The structure of the early cosmos in a Bose-Einstein condensate

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STRUCTURES CLUSTER OF EXCELLENCE

Team & publications


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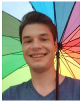
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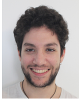
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- Celia Viermann, Marius Sparn, Nikolas Liebster, Maurus Hans, Elinor Kath, Álvaro Parra-López, Mireia Tolosa-Simeón, Natalia Sánchez-Kuntz, Tobias Haas, Helmut Strobel, Stefan Floerchinger, Markus K. Oberthaler, *Quantum field simulator for dynamics in curved spacetime*, arXiv:2202.10441
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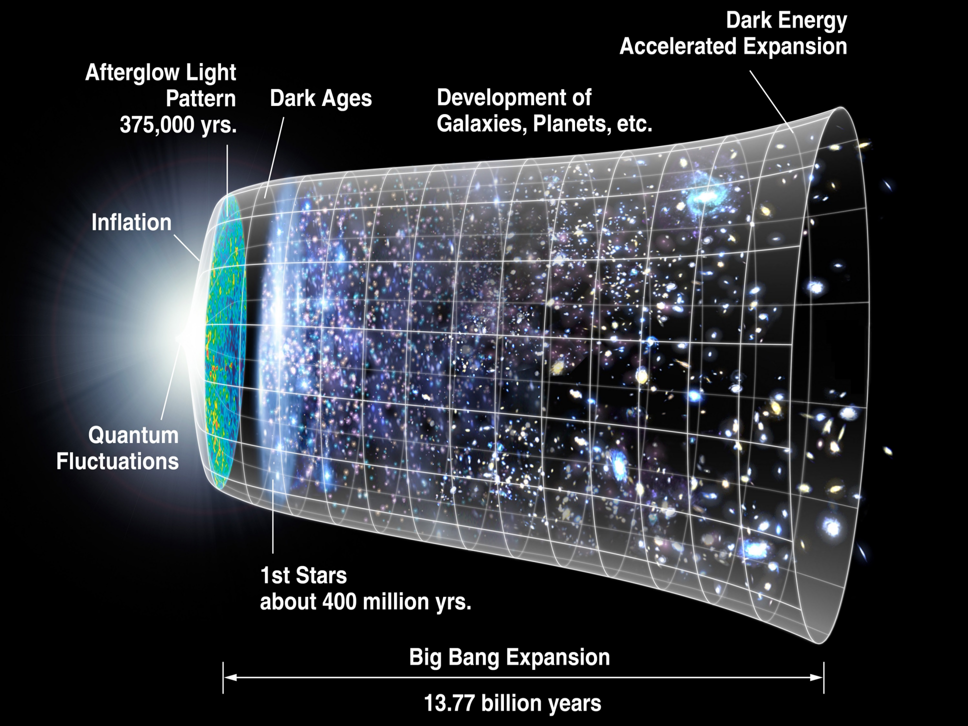
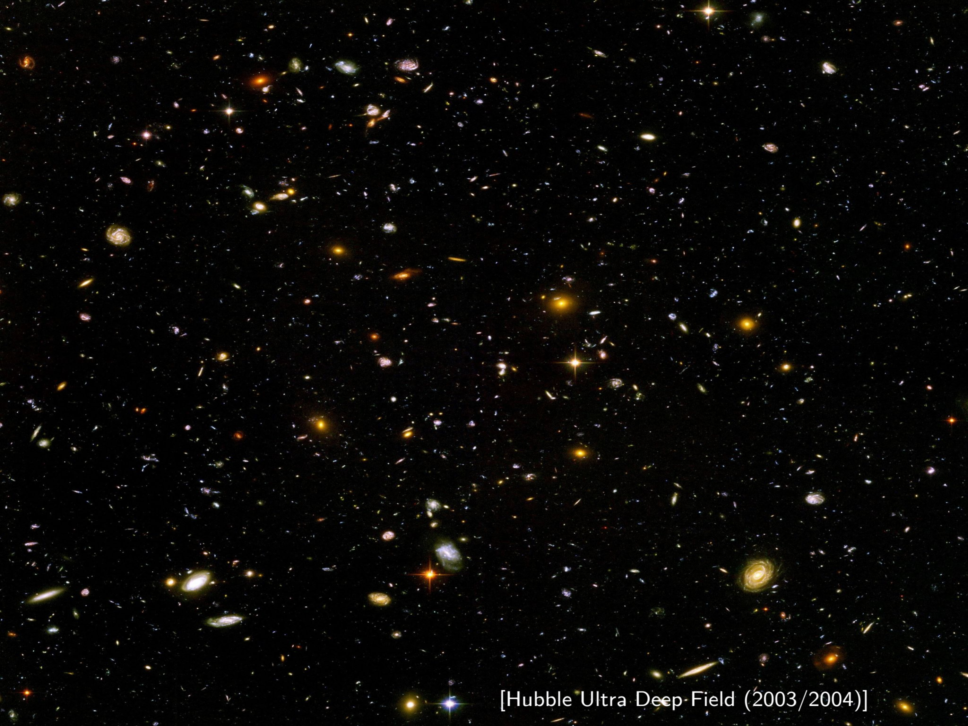
The night sky

- many bright stars and galaxies
- but overall dark
- Heinrich Wilhelm Olbers (1823):

  "Sind wirklich im ganzen unendlichen Raum Sonnen vorhanden, sie mögen nun in ungefähr gleichen Abständen von einander, oder in Milchstrassen-Systeme vertheilt sein, so wird ihre Menge unendlich, und da müsste der ganze Himmel eben so hell sein wie die Sonne. Denn jede Linie, die ich mir von unserem Auge gezogen denken kann, wird nothwendig auf irgend einen Fixstern treffen, und also müsste uns jeder Punkt am Himmel Fixsternlicht, also Sonnenlicht zusenden."
- out-of-equilibrium state needed
- the Universe expands !



Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years


[Hubble Ultra Deep Field (2003/2004)]

COSMOLOGY

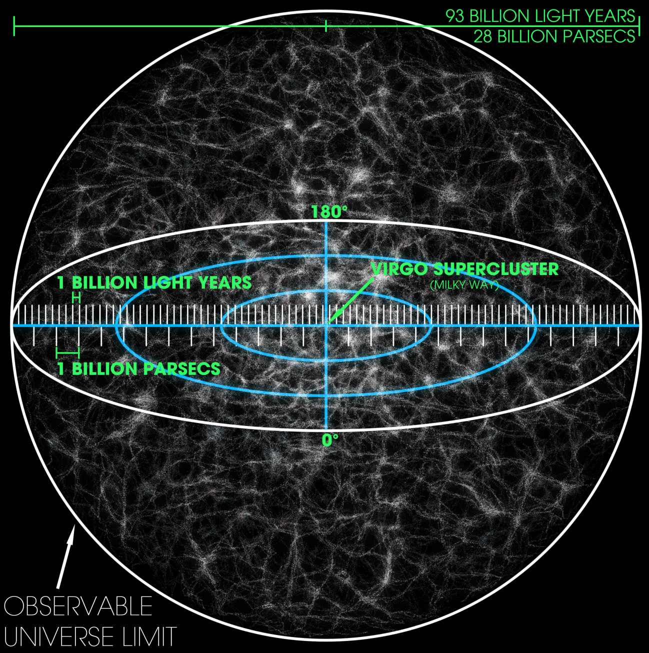
Holger Gies

— Lecture Notes —

1 Geometry & the Cosmological Principle

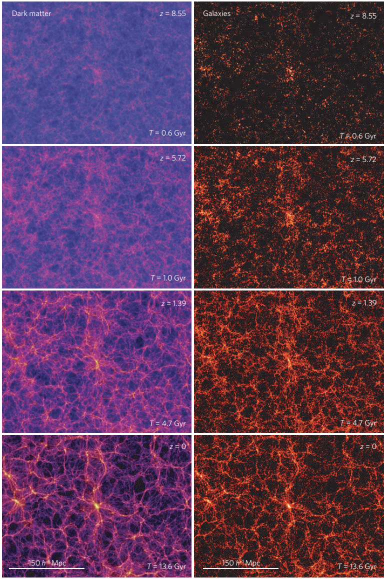
"Weißt Du wieviel Sternlein stehen...?"

Yes, there are about a hundred billion (10^{11}) stars in a galaxy (as many as neurons in our brains), and about a hundred billion galaxies in the visible universe. This makes $\sim 10^{22}$ stars in the cosmos. How could we ever hope that the evolution of the cosmos can be described by a set of simple equations?



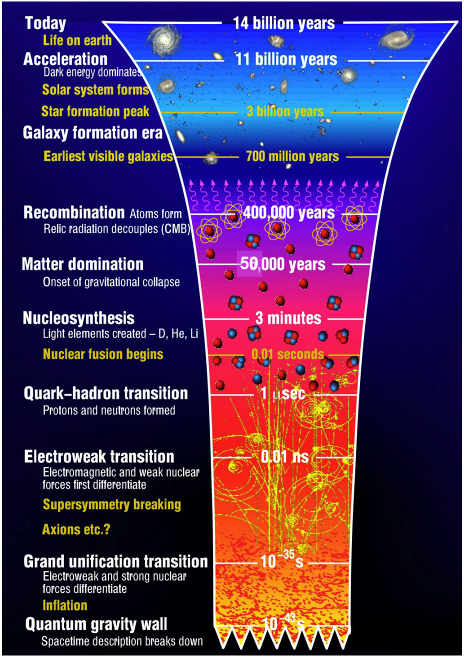
93 BILLION LIGHT YEARS
28 BILLION PARSECS
180°
VIRGO SUPERCLUSTER
(MILKY WAY)
1 BILLION LIGHT YEARS
1 BILLION PARSECS
0°
OBSERVABLE
UNIVERSE LIMIT

Evolution of cosmic large-scale structure



[Springel, Frenk & White, Nature 440, 1137 (2006)]

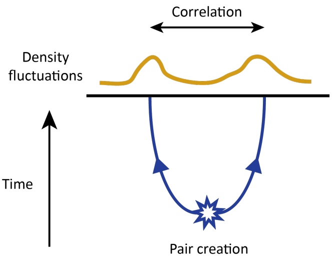
Time history



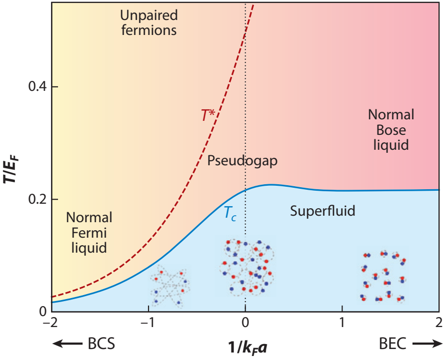
Quantum origin of fluctuations

- Universe was almost homogeneous at early times
- small fluctuations magnified by gravitational attraction
- primordial fluctuations most likely quantum fluctuations maginfied by inflation

  [Mukhanov & Chibisov (1981), Hawking (1982), Starobinsky (1982), Guth & Pi (1982), Bardeen, Steinhardt & Turner (1983), Fischler, Ratra & Susskind (1985)]



Ultracold quantum gases



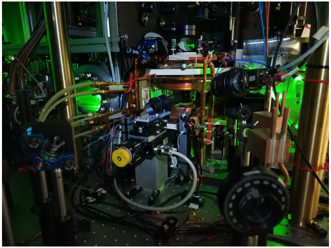
- why study ultracold quantum gases?
- develop and test understanding of quantum fields
- functional renormalization group developed for cold atoms in Heidelberg [Wetterich, Diehl, **Gies**, Pawlowski, Floerchinger, Scherer, Krahl, Schmidt, Moroz, Boettcher, Faigle-Cedzich, ... Salmhofer, Honerkamp, Metzner, Gubbels, Stoof, Dupuis, Strack, Bartosch, Kopietz, ...]

Non-relativistic quantum fields

- Bose-Einstein condensate in two dimensions
  [Gross (1961), Pitaevskii (1961)]

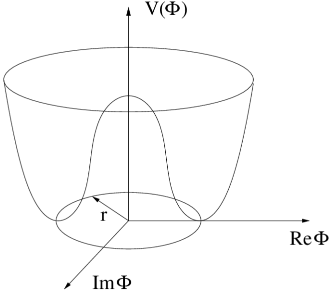
$$\Gamma[\Phi] = \int \mathrm{d}t\,\mathrm{d}^2x \Big\{ \hbar\Phi^*(t,\mathbf{x}) \left[i\frac{\partial}{\partial t} - V(t,\mathbf{x}) \right] \Phi(t,\mathbf{x}) - \frac{\hbar^2}{2m} \boldsymbol{\nabla}\Phi^*(t,\mathbf{x}) \boldsymbol{\nabla}\Phi(t,\mathbf{x}) - \frac{\lambda(t)}{2} \Phi^*(t,\mathbf{x})^2 \Phi(t,\mathbf{x})^2 \Big\}$$

- traping potential $V(t,\mathbf{x})$ and coupling strength $\lambda(t)$
- can be realized and controlled experimentally



[Oberthaler group, KIP Heidelberg]

Superfluid and small excitations



- Complex non-relativistic field can be decomposed

$$\Phi = e^{iS_0}\left(\sqrt{n_0} + \frac{1}{\sqrt{2}}\left[\phi_1 + i\phi_2\right]\right)$$

- real fields ϕ_1 and ϕ_2 describe excitations on top of the superfluid
- stationary superfluid density $n_0(\mathbf{x})$ and vanishing superfluid velocity

$$\mathbf{v} = \frac{\hbar}{m}\boldsymbol{\nabla} S_0 = 0$$

Sound waves / phonons

- small energy excitations are sound waves or phonons
- propagate with finite velocity, similar to light
- local speed of sound

$$c_S(t, \mathbf{x}) = \sqrt{\frac{\lambda(t)\, n_0(\mathbf{x})}{m}}$$

- sound waves propagate along

$$ds^2 = -dt^2 + \frac{1}{c_S(t, \mathbf{x})^2}(d\mathbf{x} - \mathbf{v}\, dt)^2 = 0$$

- acoustic metric for $\mathbf{v} = 0$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{c_S(t,\mathbf{x})^2} & 0 \\ 0 & 0 & \frac{1}{c_S(t,\mathbf{x})^2} \end{pmatrix}$$

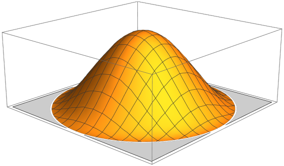
Relativistic scalar field

- Low energy theory for phonons (with $\phi = \phi_2/\sqrt{2m}$)

$$\Gamma[\phi] = \int \mathrm{d}t\, \mathrm{d}^2x\, \sqrt{g} \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\}$$

- metric determinant $\sqrt{g} = \sqrt{-\mathrm{det}(g_{\mu\nu})}$
- acoustic metric depends on space and time like the space-time metric in Einsteins theory of general relativity !
- phonons behave like real, massless, relativistic scalar field in a curved spacetime !

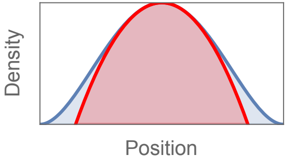
Density profiles



- assume specifically for $r = |\mathbf{x}| < R$

$$n_0(r) = \bar{n}_0 \times \left[1 - \frac{r^2}{R^2}\right]^2$$

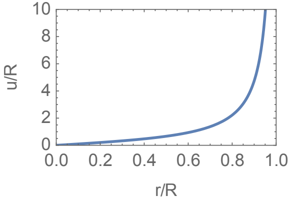
- experimental realization with optical trap and digital micromirror device
- approximate realization in harmonic trap



Acoustic spacetime geometry

- variable transform to $0 \leq u < \infty$

$$u(r) = \frac{r}{1 - \frac{r^2}{R^2}}$$



- leads to Friedmann-Lemaitre-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{du^2}{1 - \kappa u^2} + u^2 d\varphi^2\right)$$

- negative spatial curvature

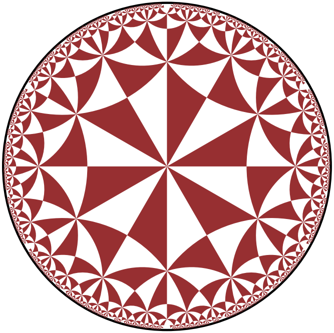
$$\kappa = -4/R^2$$

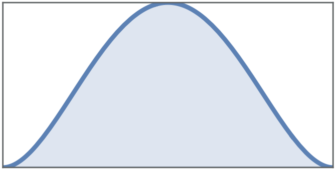
- scale factor

$$a(t) = \sqrt{\frac{m}{\bar{n}_0}\frac{1}{\lambda(t)}}$$

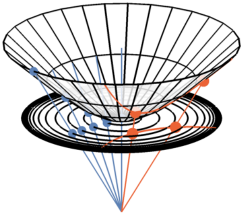
Hyperbolic geometry

Lewis Carroll (1888): "Too fanciful !"





Hyperbolic geometry in Minkowski space



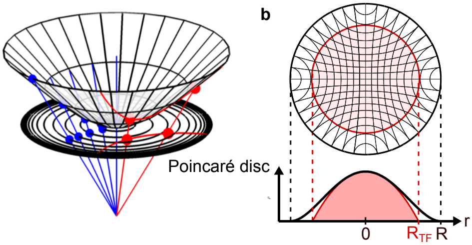
- start with Minkowski space $ds^2 = dX^2 + dY^2 - dZ^2$
- consider hyperbloid $X^2 + Y^2 - Z^2 = -R^2/4$
- stereographic projection to Poincaré disc

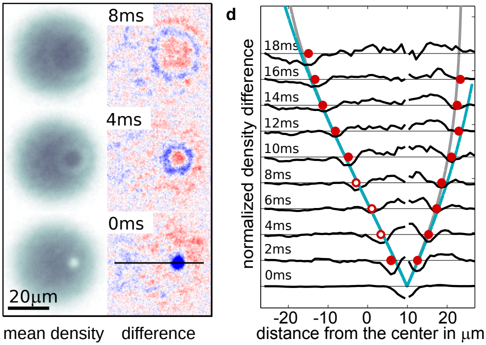
Poincaré disc and M. C. Eschers circle limit series

Circle limit III, M. C. Escher, 1959.

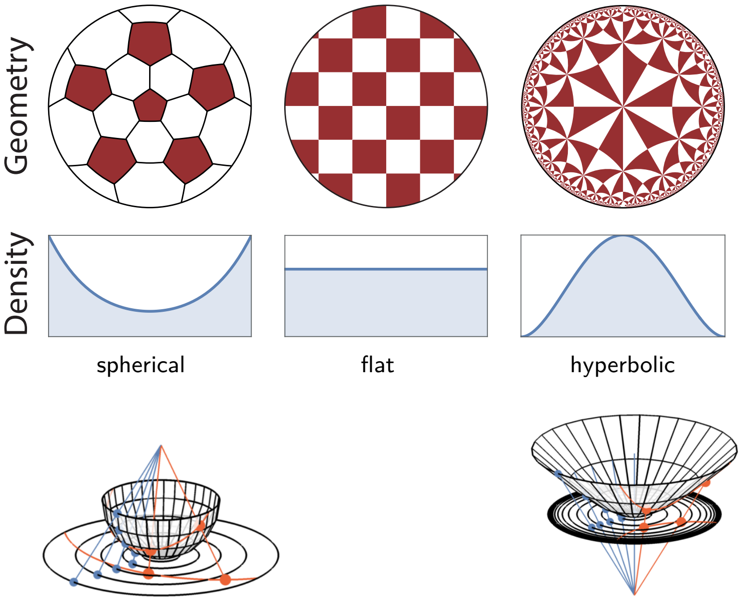
Hyperbolic geometry in biology

Experimental realization in a Bose-Einstein condensate

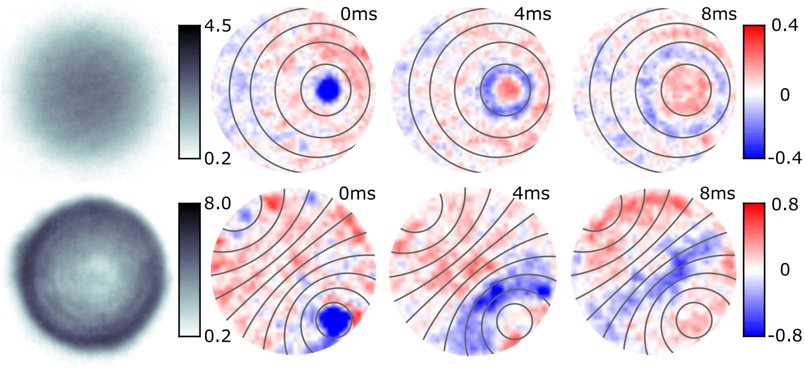




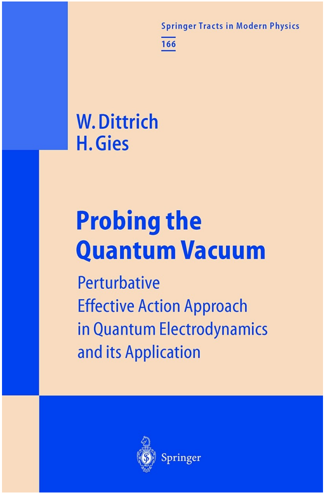
Geometries with constant spatial curvature



Propagating sound waves



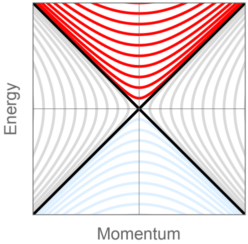
The quantum vacuum



- the vacuum in a quantum field theory is very nontrivial
- no unique vacuum in curved spacetime

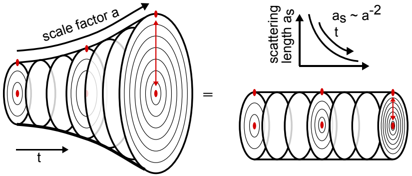
Symmetries

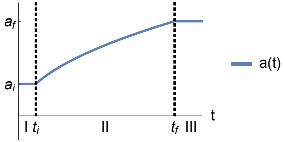
- Emmy Noether (1918): *Symmetries imply conservation laws*
- Eugene P. Wigner (1939): *Classification of particles through representations of symmetries*



- translations in space and time $\leftrightarrow$ momentum, energy, mass
- rotations and Lorentz boosts $\leftrightarrow$ spin / helicity
- further internal symmetries $\leftrightarrow$ charge, isospin, etc.
- symmetries are needed for particle concept to work properly
- what happens if they get broken?

Particle production





- time-dependent scattering length induces time-dependent metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{du^2}{1-\kappa u^2} + u^2 d\varphi^2\right)$$

- particle concept works well in regions I and III but not in region II
- vacuum state in region I leads to state with particles in region III
- expanding space produces particles !
- analytic calculations possible for power law scale factors

$$a(t) = \text{const} \times t^\gamma$$

Mode functions and Bogoliubov transforms

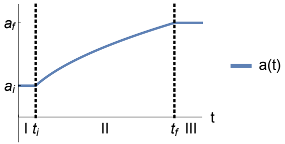
- field gets expanded in modes

$$\phi(t,u,\varphi) = \int_{k,m} \left[\hat{a}_{km}\mathcal{H}_{km}(u,\varphi)v_k(t) + \hat{a}^\dagger_{km}\mathcal{H}^*_{km}(u,\varphi)v^*_k(t)\right]$$

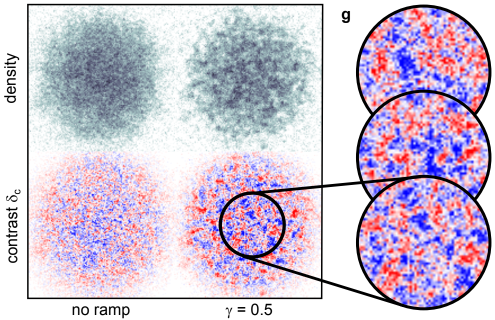
- spatial part $\mathcal{H}_{km}(u,\varphi)$ can be expressed in terms of spherical harmonics at complex angular momenta in hyperbolic geometry
- mode functions satisfy

$$\ddot{v}_k(t) + 2\frac{\dot{a}(t)}{a(t)}\dot{v}_k(t) + \frac{k^2 + |\kappa|/4}{a^2(t)}v_k(t) = 0$$

- vacuum state only unique for $\dot{a}(t) = 0$ where $v_k(t) \sim e^{-i\omega_k t}$
- Bogoliubov transforms between different choices of $\hat{a}_{km}$ and vacuum states



Observation of particle production

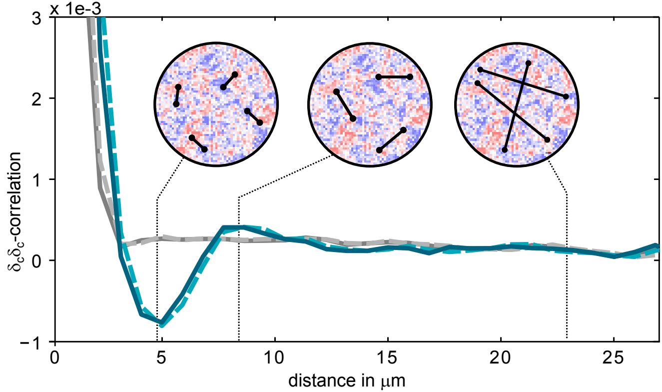


- rescaled density contrast

$$\delta_c(t,\mathbf{x}) = \sqrt{\frac{n_0(\mathbf{x})}{\bar{n}_0^3}}\,[n(t,\mathbf{x}) - n_0(\mathbf{x})] \sim \partial_t \phi(t,\mathbf{x})$$

- allows to test relativistic scalar field

Density contrast correlation function

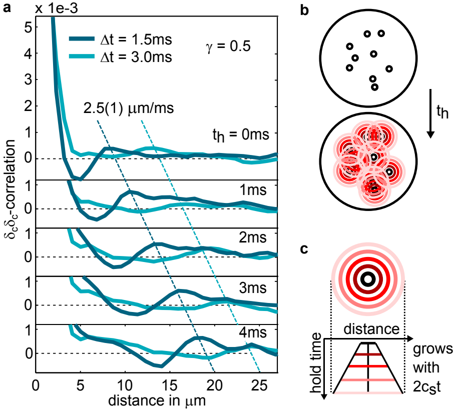


- correlation function

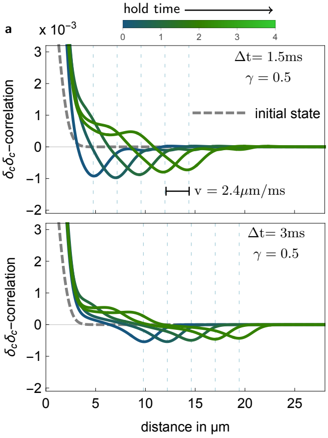
$$\langle \delta_c(\mathbf{x})\delta_c(\mathbf{y}) \rangle$$

- before and after expansion

Time dependent correlation functions after expansion



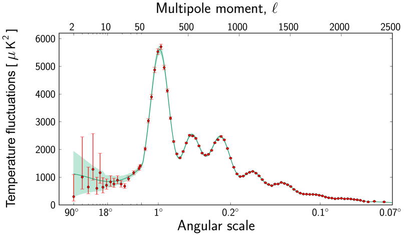
Experiment

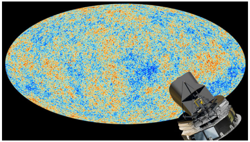


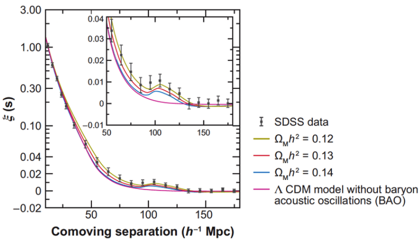
Theory

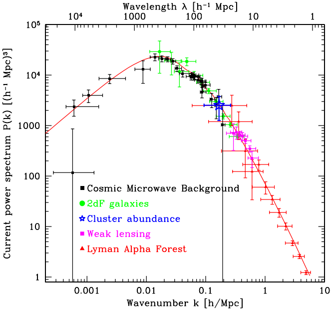
- analgous to baryon accoustic or Sakharov oscillations in cosmology
- optical resolution important for detailed shape

Baryon acoustic oscillations

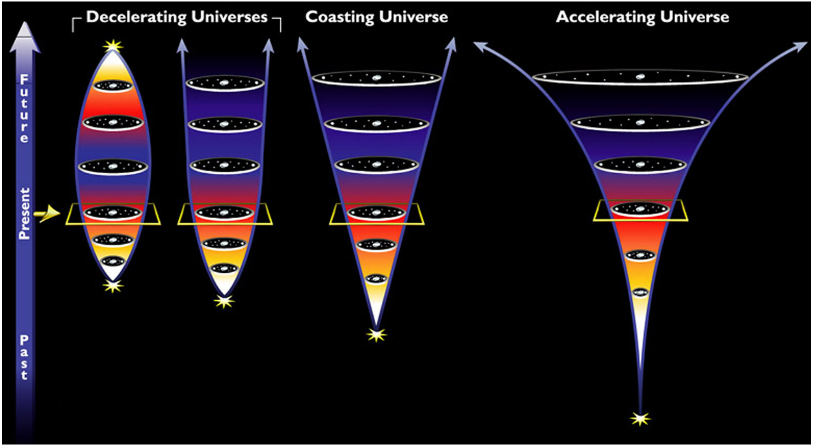




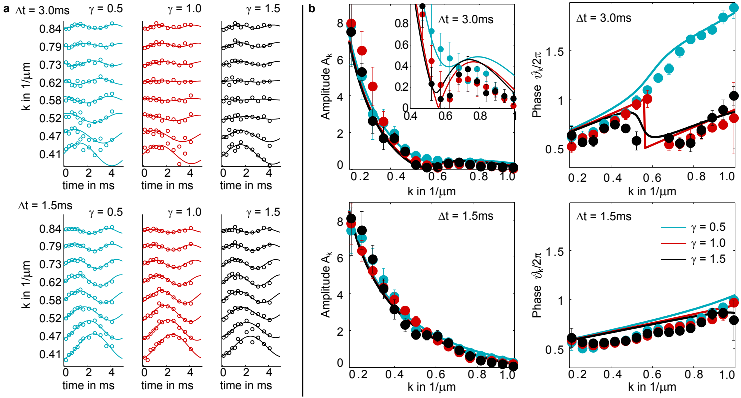




Expansion history



Oscillations in Fourier space



- Fourier spectrum of excitations

$$S_k(t) = \frac{1}{2} + N_k + A_k \cos(2\omega_k(t - t_f) + \vartheta_k)$$

- decelerated, coasting and accelerated expansion
- good agreement with analytic theory (solid lines)

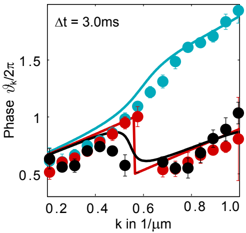
Quantum recurrences

- uniform expansion with $a(t) = Qt$ is special
- shows quantum recurrences of the incoming vacuum state at special values of wavenumber k

$$k_n = \frac{a_\mathsf{f} - a_\mathsf{i}}{\Delta t} \left[\left(\frac{n\pi}{\ln(a_\mathsf{f}/a_\mathsf{i})} \right)^2 + \frac{1}{4} \right]^{\frac{1}{2}},$$

with integer $n = 1, 2, 3, \ldots$

- at these points one has trivial Bogoliubov coefficient $\beta_k = 0$
- can be seen experimentally as a discontinuity in the phase !

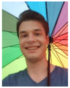


Conclusion

- Bose-Einstein condensates can be quantum simulators for quantum fields in curved spacetime
- Symmetric spaces with constant curvature can be realized with specific radial density profiles
- Experimental realization in two spatial dimensions
- Time-dependent coupling allows to simulate expansion
- Particle production by time-dependent scale factor
- Oscillations after expansion allow detailed investigations
- Quantum information theoretic aspects also accessible
- Extensions to three dimensions, other geometries, other field content, and more, are possible

Many thanks to the team...

Celia Viermann



Marius Sparn

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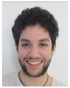


Álvaro Parra-López

Natalia Sánchez-Kuntz

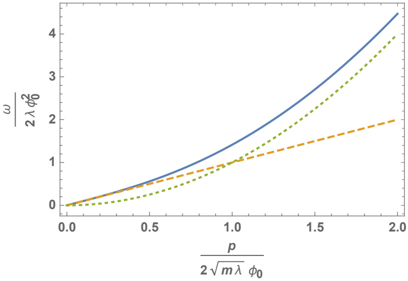
Tobias Haas

Stefan Floerchinger

... and Happy Birthday Holger Gies !

Backup

Bogoliubov dispersion relation



- Quadratic part of action for excitations

$$S_2 = \int dt\ d^3x \left\{ -\frac{1}{2}(\phi_1, \phi_2) \begin{pmatrix} -\frac{\nabla^2}{2m} + 2\lambda n_0 & \partial_t \\ -\partial_t & -\frac{\nabla^2}{2m} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \right\}$$

- Dispersion relation

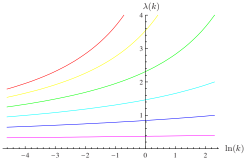
$$\omega = \sqrt{\left(\frac{\mathbf{p}^2}{2m} + 2\lambda\phi_0^2\right)\frac{\mathbf{p}^2}{2m}}$$

becomes linear for

$$\mathbf{p}^2 \ll 4\lambda m n_0 = \frac{2}{\xi^2}$$

Renormalization in two dimensions

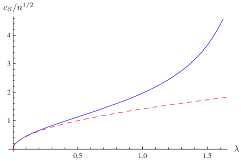
[S. Floerchinger, C. Wetterich, *Superfluid Bose gas in two dimensions*, PRA 79, 013601 (2009)]

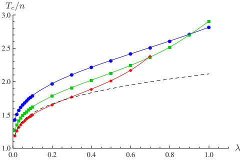


- scale-dependent coupling in two dimensions

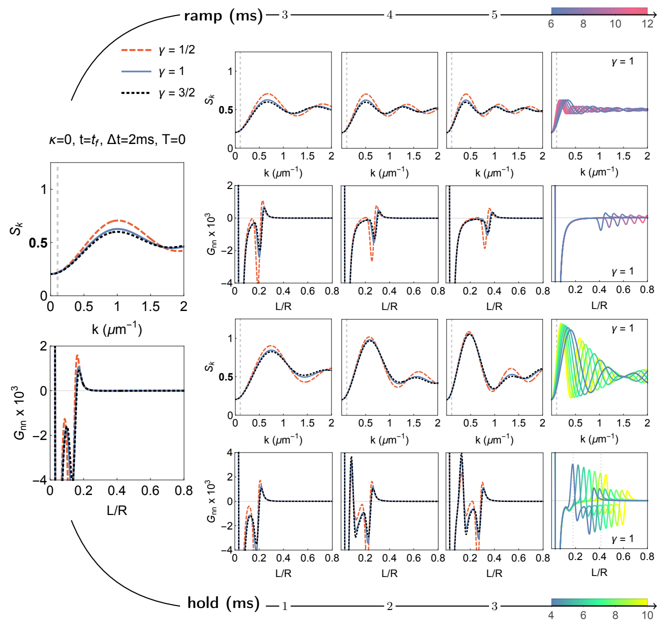
$$k\frac{\partial}{\partial k}\lambda = \frac{\lambda^2}{4\pi}$$

- sound velocity and critical temperature

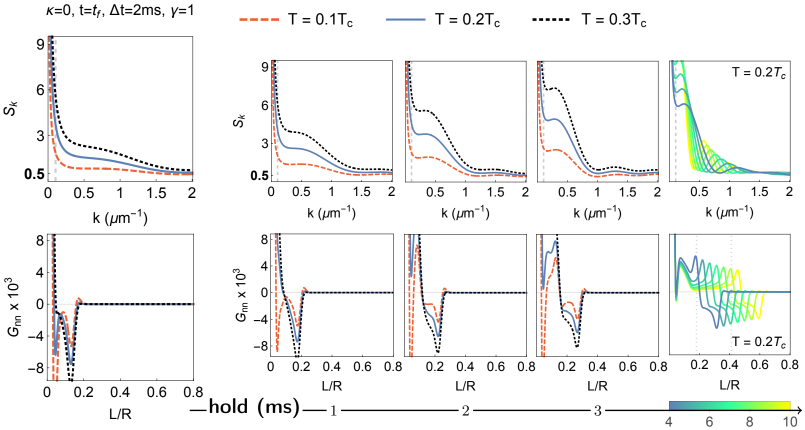




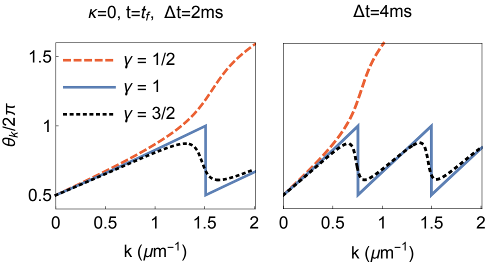
Expansion and hold time dependence



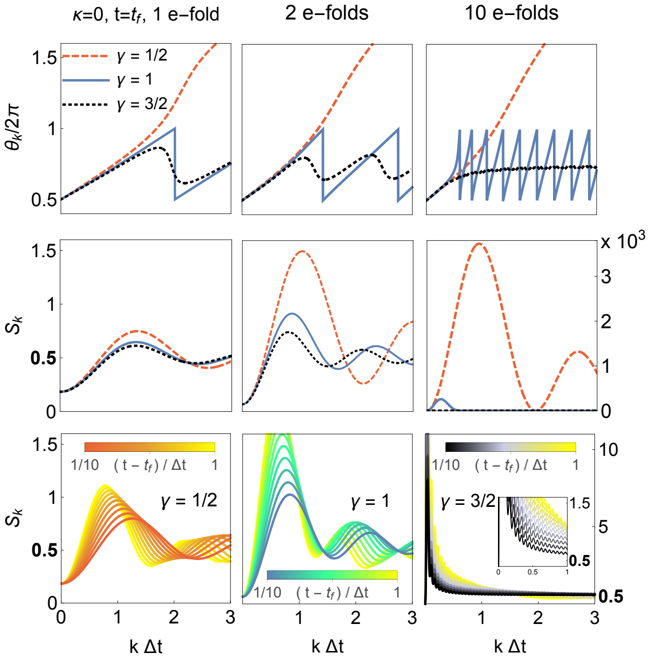
Temperature dependence



Phases

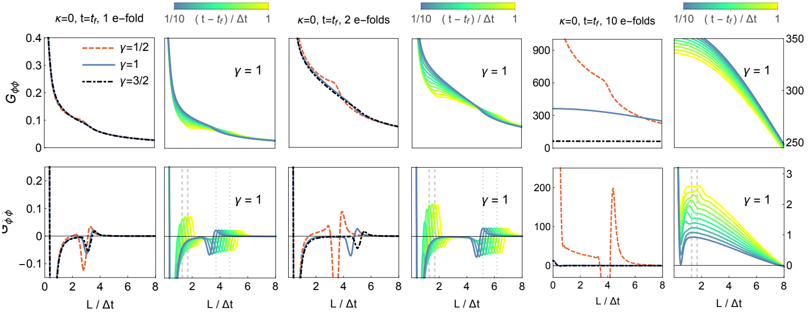


More e-folds

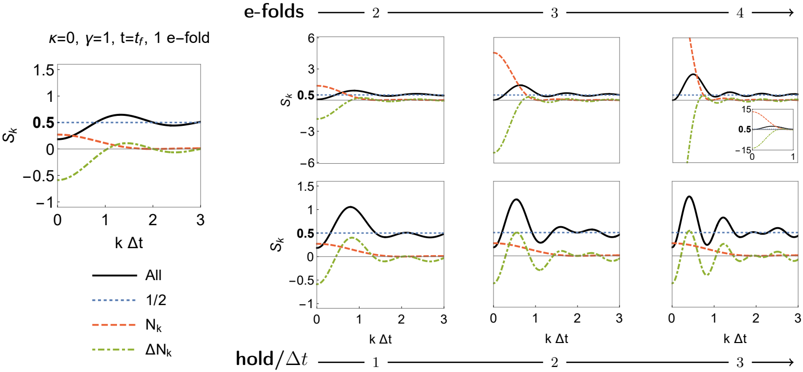


κ=0, t=t_f, 1 e−fold
2 e−folds
10 e−folds
γ = 1/2
γ = 1
γ = 3/2
θ_k/2π
S_k
x 10^3
1/10 (t − t_f) / Δt 1
k Δt

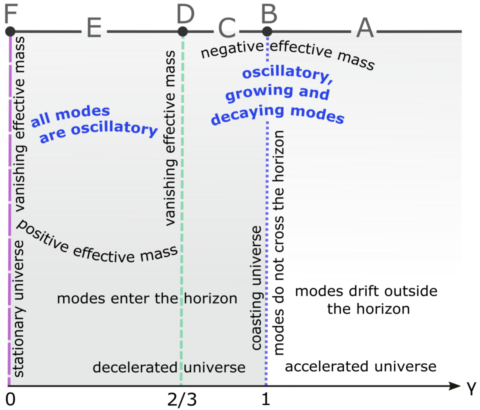
Correlation functions



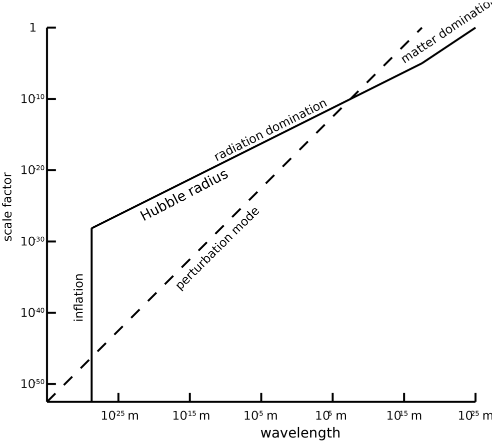
Power spectra



Horizon crossing



Horizons and inflation



Bogoliubov transforms

- in region I one has positive frequency modes v_k and corresponding operators. Define vacuum

$$\hat{a}_{km}|\Omega\rangle = 0$$

- similar in region III positive frequency modes u_k with

$$\hat{b}_{km}|\Psi\rangle = 0$$

- Bogoliubov transform mediates between them

$$u_k = \alpha_k v_k + \beta_k v_k^*, \qquad v_k = \alpha_k^* u_k - \beta_k u_k^*$$

- operators are related by

$$\hat{b}_{km} = \alpha_k^* \hat{a}_{km} - \beta_k^* (-1)^m \hat{a}_{k,-m}^\dagger$$

- condition $|\alpha_k|^2 - |\beta_k|^2 = 1$
- constant term in spectrum $N_k = |\beta_k|^2$
- oscilating term $\Delta N_k = \mathrm{Re}[\alpha_k \beta_k e^{2i\omega_k t}]$