2/dof=35/32, CL=37$\%$
$D_S(\infty) = 4.04 \pm 0.26$, $D_S(0) = 1.457 \pm 0.064$ (includes "fitting" systematic error)



Other Systematic Errors?

- Must go to finer lattice spacings. Still need to explore the phase diagram to understand where the fixed point(s) is(are), so we know how to go to finer lattices. Studies in progress indicate that going to finer lattices is possible, and that the spectral dimension approaches 4/3.

Taking the continuum limit



Comparison with previous results

Note that [Lauscher + Reuter, JHEP 0510, 050 (2005)] use renormalization group methods and find that $D_S(0) = 2$ exactly, independent of any truncation.

Rosten [arXiv:1106.2544] suggests that the value $D_S(0) = 2$ may be an artifact of truncations used so far.

Reuter + Saueressig [arXiv:1110.5224] suggest that the lattice spacing is still too coarse to resolve the actual fixed-point behavior, and that the spectral dimension is non-monotonic, with a more complicated shape and a long plateau at 4/3 that could lead lattice calculations to underestimate the value 2.

CDT result is $D_S(0) = 1.80 \pm 0.25$. [Ambjorn, et. al., PRL 95, 171301 (2005)]

Collapsed phase?

New data suggests that the “extended” phase is actually the collapsed phase, but with especially large finite-size effects.

At large volumes both the Hausdorff and spectral dimensions become large.

Still, at small volumes in the collapsed phase, the behavior of the spectral dimension is strikingly similar to the CDT calculations and RG prediction. Furthermore, it appears that within this phase one can push to arbitrarily fine lattice spacings.

Perhaps the Euclidean theory can be rescued? Tests of new ideas in progress...

Is Asymptotic Safety Correct?

Holographic argument against asymptotic safety due to Banks and Shomer (arXiv:0709.3555):
For a renormalizable theory with an ultraviolet fixed point the theory is a CFT at very high energies. The only dimensionful scale is temperature T, so

$$S \sim (RT)^{d-1}, \quad E \sim R^{d-1} T^{d}, \tag{9}$$

where R is the radius of the spacetime region, S is entropy and E is energy.
This leads to an entropy EOS

$$S \sim E^{\frac{d-1}{d}} \tag{10}$$

Holography and Asymptotic Safety

For gravity one expects that the high energy spectrum will be dominated by black holes. In asymptotically flat d dimensional spacetime, the general (Schwarzschild-Tangherlini) solution to the d dimensional Einstein equations for a spherically symmetric stationary geometry gives a horizon at

$$r^{d-3} \sim G_N M \tag{11}$$

Assuming the Beckenstein-Hawking entropy formula

$$S \sim \text{Area} \sim r^{d-2} \tag{12}$$

This is believed to hold quite generally, and is a consequence of the generalized 2nd law of thermodynamics.
This leads to an entropy EOS

$$S \sim E^{\frac{d-2}{d-3}} \tag{13}$$

Consistent?

$$S \sim E^{\frac{d-1}{d}}, \quad CFT \tag{14}$$

$$S \sim E^{\frac{d-2}{d-3}}, \quad GR \tag{15}$$

The scaling is inconsistent!

However, the effective dimension may change as we go to shorter distances. The spectral dimension is defined by a diffusion process via a heat kernal, and it is the relevant dimension for thermodynamic quantities like entropy scaling on a fractal space (Akkermans et al, PRL 105, 230407, 2010).

Consistent?

$$S \sim E^{\frac{d-1}{d}}, \quad CFT \tag{16}$$

$$S \sim E^{\frac{d-2}{d-3}}, \quad GR \tag{17}$$

For Eq. (21) the relevant dimension is the spectral dimension on a fractal space. Under the assumption that the spectral dimension is also the relevant dimension in the scaling argument for Eq. (22), the scaling becomes consistent when $d = 3/2$.

Future Directions

Evidence for a 2nd order phase transition [Ambjorn, et al, arXiv:1108.3932] and for 4-dimensional de Sitter space in CDT. Can we find the same thing in EDT with appropriate modifications?

Add matter! Scalar and gauge fields straightforward, though previous studies combined with the present work suggest they won't change the phase diagram. Fermions require defining a spin connection, so they are more challenging, but should be possible.

Backup Slides

Update Moves

Update moves are known as (p, q) moves. There are 5, since the last is its own inverse.

$$12346 + 12356 + 12456 + 13456 + 23456 \leftrightarrow 12345 \tag{18}$$

$$12345 + 12346 + 12356 + 12456 \leftrightarrow 13456 + 23456 \tag{19}$$

$$12456 + 13456 + 23456 \leftrightarrow 12345 + 12346 + 12356 \tag{20}$$

Hausdorff dimension

We look at the three-volume correlator,

$$C_{N_4}(\delta) = \sum_{\tau=1}^{t} \frac{4\langle N_3(\tau) N_3(\tau+\delta)\rangle}{N_4^2}, \tag{21}$$

where $N_3(\tau)$ is the total number of four-simplices in a spherical shell a geodesic distance τ from its center, N_4 is the total number of four-simplices in the configuration. The normalization is chosen so that

$$\sum_{\delta=0}^{t-1} C_{N_4}(\delta) = 1, \tag{22}$$

If we rescale δ by N_4^{1/d_H}, introducing $x = \frac{\delta}{N_4^{1/d_H}}$, then $C_{N_4}(x)$ should be independent of volume. Thus, we can determine d_H, the Hausdorff dimension.

Three volume correlator