A fermionic gauge theory for bosonic deconfined criticality

— quantum critical behavior of the QED₃-Gross-Neveu model —

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Thermal critical point (TCP) vs. quantum critical point (QCP)



Thermal:

Quantum:



[[]M Vojta, Rep. Progr. Phys. '03]

... driven by thermal fluctuations

... tuned by temperature

... driven by quantum fluctuations

... tuned by pressure, field, ...

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Thermal:

Quantum:



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... driven by thermal fluctuations

... tuned by temperature

... driven by quantum fluctuations

... tuned by pressure, field, ...

Any significant differences?

TCP vs. QCP: Example

Liquid-gas transition: ... in 3D $T/T_{\rm c}$ Thermosta 1.00 W 0.95 0.90 0.85 0.80 + Ne • A 0.75 ∎ Kr × Xe ∆ N-0.70 v 0₂ 🗆 CO o CH₄ 0.65 0.60 $\rho/\rho_{\rm c}$ 0.55 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 0.8 1.0 Ó) 0.2 0.4 0.6

[Grundpraktikum @ FSU Jena]

Order parameter:

$$|
ho_{
m L}-
ho_{
m G}|\propto |T-T_{
m c}|^{m eta}$$
, $m etapprox 1/3$

[Guggenheim, J. Chem. Phys. '45]

Transverse-field Ising model: ... in 2D

$$\mathcal{H} = -J\sum_{\langle ij \rangle} S_i^z S_j^z - \vec{h} \cdot \sum_i \vec{S}_i$$



Order parameter:

$$|m_z| \propto |J-J_c|^{\beta}, \quad \beta \approx 0.33$$

... and other exponents also agree

[Elliot et al., PRL '70]

TCP vs. QCP: Example

Transverse-field Ising model: Liquid-gas transition: ... in 3D ... in 2D $\mathcal{H} = -J\sum S_i^z S_j^z - \vec{h} \cdot \sum \vec{S}_i$ $T/T_{\rm c}$ Thermosta 1.00 $\langle ij \rangle$ 0.95 0.90 ۲ ک ا 0.85 0.80 + Ne 0.75 0.70 - CO o CH₄ 0.65 0.60 $h/J < (h/J)_c$ $h/J > (h/J)_c$ $\rho/\rho_{\rm c}$ 0.55 1.2 1.4 1.6 1.8 2.0 2.2 2.4 0.8 1.0 0.6 [Grundpraktikum @ FSU Jena]

Quantum-to-classical mapping:

 $TCP(d+z) \iff QCP(d)$

z ... dynamical critical exponent

Landau-Ginzburg-Wilson theory

Assumption:

Transition uniquely characterized by order-parameter fluctuations

Continuum field theory: $S[\phi] = \int d^d \vec{r} \left[\frac{1}{2} (\nabla \phi)^2 + \frac{r}{2} \phi^2 + \lambda \phi^4 + \dots \right]$

 ϕ ... order-parameter field



Renormalization group (Wilson):

Universality \iff Existence of stable RG fixed point

Landau-Ginzburg-Wilson theory: Successes

Ansatz works remarkably well ...

... magnets $(ec{arphi})$

[Wilson & Fisher, PRL '72]

... superconductors (ϕ, ϕ^*, a_μ)

[Halperin, Lubensky, Ma, PRL '74]



... Mott transition in Fermi-point systems $(\vec{\varphi}, \psi^{\dagger}, \psi)$ 2D Dirac: [Herbut, PRL '06] [Raghu, Qi, Honerkamp, Zhang, PRL '08] [Assaad & Herbut, PRX '13] [LJ & Herbut, PRB '14] [Otsuka, Yunoki, Sorella, PRX '16] ... and more

3D QBT: [Herbut & LJ, PRL '14]



Landau-Ginzburg-Wilson theory: Successes



Challenging Landau's paradigm

(1) "Fluctuation-induced" quantum criticality

... Kekulé QCP

... despite the presence of cubic term in S[ϕ] [Li, Jiang, Jian, Yao, Nat. Comm. '17]

[Classen, Herbut, Scherer, PRB '17]

... i.e., mean-field theory becomes invalid



... adjacent phase characterized by topological order ... i.e., no local order parameter

(2) "Topological" quantum criticality

... Kagome spin liquid



[Hastings, PRB '00] [He & Chen, PRL '15] [He *et al.*, PRX '17] ... topological insulator



(3) "Deconfined" quantum criticality

[Senthil, Vishwanath, Balents, Sachdev, Fisher, Science '04]

... continuous order-to-order transition ... characterized by fractionalized excitations

Deconfined quantum criticality

(1) Néel-to-Kekulé transition on honeycomb lattice

... anticommuting masses

[Sato, Hohenadler, Assaad, PRL '17]

 \rightarrow talk by F. Assaad @ SIFT17



... direct continuous transition?

... emergent SO(4)?

(2) Strong-coupling limit $U \rightarrow \infty$:

[Senthil et al., Science '04]







... isotropic!

[Nahum et al., PRL '15]

(2) Strong-coupling limit $U \rightarrow \infty$:

[Senthil et al., Science '04]



Breakdown of Landau-Ginzburg-Wilson

Continuum field theory for Néel phase:

$$S_{\vec{n}} = \frac{1}{2g} \int d^2r d\tau (\partial_{\mu}\vec{n})^2 + S_{\text{B}} \qquad \dots \text{ O(3) nonlinear } \sigma \text{ model}$$

$$\dots \text{ with nonlocal } S_{\text{B}}$$

Néel order parameter: $\vec{n} \propto (-1)^r \vec{S}_r$

Spin Berry phase:

$$S_{\mathsf{B}} = iS\sum_{r}(-1)^{r}A_{r}$$

 A_r ... area enclosed by $\vec{n_r}(\tau)$

r ... lattice site

[Senthil et al., Science '04; PRB '04]

... nonvanishing for monopole events:

... e.g., creation of skyrmion with $Q = \frac{1}{4\pi} \int d^2 r \, \vec{n} \cdot (\partial_x \vec{n} \times \partial_y \vec{n})$

Order parameters: Néel: (n_1, n_2, n_3) VBS: $(\text{Re }\mathcal{M}, \text{Im }\mathcal{M})$

 ${\mathcal M} \dots$ monopole operator





Breakdown of Landau-Ginzburg-Wilson

Continuum field theory for Néel phase:

$$S_{\vec{n}} = rac{1}{2g} \int d^2 r d au (\partial_\mu \vec{n})^2 + S_{
m B}$$
 ... O(3) nonlinear σ model ... with nonlocal $S_{
m B}$

 $\vec{n} \propto (-1)^r \vec{S}_r$ Néel order parameter:

... e.g., creation of skyrmion with $Q = \frac{1}{4\pi} \int d^2 r \, \vec{n} \cdot (\partial_x \vec{n} \times \partial_y \vec{n})$

VBS: (Re \mathcal{M} , Im \mathcal{M}) Néel: (n_1, n_2, n_3) Order parameters:

 \mathcal{M} ... monopole operator



[Senthil et al., Science '04; PRB '04]



Field theory for deconfined criticality

Reformulation:

$$\vec{n} = z^{\dagger}\vec{\sigma}z$$

 $z = (z_1, z_2) \dots$ complex "spinon"

CP¹ model:

$$S_{z} = \int d^{2}\vec{r}d\tau \left[\sum_{\alpha=1,2} |(\partial_{\mu} - ia_{\mu})z_{\alpha}|^{2} - (|z_{1}|^{2} + |z_{2}|^{2})^{2} \right]$$

 a_{μ} ... "photon"

... monopoles = instantons in a_{μ}

Senthil *et al.*:

Monopoles irrelevant at critical point!

[Senthil et al., Science '04; PRB '04]

Natural field theory: noncompact CP¹ model (NCCP¹)



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Natural field theory: noncompact CP¹ model (NCCP¹)

Deconfined QCP = critical point with fractionalized excitations

... with fractionalized excitations being "confined" in either phase



Field theory for deconfined criticality





Deconfined QCP = critical point with fractionalized excitations

... with fractionalized excitations being "confined" in either phase

Alternative formulations of deconfined QCP



Duality conjecture:

[Wang, Nahum, Metlitski, Xu, Senthil, PRX '17]

noncompact CP¹ model
$$\iff$$
 QED₃-Gross-Neveu model
 $(z_1, z_2, z_1^{\dagger}, z_2^{\dagger}, b_{\mu}) \iff (\psi_1, \psi_2, \bar{\psi}_1, \bar{\psi}_2, a_{\mu})$
 $\sum_{\alpha=1,2} |D_b z_{\alpha}| - (|z_1|^2 + |z_2|^2)^2 \iff \sum_{i=1,2} (\bar{\psi}_i \overline{\mathcal{D}}_a \psi_i + \phi \bar{\psi}_i \psi_i) + V(\phi)$
... with $V(\phi)$ tuned to criticality

Explicitly:

Citry:

$$(n_{1}, n_{2}, n_{3}, n_{4}, n_{5}) \sim (2 \operatorname{Re} \mathcal{M}_{b}, 2 \operatorname{Im} \mathcal{M}_{b}, z^{\dagger} \sigma_{x} z, z^{\dagger} \sigma_{y} z, z^{\dagger} \sigma_{z} z)$$

$$\sim \left[\operatorname{Re} (\psi_{1}^{\dagger} \mathcal{M}_{a}), -\operatorname{Im} (\psi_{1}^{\dagger} \mathcal{M}_{a}), \operatorname{Re} (\psi_{2}^{\dagger} \mathcal{M}_{a}), \operatorname{Im} (\psi_{2}^{\dagger} \mathcal{M}_{a}), \phi \right]$$

$$U(2)$$

... naturally explains emergent SO(5)!

... part of "duality web" in 2+1D: [Seiberg, Senthil, Wang, Witten, Ann. Phys. '16] [Karch & Tong, PRX '16] [Thomson & Sachdev, arXiv '17]

•••

Consequences of NCCP¹ \iff QED₃-Gross-Neveu



Predictions for critical behavior:

[Wang, Nahum, Metlitski, Xu, Senthil, PRX '17]

(1)
$$[z^{\dagger}\sigma^{z}z] = [\phi] \Rightarrow \eta_{\text{QED}_{3}-\text{GN}} = \eta_{\text{Néel}} = \eta_{\text{VBS}} \dots \text{ from } \phi \sim z^{\dagger}\sigma^{z}z$$

... $\eta_{\text{Néel}} = \eta_{\text{VBS}} \text{ consistent with QMC}$
[Sandvik, PRL '07]

(2)
$$[z^{\dagger}z] = [\phi^2] \Rightarrow \nu_{\text{QED}_3-\text{GN}} = \nu_{\text{N\'eel-VBS}} \dots \text{from } (\phi^2, \dots) \sim (z^{\dagger}\sigma^z z, z^{\dagger} z, \dots)$$

(3)
$$[\bar{\psi}\sigma^{z}\psi] = [\phi^{2}] \Rightarrow [\bar{\psi}\sigma^{z}\psi] = 3 - 1/\nu_{\text{QED}_{3}}$$
-GN ... from $\bar{\psi}\sigma^{z}\psi \sim z^{\dagger}z$
... nontrivial prediction fully within QED₃-GN

.. allows quantitative test of duality conjecture

Here: (a) Existence of QCP in QED₃-GN model? ... prerequisite for duality (b) Critical behavior? ... & comparison with NCCP¹

QED₃-Gross-Neveu model: GN limit

Lagrangian:

$$\mathcal{L}_{\psi\phi} = \sum_{i=1,2} \left[\bar{\psi}_i (\partial_\mu - i\phi_\mu) \gamma_\mu \psi_i + g\phi \bar{\psi}_i \psi_i \right] + \frac{1}{2} \phi(r - \partial_\mu^2) \phi + \lambda \phi^4$$

... in $D = 2+1$
... $i = 1,..., 2N$

Gross-Neveu limit $(e^2 \rightarrow 0)$:



Gross-Neveu theory $\lim_{r \to \infty} D = 2+\varepsilon$ $g\phi\bar{\psi}\psi + \frac{r}{2}\phi^2 \sim -\frac{g^2}{r}(\bar{\psi}\psi)^2$ UV fixed point $(g^2/r)_c \qquad g^2$

GN-QCP exists for all 2 < D < 4 and can be understood as either IR fixed point of GNY or ... UV fixed point of GN

[Zinn-Justin, NPB '91] [Braun, Gies, D Scherer, PRD '11] [LJ, Herbut, PRB '14]

QED₃-Gross-Neveu model: QED₃ limit

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[LJ, PRD '16]





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QED₃ limit $(g \rightarrow 0)$:



[LJ, PRD '16]

... QED₃ (potentially) unstable at low N! \rightarrow talk by I. Herbut @ SIFT17

[Appelquist, Nash, Wijewardhana, PRL '88] [Braun, Gies, LJ, Roscher, PRD '14] [Di Pietro *et al.*, PRL '16] [Herbut, PRD '16]

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QED₃ limit $(g \rightarrow 0)$: [LJ, PRD '16] 1000 $N_{\rm c}^{\rm conf} (2 + \epsilon \, \exp.)$ $8\sqrt[3]{2}$ Relevant for QED₃-GN? Lorentz 10 symmetry breaking chiral symmetry breaking (allowed) 2.52.03.53.04.0d

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[Appelquist, Nash, Wijewardhana, PRL '88] [Braun, Gies, LJ, Roscher, PRD '14] [Di Pietro *et al.*, PRL '16] [Herbut, PRD '16]

QED₃-GN model: Fermionic RG

Integrate out ϕ :

$$g\phi\bar{\psi}_i\psi_i+rac{r}{2}\phi^2\mapsto u(\bar{\psi}_i\psi_i)^2$$

... u will also generate other four-fermion terms

General four-fermion theory compatible with U(2N):

[Gies & LJ, PRD '10]

$$\mathcal{L}_{m{\psi}} = ar{\psi}_i \gamma_\mu (\partial_\mu - i a_\mu) \psi + u (ar{\psi}_i \psi_i)^2 + v (ar{\psi}_i \gamma_\mu \psi_i)^2$$

One-loop RG:

... at large N

$$\frac{du}{d\ln b} = -u - 8u^2 + 2e^4$$



QED₃-GN model: Fermionic RG

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One-loop RG: $\frac{du}{d \ln b} = -u - 8u^{2} + 2e^{4}$... gauge fluctuations stabilize QED₃-GN fixed point!

... in contrast to QED₃-Thirring fixed point

Fermionic RG: Flow diagram

[LJ & Y-C He, PRB '17]





u

QED₃-GN model: 4-*ε* expansion

Lagrangian:

$$\mathcal{L}_{\psi\phi} = \sum_{i=1,2} \left[\bar{\psi}_i (\partial_\mu - i e a_\mu) \gamma_\mu \psi_i + g \phi \bar{\psi}_i \psi_i \right] + \frac{1}{2} \phi (r - \partial_\mu^2) \phi + \lambda \phi^4$$

Engineering dimensions:

$$[e^2] = 4 - D,$$
 $[g] = \frac{4 - D}{2},$ $[\lambda] = 4 - D$

... become simultaneously marginal near D = 3+1!

$$\varepsilon$$
 expansion in $D = 4 - \varepsilon$ possible!

QED₃-GN model: Flow diagram in $D = 4-\varepsilon$



... fully IR stable fixed point

[LJ & Y-C He, PRB '17]

QED₃-GN model: Critical exponents

 $D = 4 - \epsilon$:

$$\begin{split} \eta_{a} &= \epsilon \\ \eta_{\phi} &= \frac{2N+9}{2N+3}\epsilon + \mathcal{O}(\epsilon^{2}) \\ \nu &= \frac{1}{2} + \frac{10N^{2}+39N+f(N)}{24N(2N+3)}\epsilon + \mathcal{O}(\epsilon^{2}) \\ [\bar{\psi}\sigma^{z}\psi] &= 3 - \frac{2N+6}{2N+3}\epsilon + \mathcal{O}(\epsilon^{2}) \\ & \dots \text{ with } f(N) \equiv \sqrt{4N^{4}+204N^{3}+1521N^{2}+2916N} \\ \dots \text{ large } \mathcal{O}(\epsilon) \text{ corrections} \end{split}$$

Combine with results from $D = 2 + \epsilon$:

$$egin{aligned} \eta_{\phi} &= 2 - (D-2) + \mathcal{O}(1/N,(D-2)^2) \ 1/
u &= (D-2) + \mathcal{O}(1/N,(D-2)^2) \ [ar{\psi}\sigma^z\psi] &= 1 + (D-2) + \mathcal{O}(1/N,(D-2)^2) \end{aligned}$$

QED₃-GN model: Critical exponents

[LJ & Y-C He, PRB '17]

N = 1:

dashed: *ɛ*-expansion results

solid: interpolation



... error: difference to plain extrapolation

Comparison: QED₃-GN vs. NCCP¹

QED_3 -GN	$SU(2) NCCP^1$	
$[\phi] \approx (1 + 1.3(9))/2$	$[z^{\dagger}\sigma^{z}z] \approx (1+0.26(3))/2$	[Sandvik, PRL '07]
	$\approx (1 + 0.35(3))/2$	[Melko & Kaul, PRL '08]
	$\approx (1 + 0.25(3))/2$	[Nahum <i>et al.</i> , PRX '15]
	$\approx (1+0.22)/2$	[Bartosch, PRB '13]
$\left[\bar{\psi}\sigma^z\psi\right] \approx 3 - 1.2(5)$	$[z^{\dagger}z] \approx 3 - 1.28(5)$	[Sandvik, PRL '07]
	$\approx 3 - 1.47(9)$	[Melko & Kaul, PRL '08]
	$\approx 3 - 1.99(4)$	[Nahum <i>et al.</i> , PRX '15]
	$\approx 3 - 1.79$	[Bartosch, PRB '13]
$[\phi^2] \approx 3 - 0.3(4)$	$[z^{\dagger}z] \approx$ —see above-	

 $\cdots [\phi] \sim [z^{\dagger} \sigma^z z] = (1+\eta)/2$ $\cdots [\bar{\psi} \sigma^z \psi] \sim [\phi^2] = 3 - 1/
u$

... $[\bar{\psi}\sigma^z\psi]$ not inconsistent, but large deviations for $[\phi]$ and $[\phi^2]$

Conclusions

 QED_3 -Gross-Neveu model ...

... interesting due to possible duality with NCCP1 $$\dots$$ i.e., theory of Néel-VBS deconfined critical point

... has a stable fixed point ... even when cQED₃ collides with another QCP ... prerequisite for duality to hold

... critical behavior computable within 4- ε expansion ... all couplings simultaneously marginal

... but higher orders necessary to test of duality

... large anomalous dimension η_{ϕ}

... however: large η_{ϕ} necessary for emergent SO(5)

[Nakayama & Ohtsuki, PRL '16]

→ poster by B. Ihrig @ SIFT17