Fluid description for high-energy nuclear collisions starting before the collisions

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The QCD Phase Transition, Physikzentrum Bad Honnef
April 04, 2023.

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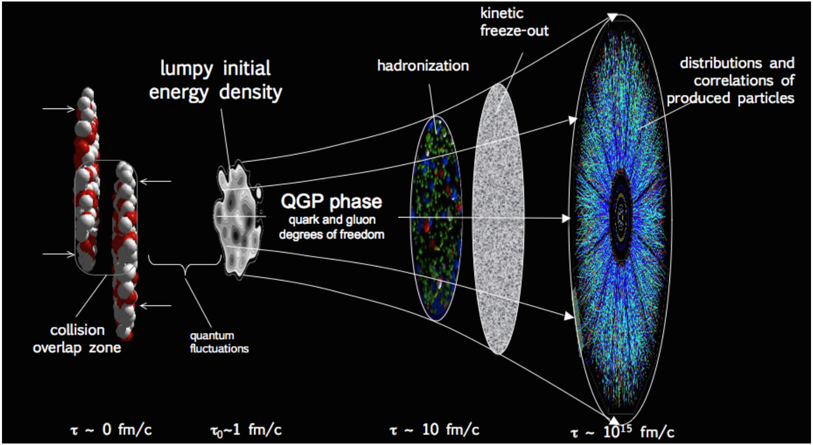
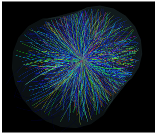
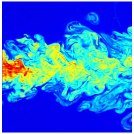
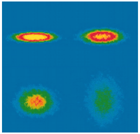
High energy nuclear collisions



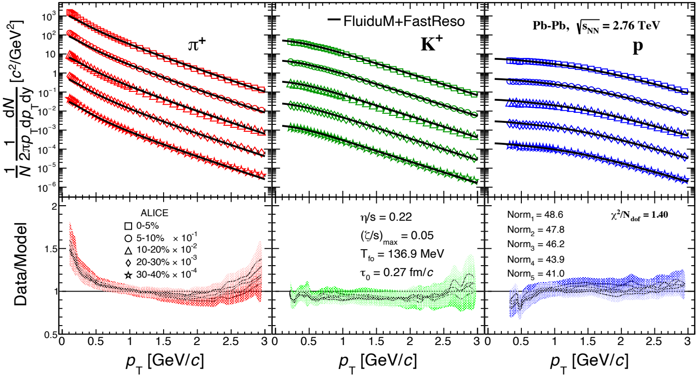
Fluid dynamics

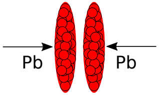
    

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear + bulk viscosity $\eta(T, \mu)$, $\zeta(T, \mu)$
  - heat conductivity $\kappa(T, \mu)$, ...
  - relaxation times, ...
  - electrical conductivity $\sigma(T, \mu)$
- fixed by **microscopic** properties encoded in Lagrangian $\mathscr{L}_{\text{QCD}}$

Particle production at the Large Hadron Collider

[Devetak, Dubla, Floerchinger, Grossi, Masciocchi, Mazeliauskas & Selyuzhenkov, JHEP 06(2020)044]





- data are very precise now - high quality theory development needed
- largest uncertainty concerns fluid fields in the **initial state**

An all-fluid description?

- we need an (approximate) description of soft QCD dynamics without uncontrolled modeling
- maybe this can be based on fluid variables, i. e. $T^{\mu\nu}(x)$ and $N_j^\mu(x)$
- investigations of different non-equilibrium approximations have shown that fluid dynamics works also outside the immediate vicinity of equilibrium (holographic models, effective kinetic theory)

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$

Covariant **conservation laws** $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for energy density ϵ
- equation for fluid velocity u^μ
- equation for particle number density n

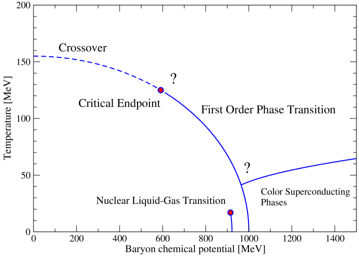
Need **further evolution equations** [e.g Israel & Stewart]

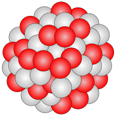
- equation for shear stress $\pi^{\mu\nu}$
- equation for bulk viscous pressure π_{bulk}

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \ldots + \pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu$$

- equation for diffusion current ν^μ
- non-hydrodynamic degrees of freedom are needed for relativistic causality!

Nuclear liquid droplet model



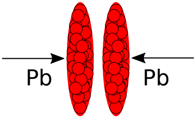


G. A. Gamow

- Gamov (*1904 Odessa) 1928: nuclei as liquid droplets
- basis of Bethe - Weizsäcker mass formula
- nuclear matter at the first order liquid-gas transition
- in thermal equilibrium with vacuum at $T = 0$ and $\mu_\mathrm{B} = \mu_c$
- energy-momentum tensor $T^{\mu\nu}(x)$ and baryon number current $N^\mu(x)$ for nucleus in vacuum easily obtained

$$T^{00} = \epsilon = \mu_c n_\mathrm{B}, \qquad N^0 = n_\mathrm{B}.$$

Superposition and Landau matching



- boosts and linear superposition gives initial state before collision

$$T^{\mu\nu}(x) = T^{\mu\nu}_{\rightarrow}(x) + T^{\mu\nu}_{\leftarrow}(x), \qquad N^{\mu}(x) = N^{\mu}_{\rightarrow}(x) + N^{\mu}_{\leftarrow}(x)$$

- resulting sum not a global equilibrium state any more!
- Landau matching decomposition

$$T^{\mu\nu} = \epsilon\, u^{\mu} u^{\nu} + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^{\mu} = n\, u^{\mu} + \nu^{\mu}$$

- uses fluid velocity u^{μ} and orthogonal projector $\Delta^{\mu\nu} = g^{\mu\nu} + u^{\mu} u^{\nu}$
- fluid velocity defined to be time-like eigenvector

$$T^{\mu}{}_{\nu} u^{\nu} = -\epsilon u^{\mu}$$

Necessary incredients

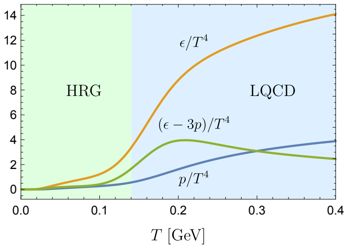
- realistic equation of state in the T-μ plane
- second order fluid dynamics
- realistic transport properties
- fluid evolution through first or second order phase transitions
- causal evolution equations also in regions with large gradients
- numerical implementation

Equation of state

- for large T and small μ: lattice QCD, e. g. Taylor expansion
  [Bazavov et al. PRD 95, 054504 (2017), Borsanyi et al. JHEP 10 (2018) 205, HotQCD 2212.09043, Borsanyi et al. PRD 105, 114504 (2022)]

$$p(T,\mu) = p(T) + \frac{1}{2!}\chi_{2B}(T)\mu^2 T^2 + \frac{1}{4!}\chi_{4B}(T)\mu^4 + \frac{1}{6!}\chi_{6B}(T)\mu^6/T^2 + \dots$$

- for small T and μ: hadron resonance gas
- around liquid-gas phase transition: nucleon-meson model



Thermodynamic equation of state at $\mu = 0$

Nucleon-meson model

Effective Lagrangian

$$\begin{aligned}\mathscr{L} =& \bar{\psi}_a\, i\gamma^\nu(\partial_\nu - ig\omega_\nu - i\mu\delta_{0\nu})\,\psi_a \\ &+ h\sqrt{2}\left[\bar{\psi}_a\left(\tfrac{1+\gamma_5}{2}\right)\phi_{ab}\psi_b + \bar{\psi}_a\left(\tfrac{1-\gamma_5}{2}\right)(\phi^\dagger)_{ab}\psi_b\right] \\ &+ \tfrac{1}{2}\phi^*_{ab}(-\partial_\mu\partial^\mu)\phi_{ab} + U_{\text{mic}}(\rho,\sigma) \\ &+ \frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu) + \frac{1}{2}m_\omega^2\,\omega_\mu\omega^\mu\end{aligned}$$

- nucleons ψ_a, scalars ϕ_{ab} and vectors ω_ν
- effective potential for scalars

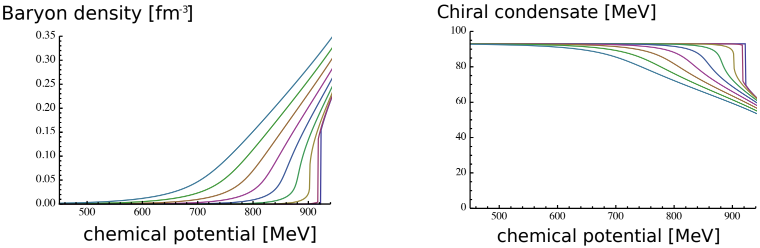
$$U_{\text{mic}}(\rho,\sigma) = \bar{U}(\rho) - m_\pi^2 f_\pi \sigma$$

with the chiral invariant combination

$$\rho = \frac{1}{2}\phi^*_{ab}\phi_{ab} = \frac{1}{2}(\sigma^2 + \boldsymbol{\pi}^2).$$

- variant of Walecka model [Walecka (1974), Floerchinger, Wetterich (2012), Drews, Hell, Klein & Weise (2013), Drews & Weise (2014)]

First order phase transition in nucleon-meson model



different curves: $T = 0 \ldots 80\,$MeV

- nucleon-meson model has liquid-gas first order phase transition to saturation density
- provides a realistic model for normal nuclear matter and equation of state in transition region

Fluid dynamics with several conserved quantum numbers

- fluid with conserved quantum number densities $c_m = (\epsilon, n_{\mathsf{B}}, n_{\mathsf{C}}, n_{\mathsf{S}}, \ldots)$
- equation of state in grand canonical ensemble in terms of Massieu potential $w(\beta, \alpha_j) = \beta p(\beta, \alpha_j)$ with $\beta = 1/T$, $\alpha_j = \mu_j/T$,

$$dw = -\epsilon d\beta + n_j d\alpha_j$$

- second derivative yields a matrix of susceptibilities with $\gamma^m = (\beta, \alpha_1, \alpha_2, \ldots)$

$$G_{mn}(\gamma) = \frac{\partial^2 w}{\partial \gamma^m \partial \gamma^n}$$

- fluid evolution equations from conservation laws

$$u^\mu \partial_\mu c_m + f_m = 0$$

- can be written with inverse susceptibility matrix as

$$u^\mu \partial_\mu \gamma^n + (G^{-1}(\gamma))^{nm} f_m = 0$$

First order transitions

- consider some first order transition surface with normal vector $n_m(\gamma)$
- during a first order transition: phase coexistence
- bubble nucleation or spinodal decomposition
- here: macroscopic description with volume mixing parameter $0 < t < 1$
- write the Massieu potential density as

$$w(\gamma) = [1-t]\, w'(\gamma) + t\, w''(\gamma)$$

- matrix $G^{mn}(\gamma, t)$ is the inverse of

$$\left[(1-t)\, G'_{mn}(\gamma) + t\, G''_{mn}(\gamma)\right]$$

- during the transition no change orthogonal to phase transition surface

$$n_m(\gamma)\, d\gamma^m = 0$$

- leads to evolution equation for mixing parameter

$$u^\mu \partial_\mu t = -\frac{n_p(\gamma)\, G^{pq}(\gamma, t) f_q}{n_r(\gamma)\, G^{rs}(\gamma, t)[c''_s(\gamma) - c'_s(\gamma)]}$$

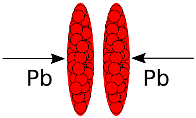
- leads in turn to full fluid evolution equations in coordinates $\gamma_m = (\beta, \alpha_j)$

Longitudinal dynamics

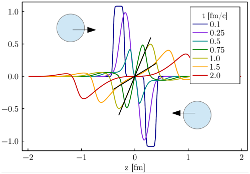
- at very early times the collision dynamics is longitudinal
- transverse expansion starts later
- concentrate here on reduced model involving only t, z
- neglect gradients with respect to x, y
- relevant fluid fields are

$$v_z(t,z), \quad T(t,z), \quad \mu(t,z), \quad \pi^{zz}(t,z), \quad \pi_{\text{bulk}}(t,z)$$

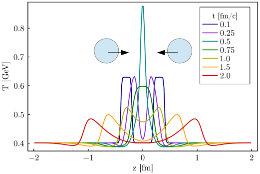
- transverse dynamics can be addressed in next step



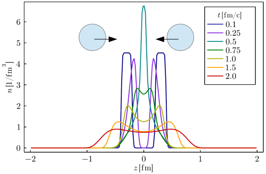
First preliminary results



longitudinal fluid velocity



temperature

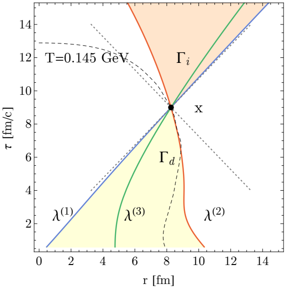


baryon density

- emergent Bjorken flow $v = z/t$ close to collision point
- plateaus in temperature and baryon density
- more refined analysis in progress
  [together with Federica Capellino, Alaric Erschfeld, Eduardo Grossi and Andreas Kirchner]

Causality

[Floerchinger & Grossi, JHEP 08 (2018) 186]



- dissipative fluid equations *can* be of hyperbolic type
- characteristic velocities depend on fluid fields
- need $|\lambda^{(j)}| < c$ for relativistic causality

Causality in radial expansion

[Floerchinger & Grossi, JHEP 08 (2018) 186]

- for radial expansion the characteristic velocities are

$$\lambda^{(1)} = \frac{v + \tilde{c}}{1 + \tilde{c}v}, \qquad \lambda^{(2)} = \frac{v - \tilde{c}}{1 - \tilde{c}v}, \qquad \lambda^{(3)} = \lambda^{(4)} = \lambda^{(5)} = v$$

- effective speed of sound

$$\tilde{c} = \sqrt{c_s^2 + d},$$

- with thermal speed of sound

$$c_s^2 = \frac{\partial p}{\partial \epsilon}$$

- and correction

$$d = \frac{\frac{4\eta}{3\tau_{\text{shear}}} + \frac{\zeta}{\tau_{\text{bulk}}}}{\epsilon + p + \pi_{\text{bulk}} - \pi^\phi_\phi - \pi^\eta_\eta}$$

- relaxation times τ_{shear} and τ_{bulk} must be large enough

Fluid equations from geometric gauge fields (?)

[Floerchinger & Grossi, PRD 105, 085015 (2022)]

- gauge symmetry of diffeomorphisms $g_{\mu\nu} \to g_{\mu\nu} + \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu$ imlies energy-momentum conservation

$$\nabla_\mu T^{\mu\nu} = 0$$

- are there more useful geometric gauge fields?
- contorsion $C_\mu{}^\rho{}_\sigma$ or spin connection for local Lorentz transformation

$$\nabla_\mu S^{\mu\rho\sigma} = \mathscr{T}^{\rho\sigma} - \mathscr{T}^{\sigma\rho}$$

- Weyl gauge field B_μ for local dilatations

$$\nabla_\mu W^\mu = \frac{2}{d} \left[T^\rho{}_\rho - \mathscr{U}^\rho{}_\rho \right]$$

- proper non-metricity $\hat{B}_\mu{}^\rho{}_\sigma$ for local shear transformations

$$\nabla_\mu Q^{\mu\rho\sigma} = 2 \left[T^{\rho\sigma} - \mathscr{U}^{\rho\sigma} - \frac{g^{\rho\sigma}}{d} (T^\nu{}_\nu - \mathscr{U}^\nu{}_\nu) \right]$$

Conclusions

- fluid description of entire collision event is under development
- towards universal description of soft QCD
- not clear how well it will work eventually
- many elements must be brought together
- fluid evolution in the whole phase diagram
- phase transitions
- causality
- a challenge to theory, but could be worth it