Lattice simulations and extensions of the standard model with strong interactions

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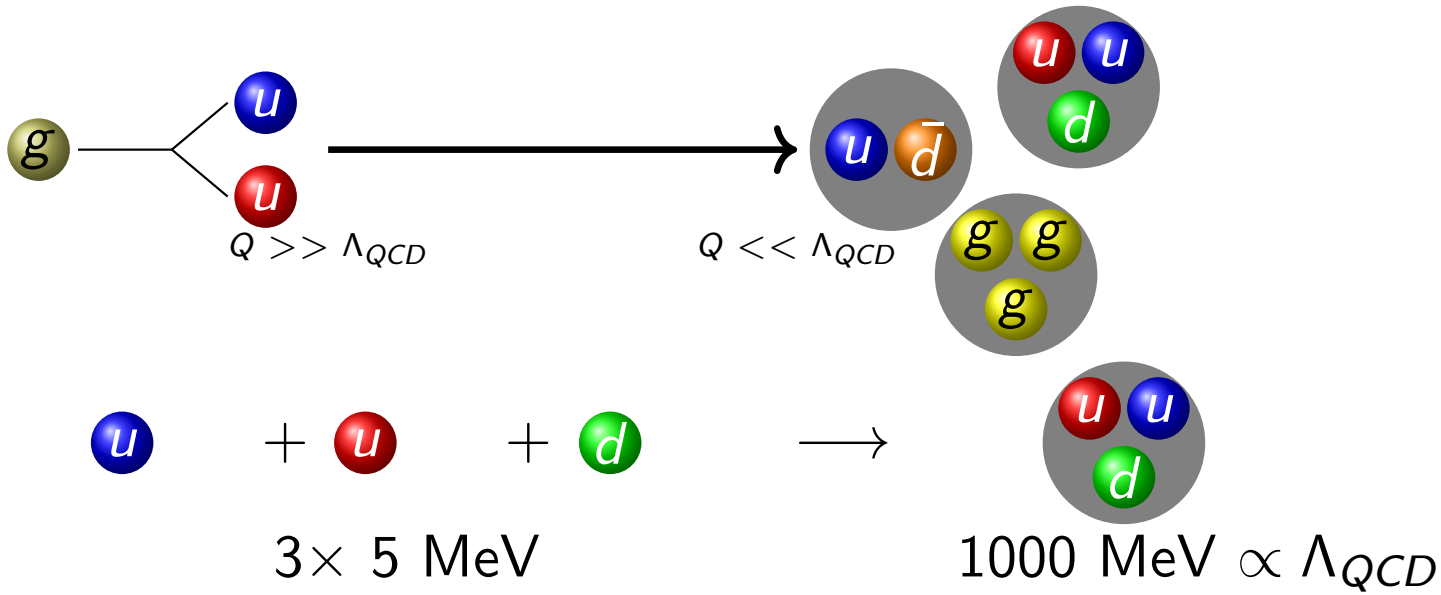
Strong interactions

Features of strong interactions:

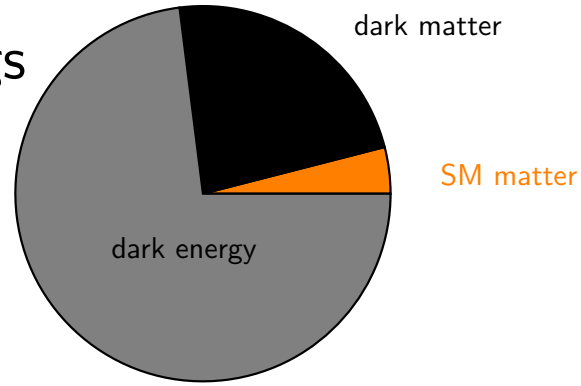
- formations of bound states of hadrons and nuclear matter
- create most of the mass of the bayonic matter in the universe
- dynamical scale generation

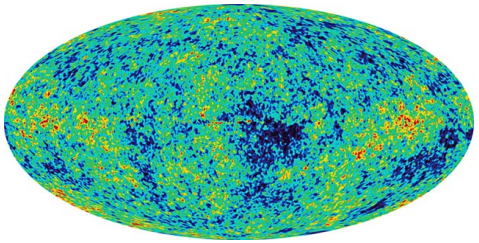


The extension/completion/explanation of the Standard Model

Why extend the standard model of particle physics:

- add missing explanation for Dark Matter
- solve hierarchy problem and explain small Higgs mass
- explain standard model parameters: flavour, mass parameters / Yukawa couplings
- may other motivations: neutrino physics, quantum gravity, unification of forces





The extension/completion/explanation of the Standard Model

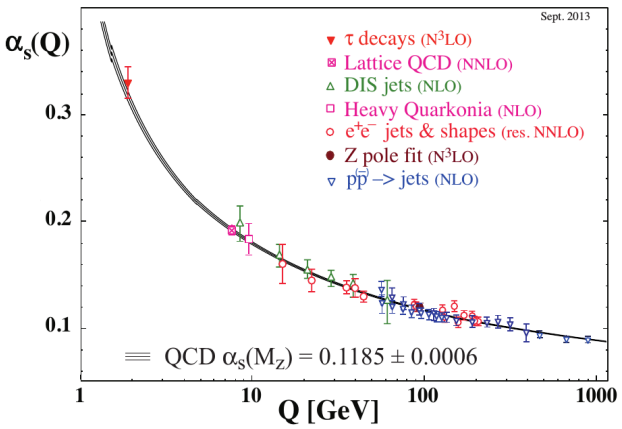
General paradigms for the introduction of new physics:

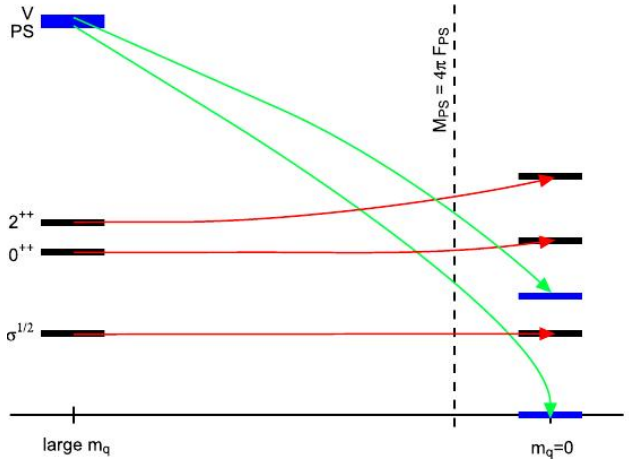
1. new physics means new parameters and scales,
   less "explanation"
2. new physics adds natural explanation of scales by dynamics or symmetry
3. scales and parameters are explained by unknown quantum gravity

Strong interactions: dynamic scale generation

Massless QCD has no free parameters:

- hadronic bound states are dynamical generated
- scale of the masses not a free parameter of the theory
- theory explains, not parametrizes, the masses
- generates naturally large scale separation
  massless pions (Goldstone bosons),
  massive bound states





[B. Lucini]

Specific extensions of the Standard Model

Composite or Technicolour models:

- new strong dynamics beyond the standard model
- Higgs generated as bound state, natural due to scale of additional strong interactions

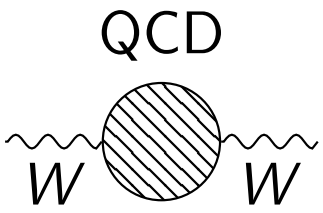
Supersymmetry:

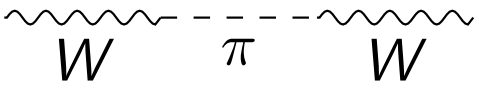
- natural explanation by symmetry
- Higgs mass corrections canceled by fermionic partners
- interesting theoretical concept

The old idea of Technicolour

QCD + weak interactions without Higgs

- QCD corrections to W and Z propagators
- dominant contribution in massless limit: pion propagator
- effectively generates mass for W and Z bosons





$$m_W^2 = \frac{1}{4} g f_\pi^2$$

W mass correction by pion decay constant [Weinberg '76],[Susskind '79]

$$SU(N_{TC}) \times SU(3) \times SU(2)_L \times U_Y$$

Two TC flavours: $\pi_T^\pm$,π_T^0 become longitudinal part of W, Z

The new idea of Technicolour or Composite models

Problems:

- Higgs particle observed
- Higgs generates fermion masses via Yukawa interactions

Requirements:

- need light Higgs particle, large scale separation to other states
- need effective interactions with via Extended Technicolour

Approaches:

1. Walking Technicolour approach
2. Composite models
3. Partial compositeness

The challenges for the constructions of Technicolour or Composite models

You have to explain:

- fermion mass generation
- electroweak precision data
- flavour
- Higgs mass
- ...

Since everything should be naturally generated by strong interactions you have to explain everything at once ... May be this is not a good idea.

Supersymmetry

- only possible non-trivial extension of the space-time symmmetries (Coleman-Mandula theorem)
- supersymmetry: every bosonic particle has fermionic partner with same mass

Where are the partner particles?

- supersymmetry must be broken at lower scales
- without knowledge of breaking mechanism: many possible breaking terms and parameters, less predictive

Strong interactions from lattice simulations

- still no analytic method to compute low energy features of strongly interacting theories
- first principles method: Lattice simulations

Well established method for the investigations of QCD, but can we apply it to other theories?

- QCD: we have a good feeling about parameters and uncertainties
- BSM: we need to redo and even re-think the analysis

Why needs Supersymmetry non-perturbative methods?

Two main motivations:

1. Supersymmetric particle physics requires breaking terms based on an unknown non-perturbative mechanism.
   $\Rightarrow$ need to understand non-perturbative SUSY
2. Supersymmetry as theoretical concept: (Extended) SUSY simplifies theoretical analysis and leads to new non-perturbative approaches.
   $\Rightarrow$ need to bridge the gap between "beauty" and **"reality"**

The challenges for supersymmetry on the lattice

- discretize, cutoff in momentum space
- (controlled) breaking of space-time symmetries
  $\Rightarrow$ uncontrolled SUSY breaking
- derivative operators replaced by difference operators with no Leibniz rule
  $\Rightarrow$ breaks SUSY
- fermonic doubling problem, Wilson mass term
- Nielsen-Ninomiya theorem:
  No-Go for (naive) lattice chiral symmetry
  $\Rightarrow$ breaks SUSY
- gauge fields represented as link variables
  $\Rightarrow$ different for fermions and bosons

[G.B., S. Catterall, arXiv:1603.04478]

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$\mathcal{N}=1$ supersymmetric Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\lambda}\not{D}\lambda - \frac{m_g}{2}\bar{\lambda}\lambda\right]$$

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- λ Majorana fermion in the adjoint representation

Supersymmetric Yang-Mills theory: Symmetries

SUSY

- gluino mass term $m_g \Rightarrow$ soft SUSY breaking

$U_R(1)$ symmetry, "chiral symmetry": $\lambda \to e^{-i\theta\gamma_5}\lambda$

- $U_R(1)$ anomaly: $\theta = \frac{k\pi}{N_c}$, $U_R(1) \to \mathbb{Z}_{2N_c}$
- $U_R(1)$ spontaneous breaking: $\mathbb{Z}_{2N_c} \overset{\langle\lambda\lambda\rangle\neq 0}{\to} \mathbb{Z}_2$

Supersymmetric Yang-Mills theory: effective actions

symmetries + confinement $\rightarrow$ low energy effective theory

- low energy effective actions:
  1. multiplet[1]:
  mesons : $a - f_0$ and $a - \eta'$
  fermionic gluino-glue
- 2. multiplet[2]:
  glueballs: 0^{++} and 0^{-+}
  fermionic gluino-glue

Supersymmetry

All particles of a multiplet must have the same mass.

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]

[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

Supersymmetric Yang-Mills theory on the lattice

Lattice action:

$$\mathcal{S}_L = \beta \sum_P \left(1 - \frac{1}{N_c} \Re U_P\right) + \frac{1}{2} \sum_{xy} \bar{\lambda}_x \left(\mathrm{D}_w(m_g)\right)_{xy} \lambda_y$$

- Wilson fermions:

$$\mathrm{D}_w = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)_{\alpha,\beta} T_\mu + (1+\gamma_\mu)_{\alpha,\beta} T_\mu^\dagger\right]$$

gauge invariant transport: $T_\mu \lambda(x) = V_\mu \lambda(x + \hat{\mu})$;

$$\kappa = \frac{1}{2(m_g + 4)}$$

- links in adjoint representation: $(V_\mu)_{ab} = 2\mathrm{Tr}[U_\mu^\dagger T^a U_\mu T^b]$ of $SU(2)$, $SU(3)$

Recovering symmetry

Fine-tuning (Veneziano, Curci)[1]:

chiral limit = SUSY limit $+O(a)$, obtained at critical $\kappa(m_g)$

Wilson fermions:

- explicit breaking of symmetries: ~~chiral Sym. ($U_R(1)$)~~, ~~SUSY~~
- restored in chiral continuum limit

practical determination of critical κ:

- limit of zero mass of adjoint pion $(a-\pi)$

$\Rightarrow$ definition of gluino mass: $\propto (m_{a-\pi})^2$

[1] [Veneziano, Curci, Nucl.Phys.B292 (1987)]

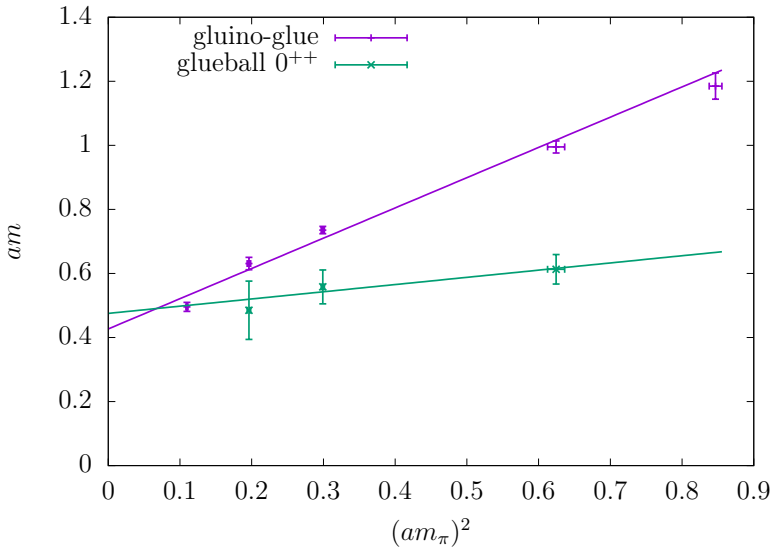
Supersymmetric Yang-Mills theory: Scale and mass determination

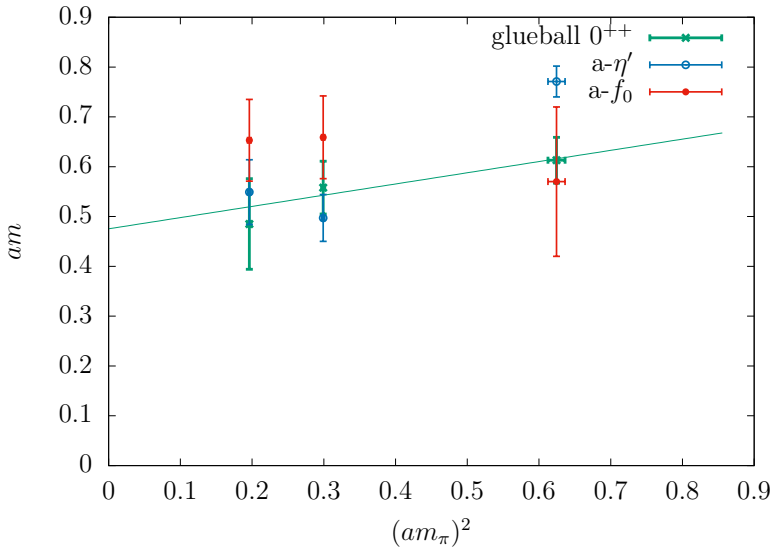
- scale setting:
  - parameters: r_0, w_0
  - can be used for comparison to Yang-Mills theory, QCD
- meson states (singlet)
  - a–f_0 ($\bar{\lambda}\lambda$), a–η' ($\bar{\lambda}\gamma_5\lambda$)
  - methods for disconnected contributions [PoS LATTICE 2011]
- glueballs
- spin-1/2 state (gluino-glue)

$$\sum_{\mu,\nu} \sigma_{\mu\nu} \mathrm{tr}\left[F^{\mu\nu}\lambda\right]$$

Technical challenge: Unfortunately all states are quite challenging to determine!

The particle masses of $\mathcal{N} = 1$ SU(3) supersymmetric Yang-Mills theory





- situation similar to the SU(2) case with comparable lattice spacing
- indication for a multiplet formation and *restored SUSY* in chiral limit

The status of the project

- verification of VC scenario: chiral symmetry breaking and supersymmetric Ward identities
- multiplet formation found in the continuum limit of SU(2) SYM [JHEP 1603, 080 (2016)]
- SYM thermodynamics: coincidence of chiral and deconfinement transition [JHEP **1411**, 049 (2014)]
- SYM compactification: Witten index and absence of deconfinement [JHEP **1412**, 133 (2014)]
- first indication of multiplet formation in the excited states
- first SU(3) results with clover improved Wilson fermions

SU(3) very similar to SU(2) case: Multiplet formation.

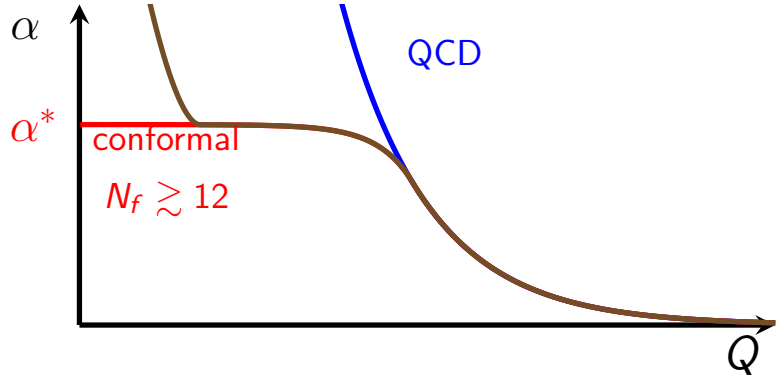
Walking Technicolour and the conformal window

Walking Technicolour scenario:

- near conformal running of the gauge coupling to accommodate fermion mass generation and absence of FCNC
- approximate conformal symmetry might also lead to natural light scalar particle (Higgs)

Interesting general question:

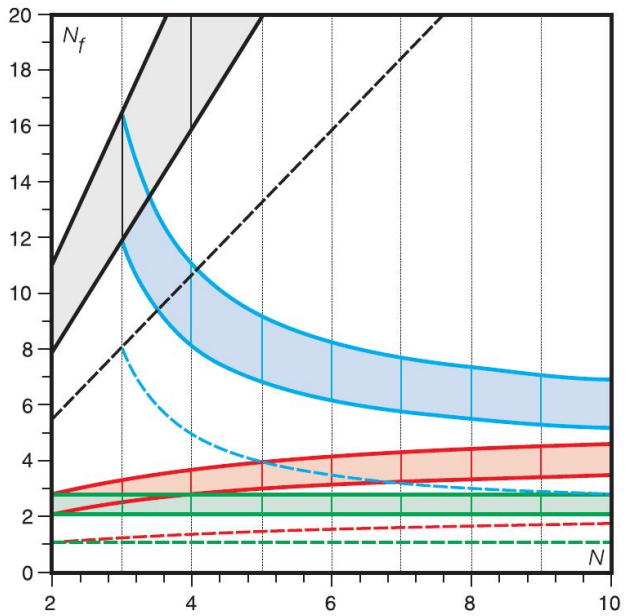
- conformal window, strong dynamics different from QCD
- conformal mass spectrum: $M \sim m^{1/(1+\gamma_m)}$ characterised by constant mass ratios



Conformal window for adjoint QCD

Gauge theories in higher representation:

- smaller number of fermions needed
- here: conformal window for adjoint representation
- mass anomalous dimension $\gamma_*(N_f)$



[Dietrich, Sannino, hep-ph/0611341]

Adjoint QCD

adjoint N_f flavour QCD:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_i^{N_f} \bar{\psi}_i(\not{D} + m)\psi_i\right]$$

$$D_\mu\psi = \partial_\mu\psi + ig[A_\mu, \psi]$$

- ψ Dirac-Fermion in the adjoint representation
- adjoint representation allows Majorana condition $\psi = C\bar{\psi}^T$

$\Rightarrow$ half integer values of N_f: $2N_f$ Majorana flavours

Chiral symmetry breaking:

$$Z_{2N_c} \times \mathrm{SU}(2N_f) \to Z_2 \times \mathrm{SO}(2N_f)$$

Particle states and lattice action

Lattice action

- Wilson fermion action + stout smearing
- tree level Symanzik improved gauge action
- some results: clover improvement

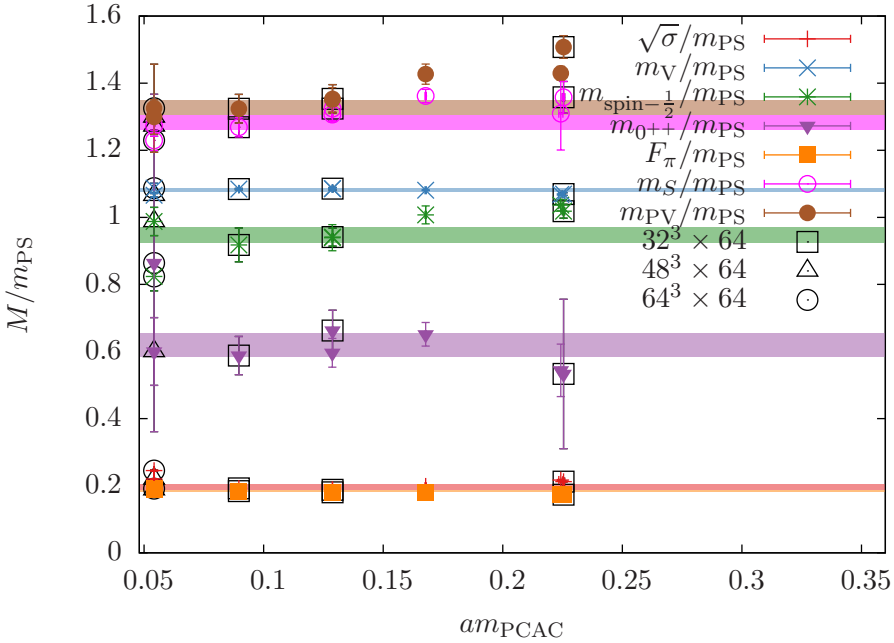
Particle states

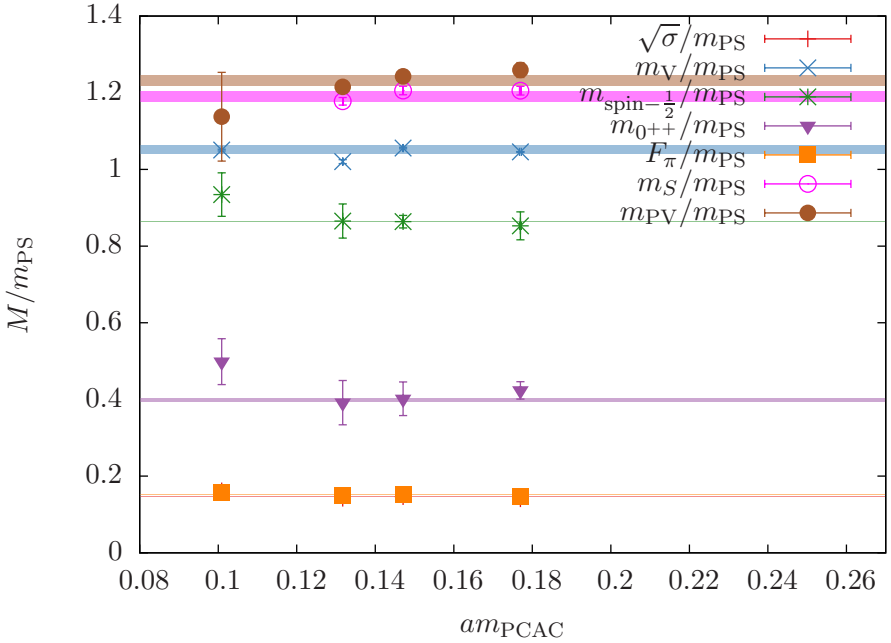
- triplet mesons m_{PS}, m_{S}, m_{V}, m_{PV}
- glueball 0^{++}
- spin-$1/2$ mixed fermion-gluon state

$$\sum_{\mu,\nu} \sigma_{\mu\nu} \mathrm{tr}\left[F^{\mu\nu}\lambda\right]$$

- singlet mesons $m_{\mathrm{a}-f_0}$, $m_{\mathrm{a}-\eta'}$

$N_f = 2$ AdjQCD, Minimal Walking Technicolour





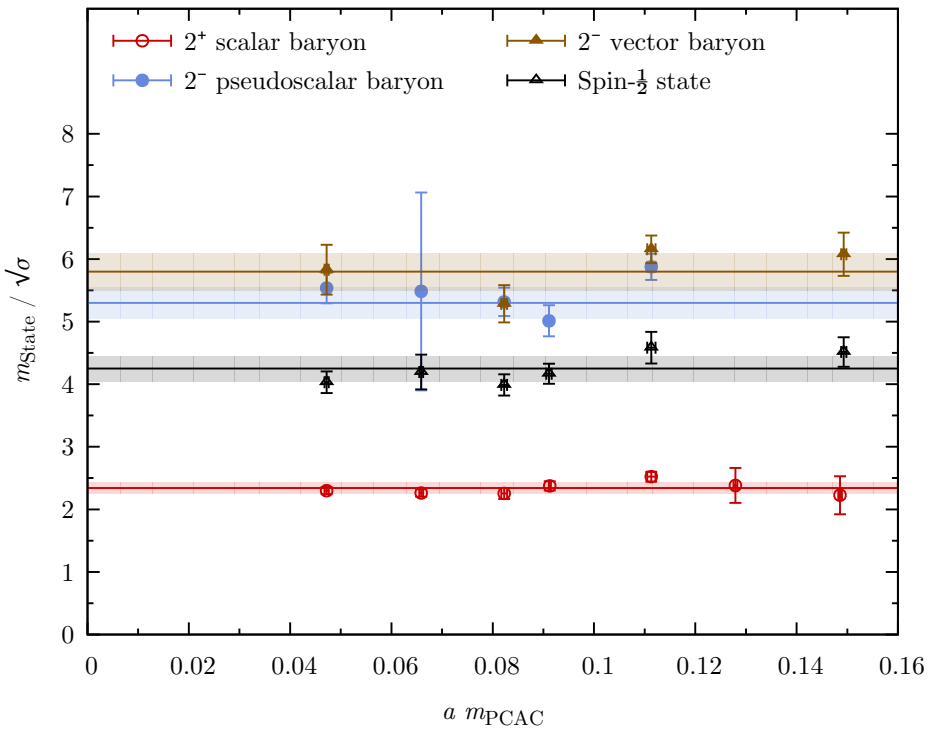
Expected behaviour of a (near) conformal theory:

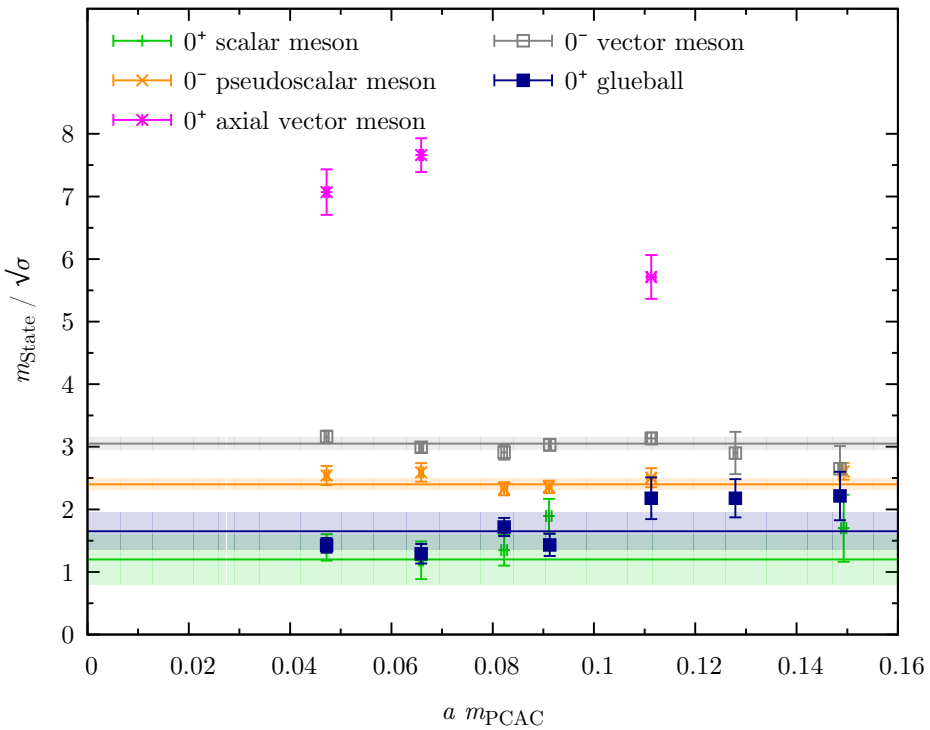
- constant mass ratios
- light scalar (0^{++})
- no light Goldstone ($m_{\rm PS}$)

Well established results: [Debbio, Lucini, Patella, Pica, 2016],

[GB, Giudice, Münster, Montvay, Piemonte, 2017]

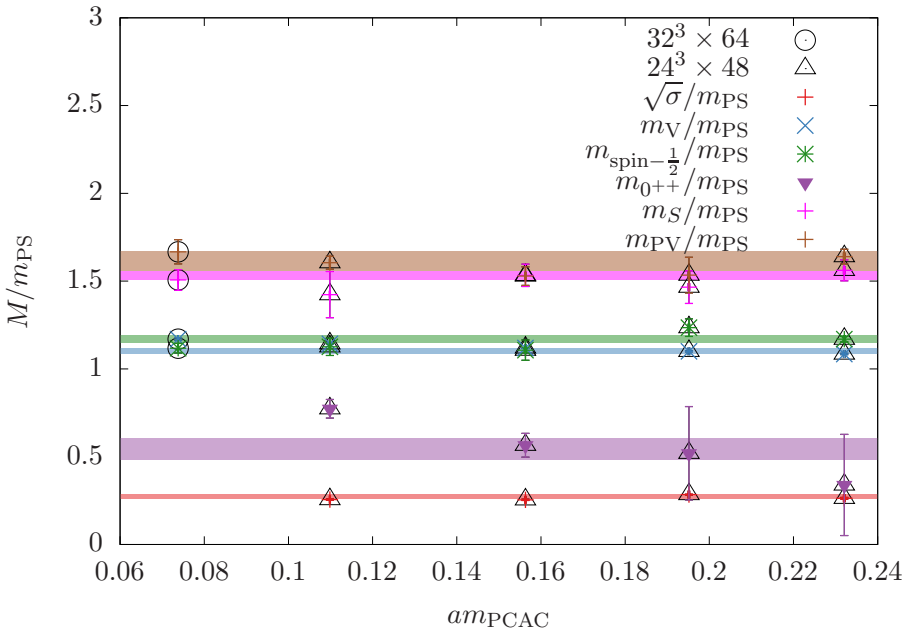
Results for $N_f = 1$ adjoint QCD

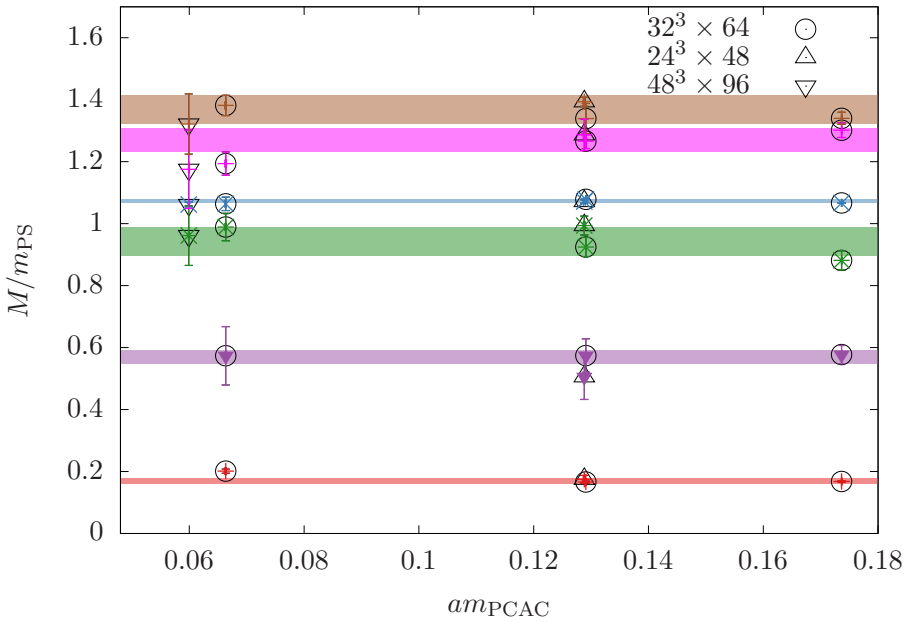




[Athenodorou, Bennett, GB, Lucini, arXiv:1412.5994]

Results for $N_f = 3/2$ adjoint QCD

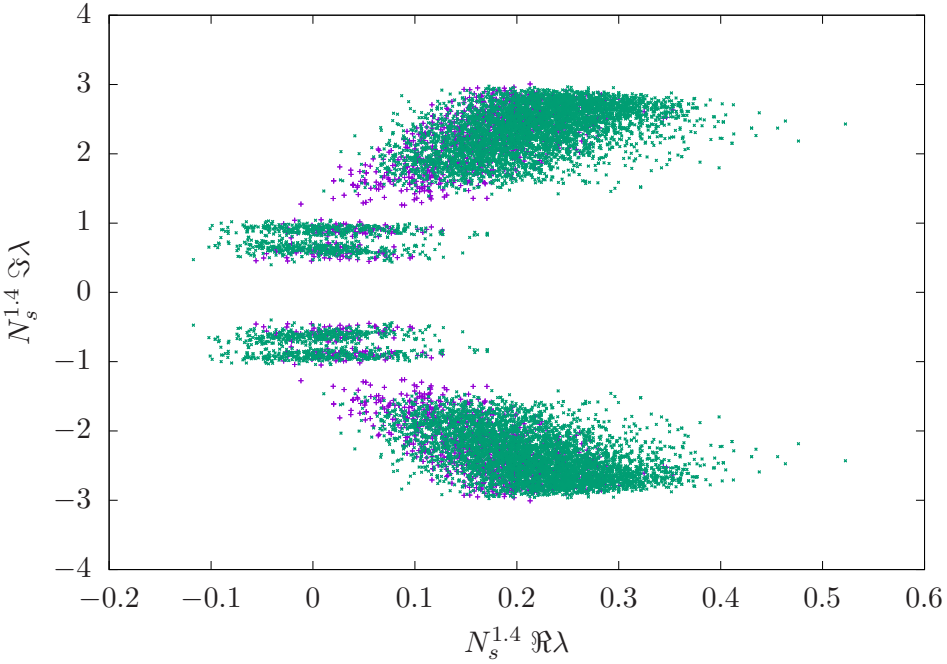




N_f Dirac fermions $\rightarrow$ $2N_f$ Majorana fermions

- with our experience of supersymmetric Yang-Mills theory, we can simulate half integer fermion numbers
- requires the Pfaffian sign

Pfaffian in $N_f = 3/2$ adjoint QCD



- even at the critical parameters: no sign fluctuations of Pfaffian

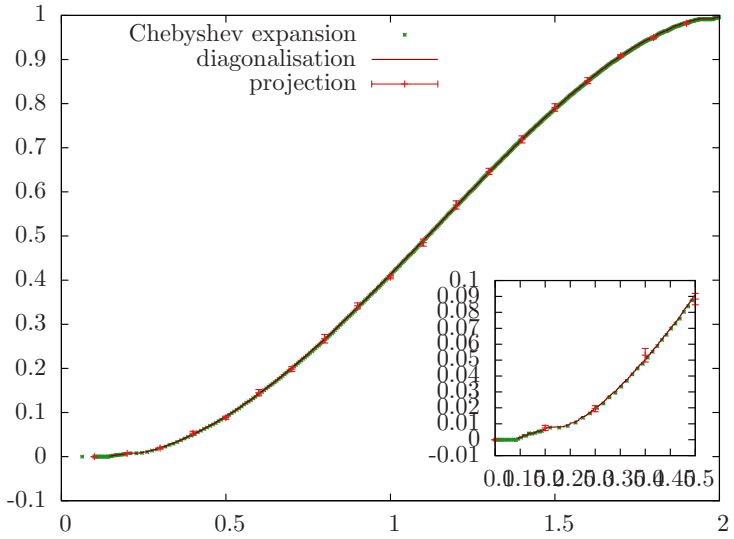
Mass anomalous dimension for Minimal Walking Technicolour: Methods

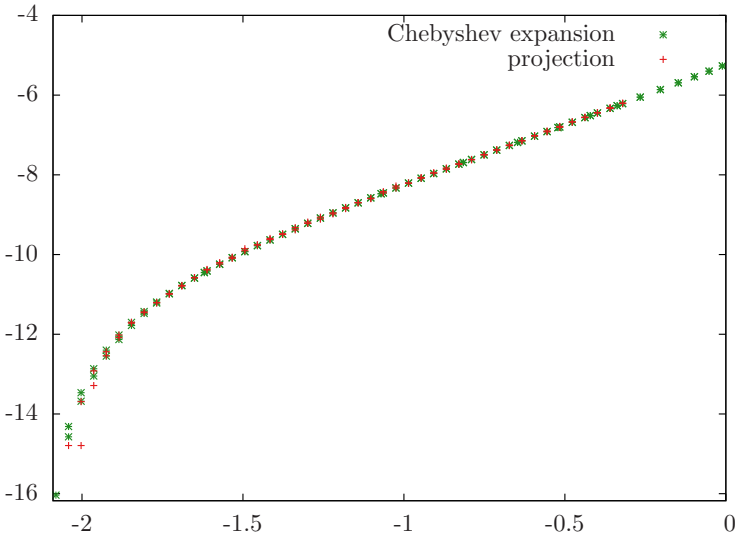
Methods for determination of γ_*:

- scaling of mass spectrum
- mode number (integrated spectral density of $D^\dagger D$)

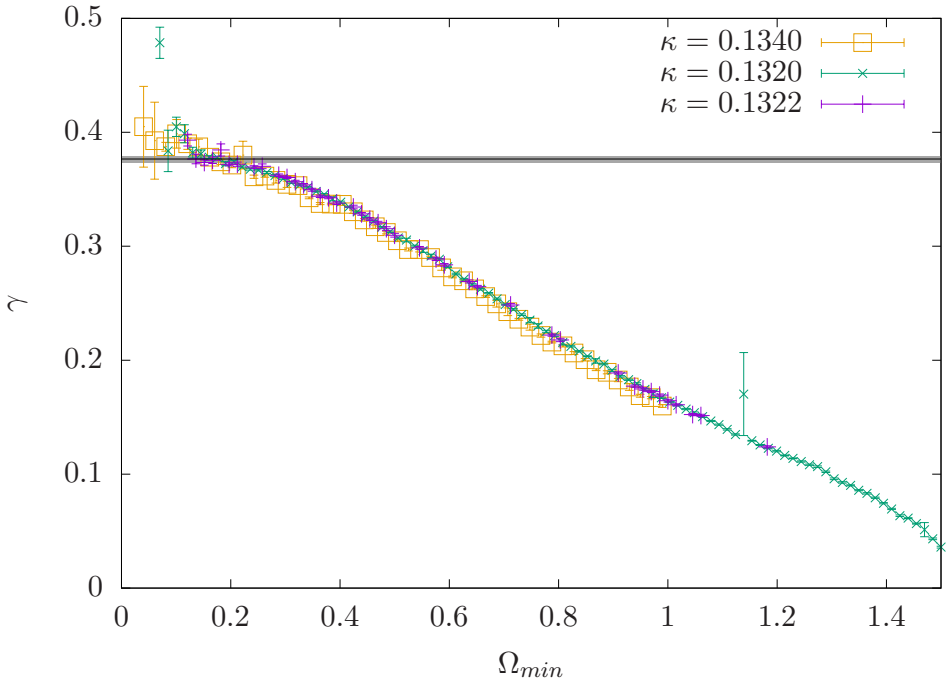
Methods for mode number determination:

- Chebyshev expansion of the spectral density
- consistency with [Giusti, Lüscher, 0812.3638] checked





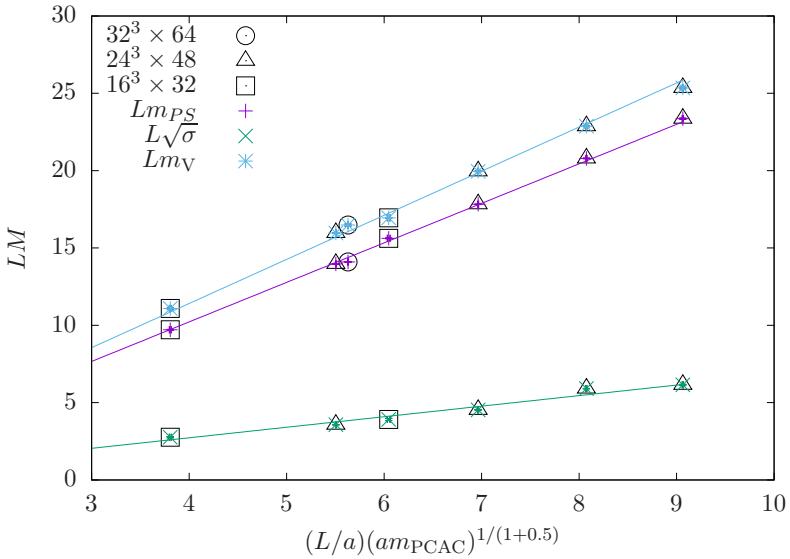
Mass anomalous dimension for $N_f = 3/2$

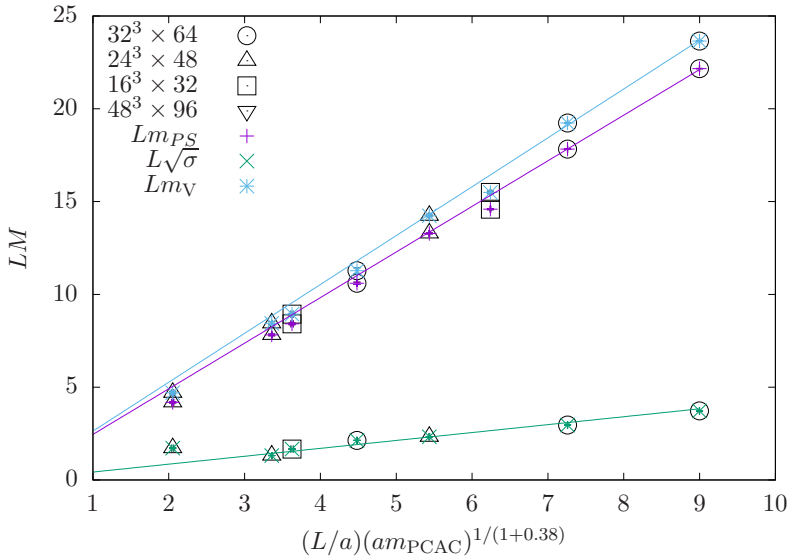


$$\nu(\Omega) = 2\int_0^{\sqrt{\Omega^2 - m^2}} \rho^H(\lambda) d\lambda \; ; \quad \nu(\Omega) = \nu_0 + a_1(\Omega^2 - a_2^2)^{\frac{2}{1+\gamma}}$$

- mass anomalous dimension around 0.38(2)

Mass anomalous dimension for $N_f = 3/2$





- check with the hyperscaling of the spectrum

Comparison with $N_f = 1$ and $N_f = 1/2$

Theory	**scalar particle**	γ_* small β	γ_* larger β
$N_f = 1/2$ SYM	part of multiplet	–	–
$N_f = 1$ adj QCD	light	0.92(1)	0.75(4)*
$N_f = 3/2$ adj QCD	light	0.50(5)*	0.38(2)*
$N_f = 2$ adj QCD	light	0.376(3)	0.274(10)

($*$ preliminary)

- remnant β dependence: γ_* not real IR fixed point values
- final results require inclusion of scaling corrections
- investigation of (near) conformal theory requires careful consideration of lattice artefacts and finite size effects

What can we learn for realistic models?

Next to Minimal Walking Technicolour: $N_f = 2$ in sextet representation [Bergner, Ryttov, Sannino, 1510.01763]

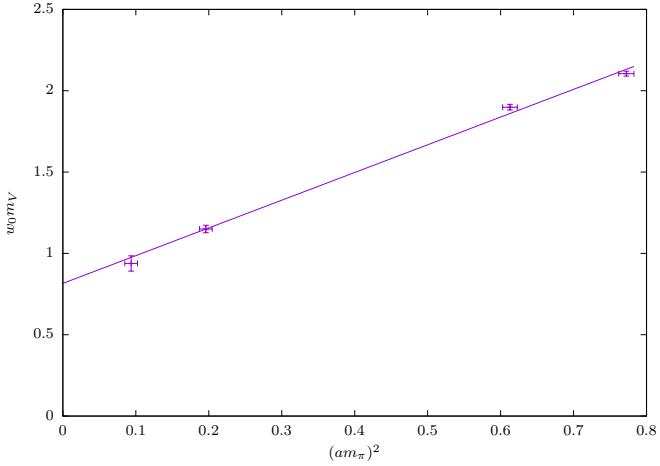
- conformal window for adjoint fermions approximately independent of N_c
- large N_c up to factor 2 equivalence between symmetric, adjoint, and antisymmetric representation
- small $N_c = 2$ equivalence between symmetric and adjoint representation
- conformal behaviour of $N_f = 1$ indicates conformality of NMWT

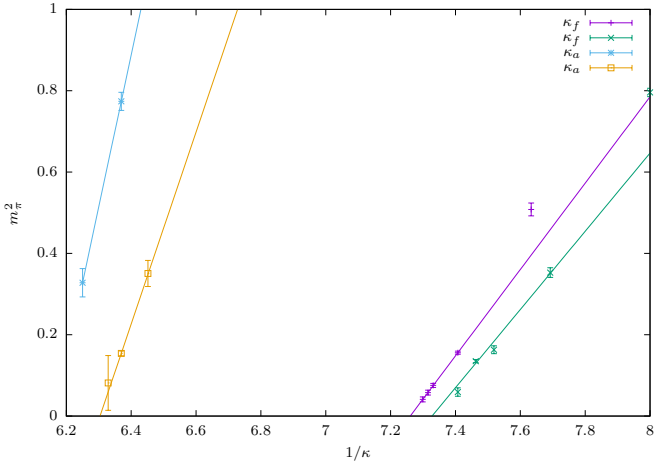
Towards more realistic theories: Ultra Minimal Walking Technicolour

- mass anomalous dimension too small in MWT to be a realistic candidate
- $N_f = 1$ has large mass anomalous dimension, but not the required particle content
- $N_f = 1$ in adjoint + $N_f = 2$ in fundamental representation of SU(2) has been conjectured to be ideal candidate (UMWT)
- $N_f = 1/2$ adjoint + $N_f = 2$ in fundamental might also be close enough to conformality
- interesting also for further investigations: SQCD without scalar fields
- test of mixed representation setup for partial compositeness models

First investigations in mixed representation setup

- difficult tuning: two bare mass parameters have to be tuned
- one-loop clover improved action with Wilson fermions in fundamental and adjoint representations
- investigate the influence of the adjoint fermions on fundamental sector





Conclusions and outlook

- interesting strongly interacting extensions of the Standard Model with dynamics different from QCD
- close connection between the investigations of conformal, or near conformal models and supersymmetric theories
- $\mathcal{N} = 1$ SU(3) supersymmetric Yang-Mills theory shows supersymmetric particle spectrum
- interesting new investigations: adjoint+fundamental SU(2), adjoint $N_f = 1$ coupled to scalar