Conserved and non-conserved Noether currents from the quantum effective action

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FunQCD Workshop, Online
March 28, 2021.

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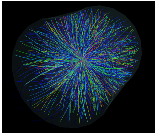
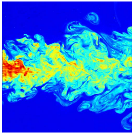
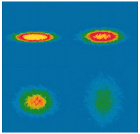
Fluid dynamics

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear + bulk viscosity $\eta(T, \mu)$, $\zeta(T, \mu)$
  - heat conductivity $\kappa(T, \mu)$, ...
  - relaxation times, ...
  - electrical conductivity $\sigma(T, \mu)$
- fixed by **microscopic** properties encoded in Lagrangian $\mathscr{L}_{\text{QCD}}$
- old dream of condensed matter physics: understand the fluid properties!

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$

Covariant **conservation laws** $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for energy density ϵ
- equation for fluid velocity u^μ
- equation for particle number density n

Need **further evolution equations** [e.g Israel & Stewart]

- equation for shear stress $\pi^{\mu\nu}$
- equation for bulk viscous pressure π_{bulk}

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \ldots + \pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu$$

- equation for diffusion current ν^μ
- non-hydrodynamic degrees of freedom are needed for relativistic causality!

Remarks

- derivation from quantum effective action $\Gamma[\phi]$ wanted
  [Floerchinger, JHEP 1205, 021 (2012); JHEP 1609, 099 (2016)]
- expectation values *and* correlation functions of interest
- underlying principle: most excitations or modes relax quickly
  [Kadanoff & Martin (1963)]
- exception: conserved quantities like energy, momentum or particle density ("hydrodynamic modes")
- but: some non-hydrodynamic modes are needed for causality
- how to obtain additional equations of motion for them?

Covariant energy-momentum conservation

- quantum effective action $\Gamma[\phi, g]$ at stationary matter fields

$$\frac{\delta}{\delta\phi(x)}\Gamma[\phi, g] = 0$$

- diffeomorphism is gauge transformation of metric

$$g_{\mu\nu}(x) \to g_{\mu\nu}(x) + \nabla_\mu \varepsilon_\nu(x) + \nabla_\nu \varepsilon_\mu(x)$$

- energy-momentum tensor defined by

$$\delta\Gamma[\phi, g] = \frac{1}{2}\int d^d x \sqrt{g}\, T^{\mu\nu}(x)\delta g_{\mu\nu}(x)$$

- from invariance of $\Gamma[\phi, g]$ under diffeomorphisms

$$\nabla_\mu T^{\mu\nu}(x) = 0$$

- worked here in Riemann geometry with Levi-Civita connection

$$\delta\Gamma_\mu{}^\rho{}_\nu = \frac{1}{2}g^{\rho\lambda}\left(\nabla_\mu \delta g_{\nu\lambda} + \nabla_\nu \delta g_{\mu\lambda} - \nabla_\lambda \delta g_{\mu\nu}\right)$$

Hypermomentum current

[von der Heyde, Kerlick & Hehl (1976)] [Floerchinger & Grossi, arXiv:2102.11098]

- connection can be varied independent of the metric

$$\delta\Gamma = \int d^d x \sqrt{g} \left\{ \frac{1}{2} \mathscr{U}^{\mu\nu}(x) \delta g_{\mu\nu}(x) - \frac{1}{2} \mathscr{S}^{\mu}{}_{\rho}{}^{\sigma}(x) \delta \Gamma_{\mu}{}^{\rho}{}_{\sigma}(x) \right\}$$

  with new symmetric tensor $\mathscr{U}^{\mu\nu}$ and *hypermomentum* current $\mathscr{S}^{\mu}{}_{\rho}{}^{\sigma}$
- hypermomentum current can be decomposed further

$$\mathscr{S}^{\mu}{}_{\rho}{}^{\sigma} = Q^{\mu}{}_{\rho}{}^{\sigma} + W^{\mu}\, \delta_{\rho}{}^{\sigma} + S^{\mu}{}_{\rho}{}^{\sigma} + S^{\sigma\mu}{}_{\rho} + S_{\rho}{}^{\mu\sigma}$$

  with
  - spin current $S^{\mu\rho\sigma} = -S^{\mu\sigma\rho}$
  - dilatation current W^{μ}
  - shear current $Q^{\mu\rho\sigma} = Q^{\mu\sigma\rho}, \qquad Q^{\mu\rho}{}_{\rho} = 0$

Equations of motion for dilatation and shear current

[Floerchinger & Grossi, arXiv:2102.11098]

- variation of connection contains Levi-Civita part and non-Riemannian part

$$\delta\Gamma_\mu{}^\rho{}_\sigma = \frac{1}{2}g^{\rho\lambda}\left(\nabla_\mu\delta g_{\sigma\lambda} + \nabla_\sigma\delta g_{\mu\lambda} - \nabla_\lambda\delta g_{\mu\sigma}\right) + \delta C_\mu{}^\rho{}_\sigma + \delta D_\mu{}^\rho{}_\sigma$$

- variation at $\delta C_\mu{}^\rho{}_\sigma = \delta D_\mu{}^\rho{}_\sigma = 0$ gives energy-momentum tensor

$$T^{\mu\nu} = \mathscr{U}^{\mu\nu} + \frac{1}{2}\nabla_\rho\left(Q^{\rho\mu\nu} + W^\rho g^{\mu\nu}\right)$$

- new equation of motion for dilatation or Weyl current

$$\nabla_\rho W^\rho = \frac{2}{d}(T^\mu{}_\mu - \mathscr{U}^\mu{}_\mu)$$

- new equation of motion for shear current

$$\nabla_\rho Q^{\rho\mu\nu} = 2\left[T^{\mu\nu} - \mathscr{U}^{\mu\nu} - \frac{g^{\mu\nu}}{d}(T^\sigma{}_\sigma - \mathscr{U}^\sigma{}_\sigma)\right]$$

- non-conserved Noether currents

Spin current

[..., Floerchinger & Grossi, arXiv:2102.11098]

- tetrad formalism: vary tetrad ${V_\mu}^A$ and spin connection ${\Omega_\mu}^{AB}$

$$\delta\Gamma = \int d^d x \sqrt{g} \left\{ \mathscr{T}^\mu{}_A(x) \delta {V_\mu}^A(x) - \frac{1}{2} S^\mu{}_{AB}(x) \delta {\Omega_\mu}^{AB}(x) \right\}$$

  with
  - canonical energy-momentum tensor $\mathscr{T}^\mu{}_A$
  - spin current $S^\mu{}_{AB}$
- symmetric energy-momentum tensor in Belinfante-Rosenfeld form

$$T^{\mu\nu}(x) = \mathscr{T}^{\mu\nu}(x) + \frac{1}{2} \nabla_\rho \left[S^{\rho\mu\nu}(x) + S^{\mu\nu\rho}(x) + S^{\nu\mu\rho}(x) \right]$$

- equation of motion for spin current

$$\nabla_\mu S^{\mu\rho\sigma} = \mathscr{T}^{\sigma\rho} - \mathscr{T}^{\rho\sigma}$$

- non-conserved Noether current

Implications for relativistic fluid dynamics

- dilatation current, shear current and spin current provide additional information about quantum fields out-of-equilibrium
- their contribution to energy-momentum tensor vanishes in equilibrium or for ideal fluids
- new equations of motion for "non-hydrodynamic" modes

Non-Riemannian geometry

[Floerchinger & Grossi, arXiv:2102.11098]

- general connection

$$\begin{aligned}\Gamma_{\mu}{}^{\rho}{}_{\sigma} =& \frac{1}{2} g^{\rho\lambda} \left(\partial_\mu g_{\sigma\lambda} + \partial_\sigma g_{\mu\lambda} - \partial_\lambda g_{\mu\sigma} \right) + C_{\mu}{}^{\rho}{}_{\sigma} \\ & + \hat{B}_{\mu}{}^{\rho}{}_{\sigma} + \hat{B}_{\sigma\mu}{}^{\rho} - \hat{B}^{\rho}{}_{\mu\sigma} + B_\mu \delta^{\rho}{}_{\sigma} + B_\sigma \delta_{\mu}{}^{\rho} - B^\rho g_{\mu\sigma}.\end{aligned}$$

- contorsion $C_{\mu}{}^{\rho}{}_{\sigma}$ = gauge field for local Lorentz transformations
- Weyl gauge field B_μ = gauge field for local dilatations
- proper non-metricity $\hat{B}_{\mu}{}^{\rho}{}_{\sigma}$ = gauge field for local shear transformations
- local Lorentz transformations, dilatations and shear transformations together form the group GL(d) of basis changes in tangent space / the frame bundle
- all these transformations are *extended symmetries*: they change the quantum effective action Γ but in a specific way
- *extended symmetries* $\Rightarrow$ *non-conserved* Noether currents

Extended symmetries 1

[Floerchinger & Grossi, arXiv:2102.11098]

- consider transformation of fields

$$\phi(x) \to \phi(x) + i d\xi^j(x)\, T_j \phi(x)$$

- might be non-Abelian with structure constants

$$[T_k, T_l] = i f_{kl}{}^j T_j$$

- introduce external gauge field and covariant derivative

$$D_\mu \phi(x) = \left(\nabla_\mu - i A_\mu^j(x) T_j \right) \phi(x)$$

- gauge field transforms as usual

$$A_\mu^j(x) \to A_\mu^j(x) + f_{kl}{}^j A_\mu^k(x) d\xi^l(x) + \nabla_\mu d\xi^j(x)$$

Extended symmetries 2

[Floerchinger & Grossi, arXiv:2102.11098]

- change of effective action $\Gamma[\phi, A]$

$$\Gamma[\phi + id\xi^j T_j \phi, A^j_\mu + f_{kl}{}^j A^k_\mu d\xi^l + \nabla_\mu d\xi^j] = \Gamma[\phi] + \int d^d x \sqrt{g} \left\{ \mathcal{I}_j(x) \, d\xi^j(x) \right\}$$

- define current through

$$\mathscr{J}^\mu_j(x) = \frac{1}{\sqrt{g}} \frac{\delta \Gamma}{\delta A^j_\mu(x)}$$

- obtain conservation-type relation (for $\delta\Gamma/\delta\phi = 0$)

$$D_\mu \mathscr{J}^\mu_j(x) = \nabla_\mu \mathscr{J}^\mu_j(x) + f_{jk}{}^l A^k_\mu(x) \mathscr{J}^\mu_l(x) = -\mathcal{I}_j(x)$$

- global symmetry $\mathcal{I}_j(x) = 0 \Rightarrow$ conserved Noether current
- extended symmetry $\mathcal{I}_j(x) \neq 0$ but known at macroscopic level $\Rightarrow$ non-conserved Noether current

Conclusions

- new divergence-type equations of motion
- dilatation current, shear current and spin current
- quantum field theory in non-Riemannian geometry
- extended symmetries $\Rightarrow$ non-conserved Noether currents

Backup

One-particle irreducible or quantum effective action

- partition function $Z[J]$, Schwinger functional $W[J]$

$$Z[J] = \int D\chi \, e^{iS[\chi] - i \int_x \{J(x)\chi(x)\}}$$

- **quantum effective action** $\Gamma[\phi]$ defined by Legendre transform

$$\Gamma[\phi] = \int_x J(x)\phi(x) - W[J]$$

  with expectation values $\phi(x) = \delta W[J]/\delta J(x)$

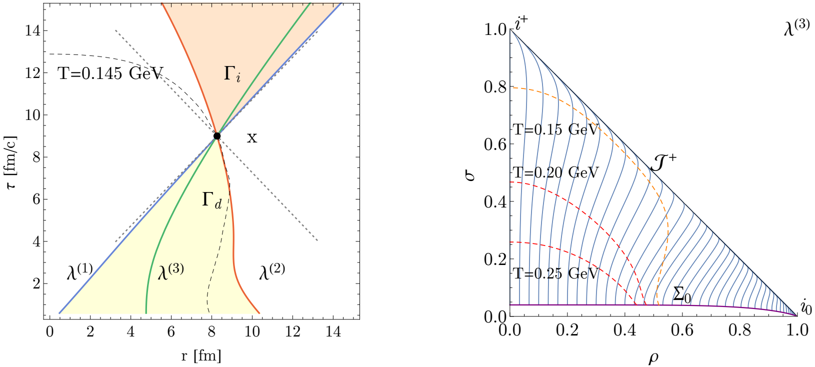
- includes all quantum and statistical fluctuations !
- equation of motion for field expectation values

$$\frac{\delta}{\delta\phi(x)}\Gamma[\phi] = J(x)$$

- functional renormalization group: flow equation for $\Gamma[\phi]$
- can be used in and out of equilibrium
  [e. g. Floerchinger, JHEP 1205, 021 (2012); JHEP 1609, 099 (2016)]

Causality

[Floerchinger & Grossi, JHEP 08 (2018) 186]



- inequalities for relativistic causality
- dissipative fluid equations *can* be of hyperbolic type
- characteristic velocities depend on fluid fields
- need $|\lambda^{(j)}| < c$ for relativistic causality