A quantum information perspective on relativistic fluid dynamics and quantum fields out-of-equilibrium

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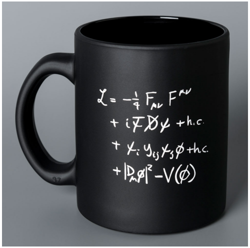
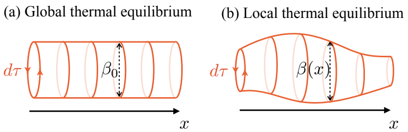
Understanding quantum field dynamics



- microscopic Lagrangian for many phenomena is known
- quantum field theories change with scale!
- need to understand quantum field dynamics away from simple limits
- important for condensed matter, optics, atomic physics, astrophysics, nuclear physics, cosmology, ...

What are the macroscopic evolution equations for quantum fields ?

Local equilibrium & partition function



- partition function $Z[J]$, Schwinger functional $W[J]$

$$Z[J] = e^{W[J]} = \int D\phi\, e^{-S[\phi]+\int_x J\phi}$$

- local equilibrium with $T(x)$ and $u^\mu(x)$
  [Floerchinger, JHEP 1609, 099 (2016)]

$$\beta^\mu(x) = \frac{u^\mu(x)}{T(x)}$$

- includes global equilibrium and vacuum as special cases

One-particle irreducible or quantum effective action

- **quantum effective action** $\Gamma[\phi]$ defined by Legendre transform

$$\Gamma[\Phi] = \int_x J_a(x)\Phi_a(x) - W[J]$$

with expectation values

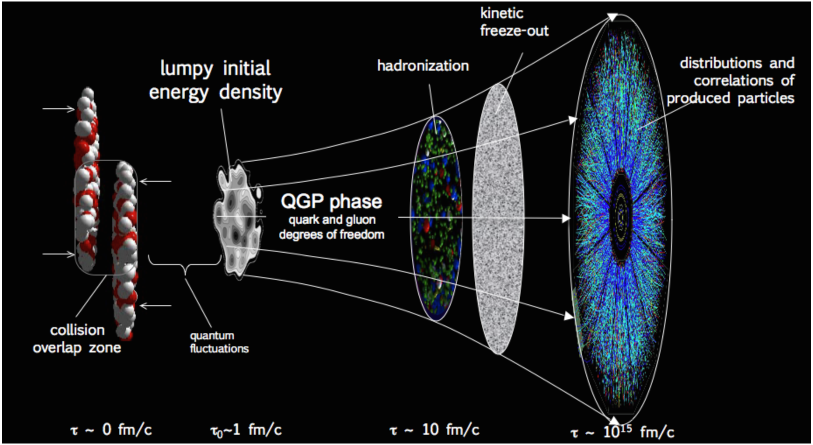
$$\Phi_a(x) = \frac{\delta}{\delta J_a(x)} W[J]$$

- Euclidean field equation

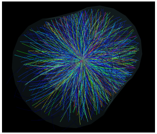
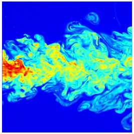
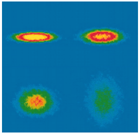
$$\frac{\delta}{\delta \Phi_a(x)} \Gamma[\Phi] = J(x)$$

- use **analytic continuation** to obtain **macroscopic evolution equations**
  [Floerchinger, JHEP 1205, 021 (2012); JHEP 1609, 099 (2016)]
- includes quantum and statistical fluctuations !
- imaginary terms lead to effective dissipation
- functional renormalization group: flow equation for $\Gamma[\Phi]$

High energy nuclear collisions: QCD fluid



Fluid dynamics

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear + bulk viscosity $\eta(T, \mu)$, $\zeta(T, \mu)$
  - heat conductivity $\kappa(T, \mu)$, ...
  - relaxation times, ...
  - electrical conductivity $\sigma(T, \mu)$
- fixed by **microscopic** properties encoded in Lagrangian $\mathscr{L}_{\text{QCD}}$
- old dream of condensed matter physics: understand the fluid properties!

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$

Covariant **conservation laws** $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for energy density ϵ
- equation for fluid velocity u^μ
- equation for particle number density n

Need in addition **constitutive relations** [e.g Israel & Stewart]

- equation for shear stress $\pi^{\mu\nu}$
- equation for bulk viscous pressure π_{bulk}

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \ldots + \pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu$$

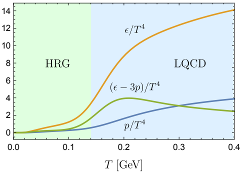
- equation for diffusion current ν^μ

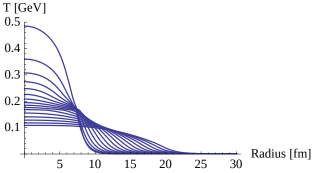
Flow and fluctuations in heavy ion collisions

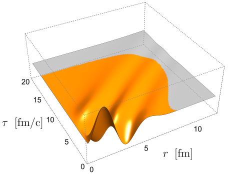
FluiduM: Fluid dynamics of heavy ion collisions with Mode expansion

[Floerchinger & Wiedemann, PLB 728, 407 (2014), PRC 88, 044906 (2013), 89, 034914 (2014)]

[Floerchinger, Grossi & Lion, PRC 100, 014905 (2019)]



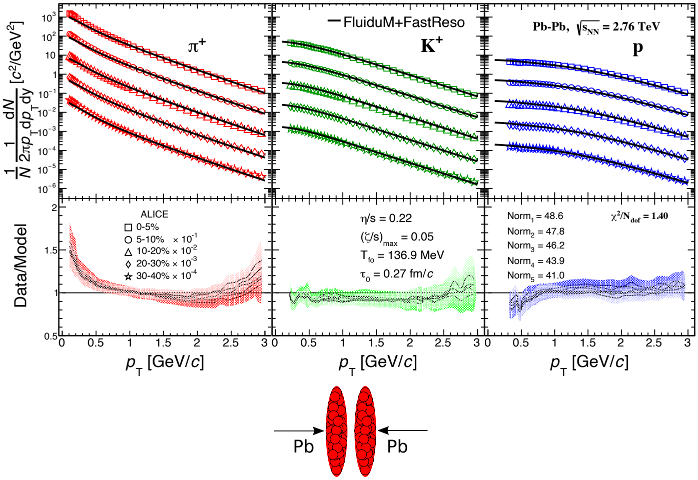




- background-fluctuation splitting + mode expansion
- analogous to cosmological perturbation theory
- substantially improved numerical performance (pseudospectral method)
- resonance decays included
  [Mazeliauskas, Floerchinger, Grossi & Teaney, EPJC 79, 284 (2019)]
- allows fast and precise comparison between theory and experiment

Particle production at the Large Hadron Collider

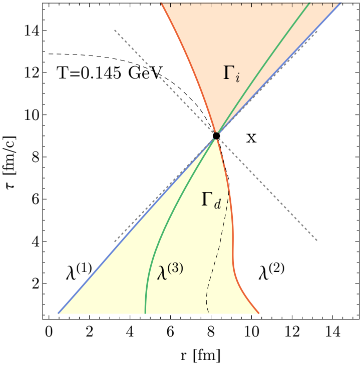
[Devetak, Dubla, Floerchinger, Grossi, Masciocchi, Mazeliauskas & Selyuzhenkov, 1909.10485]

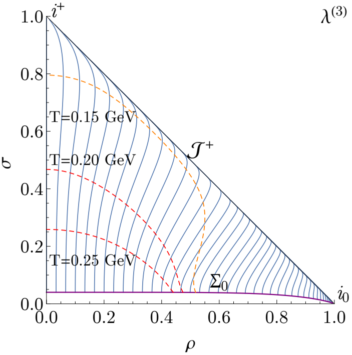


- data are very precise now - high quality theory development needed!
- next step: include coherent fields / condensates

Causality

[Floerchinger & Grossi, JHEP 08 (2018) 186]





- inequalities for relativistic causality
- dissipative fluid equations *can* be of hyperbolic type
- characteristic velocities depend on fluid fields
- need $|\lambda^{(j)}| < c$ for relativistic causality

Entropy current, local dissipation and unitarity

- local dissipation = local entropy production

$$\nabla_\mu s^\mu(x) \geq 0$$

- e. g. from analytically continued quantum effective action
[Floerchinger, JHEP 1609, 099 (2016)]
- fluid dynamics in Navier-Stokes approximation

$$\nabla_\mu s^\mu = \frac{1}{T}\left[2\eta\sigma_{\mu\nu}\sigma^{\mu\nu} + \zeta(\nabla_\rho u^\rho)^2\right] \geq 0$$

- unitary time evolution conserves von-Neumann entropy

$$S = -\mathsf{Tr}\{\rho \ln \rho\} = -\mathsf{Tr}\{(U\rho U^\dagger)\ln(U\rho U^\dagger)\} \qquad \Rightarrow \qquad \frac{d}{dt}S = 0$$

quantum information is globally conserved

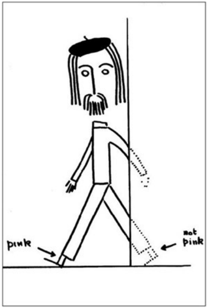
What is local dissipation in isolated quantum systems ?

Quantum entanglement

- *Can quantum-mechanical description of physical reality be considered complete?* [Einstein, Podolsky, Rosen (1935), Bohm (1951)]

$$\psi = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B \right)$$
$$= \frac{1}{\sqrt{2}} \left(|\rightarrow\rangle_A |\leftarrow\rangle_B - |\leftarrow\rangle_A |\rightarrow\rangle_B \right)$$

- *Bertlemann's socks and the nature of reality* [Bell (1980)]



Bell's inequalities and Bell tests

[John Stewart Bell (1966)]

- most popular version [Clauser, Horne, Shimony, Holt (1969)]

$$S = |E(a,b) - E(a,b') + E(a',b) + E(a',b')| \leq 2$$

  holds for local hidden variable theories
- expectation value of product of two observables

$$E(a,b) = \langle A(a)B(b) \rangle$$

  with possible values $A = \pm 1$, $B = \pm 1$.
- depending on measurement settings a, a' and b, b' respectively
- quantum mechanical bound is $S \leq 2\sqrt{2}$
- experimental values $2 < S \leq 2\sqrt{2}$ rule out local hidden variables
- one measurement setting but at different times [Leggett, Garg (1985)]

Entanglement in high energy (QCD) physics

- entanglement of *quantum fields* instead of *particles*
- entanglement on sub-nucleonic scales
- entanglement in non-Abelian gauge theory / color / confinement
- discussions in mathematical physics [e. g. Witten (2018)]
- connections to black holes and holography [Ryu & Takayanagi (2006)]
- thermalization in closed quantum systems

Classical statistics

- consider system of two random variables x and y
- joint probability $p(x, y)$, joint entropy

$$S = -\sum_{x,y} p(x, y) \ln p(x, y)$$

- reduced or marginal probability $p(x) = \sum_y p(x, y)$
- reduced or marginal entropy

$$S_x = -\sum_x p(x) \ln p(x)$$

- one can prove: **joint entropy is greater than** or equal to **reduced entropy**

$$S \geq S_x$$

- **globally pure** state $S = 0$ is also **locally pure** $S_x = 0$

Quantum statistics

- consider system with two subsystems A and B
- combined state ρ , combined or full entropy

$$S = -\mathsf{Tr}\{\rho \ln \rho\}$$

- reduced density matrix $\rho_A = \mathsf{Tr}_B\{\rho\}$
- reduced or **entanglement entropy**

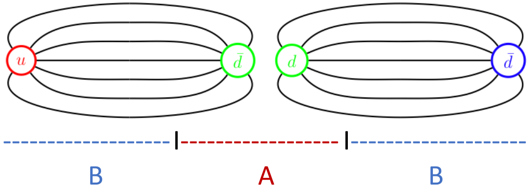
$$S_A = -\mathsf{Tr}_A\{\rho_A \ln \rho_A\}$$

- pure **product** state $\rho = \rho_A \otimes \rho_B$ leads to $S_A = 0$
- pure **entangled** state $\rho \neq \rho_A \otimes \rho_B$ leads to $S_A > 0$
- for quantum systems **entanglement makes a difference**

$$S \ngeq S_A$$

- **coherent information** $I_{B\rangle A} = S_A - S$ can be **positive**!
- **globally pure** state $S = 0$ can be **locally mixed** $S_A > 0$

Entanglement, QCD strings and thermalization



- hadronization in Lund string model (e. g. PYTHIA)
- reduced density matrix for region A

$$\rho_A = \mathsf{Tr}_B\{\rho\}$$

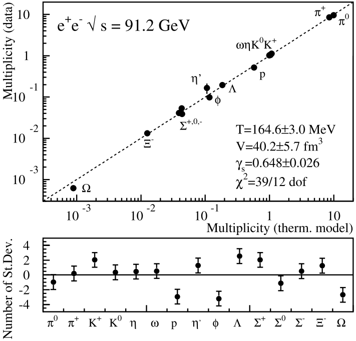
has entanglement entropy

$$S_A = -\mathsf{Tr}_A\{\rho_A \ln \rho_A\} > 0$$

- could this lead to thermal-like effects?

The thermal model puzzle

- elementary particle collision experiments such as $e^+ \ e^-$ collisions show some thermal-like features [see also Fischer & Sjöstrand (2017)]
- particle multiplicities well described by thermal model



[Becattini, Casterina, Milov & Satz, EPJC 66, 377 (2010)]

- conventional thermalization by collisions unlikely
- alternative explanations needed

Microscopic model

- QCD in 1+1 dimensions described by 't Hooft model

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - i g \mathbf{A}_\mu) \psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{2} \mathrm{tr}\, \mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu}$$

- fermionic fields ψ_i with sums over flavor species $i = 1, \ldots, N_f$
- SU(N_c) gauge fields $\mathbf{A}_\mu$ with field strength tensor $\mathbf{F}_{\mu\nu}$
- gluons are not dynamical in two dimensions
- gauge coupling g has dimension of mass
- non-trivial, interacting theory, cannot be solved exactly
- spectrum of excitations known for $N_c \to \infty$ with $g^2 N_c$ fixed
  ['t Hooft (1974)]

Schwinger model

- QED in 1+1 dimension

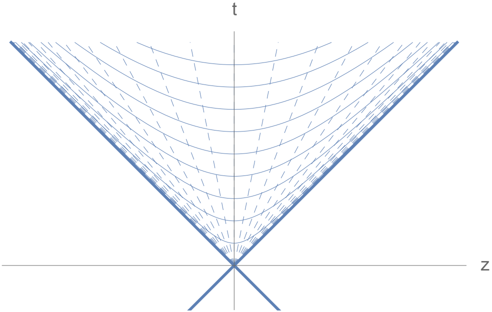
$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - iqA_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- geometric confinement
- U(1) charge related to string tension $q = \sqrt{2\sigma}$
- for single fermion one can **bosonize theory** exactly
  [Coleman, Jackiw, Susskind (1975)]

$$S = \int d^2x \sqrt{g} \Big\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 \phi^2 \\ - \frac{m\, q\, e^\gamma}{2\pi^{3/2}} \cos\left(2\sqrt{\pi}\phi + \theta\right) \Big\}$$

- Schwinger bosons are dipoles $\phi \sim \bar{\psi}\psi$
- scalar mass related to U(1) charge by $M = q/\sqrt{\pi} = \sqrt{2\sigma/\pi}$
- massless Schwinger model $m = 0$ leads to free bosonic theory

Expanding string solution



- external quark-anti-quark pair on trajectories $z = \pm t$
- coordinates: Bjorken time $\tau = \sqrt{t^2 - z^2}$, rapidity $\eta = \text{arctanh}(z/t)$
- metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$
- symmetry with respect to longitudinal boosts $\eta \to \eta + \Delta\eta$

Expanding string solution 2

- Schwinger boson field depends only on τ

$$\bar{\phi} = \bar{\phi}(\tau)$$

- equation of motion

$$\partial_\tau^2 \bar{\phi} + \frac{1}{\tau} \partial_\tau \bar{\phi} + M^2 \bar{\phi} = 0.$$

- Gauss law: electric field $E = q\phi/\sqrt{\pi}$ must approach the U(1) charge of the external quarks $E \to q_\mathsf{e}$ for $\tau \to 0_+$

$$\bar{\phi}(\tau) \to \frac{\sqrt{\pi} q_\mathsf{e}}{q} \qquad (\tau \to 0_+)$$

- solution of equation of motion [Loshaj, Kharzeev (2011)]

$$\bar{\phi}(\tau) = \frac{\sqrt{\pi} q_\mathsf{e}}{q} J_0(M\tau)$$

Gaussian states

- theories with quadratic action often have Gaussian density matrix
- fully characterized by field expectation values

$$\bar{\phi}(x) = \langle \phi(x) \rangle, \qquad \bar{\pi}(x) = \langle \pi(x) \rangle$$

  and connected two-point correlation functions, e. g.

$$\langle \phi(x)\phi(y) \rangle_c = \langle \phi(x)\phi(y) \rangle - \bar{\phi}(x)\bar{\phi}(y)$$

- if ρ is Gaussian, also reduced density matrix ρ_A is Gaussian

Entanglement entropy for Gaussian state

- entanglement entropy of Gaussian state in region A
  [Berges, Floerchinger, Venugopalan, JHEP 1804 (2018) 145]

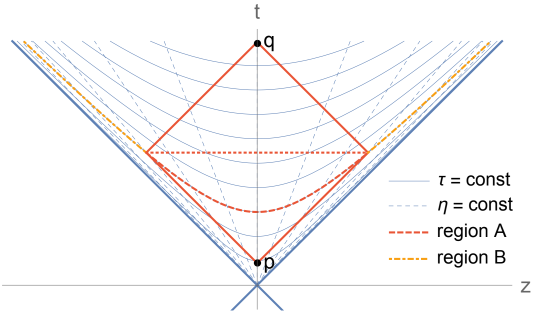
$$S_A = \frac{1}{2} \mathsf{Tr}_A \left\{ D \ln(D^2) \right\}$$

- operator trace over region A only
- matrix of correlation functions

$$D(x,y) = \begin{pmatrix} -i\langle\phi(x)\pi(y)\rangle_c & i\langle\phi(x)\phi(y)\rangle_c \\ -i\langle\pi(x)\pi(y)\rangle_c & i\langle\pi(x)\phi(y)\rangle_c \end{pmatrix}$$

- involves connected correlation functions of field $\phi(x)$ and canonically conjugate momentum field $\pi(x)$
- expectation value $\bar{\phi}$ does not appear explicitly
- coherent states and vacuum have equal entanglement entropy S_A

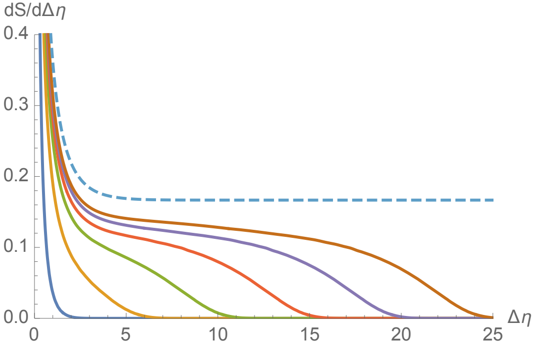
Rapidity interval



- consider rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ at fixed Bjorken time τ
- entanglement entropy does not change by unitary time evolution with endpoints kept fixed
- can be evaluated equivalently in interval $\Delta z = 2\tau \sinh(\Delta\eta/2)$ at fixed time $t = \tau \cosh(\Delta\eta/2)$
- need to solve eigenvalue problem with correct **boundary conditions**

Bosonized massless Schwinger model

- entanglement entropy understood numerically for free massive scalars [Casini, Huerta (2009)]
- entanglement entropy density $dS/d\Delta\eta$ for bosonized massless Schwinger model ($M = \frac{q}{\sqrt{\pi}}$)



$M\tau = 1,\ 10^{-1},\ 10^{-2},\ 10^{-3},\ 10^{-4}$, and 10^{-5}

Conformal limit

- For $M\tau \to 0$ one has conformal field theory limit
  [Holzhey, Larsen, Wilczek (1994)]

$$S(\Delta z) = \frac{c}{3} \ln\left(\Delta z/\epsilon\right) + \text{constant}$$

  with small length ϵ acting as UV cutoff.

- Here this implies

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln\left(2\tau \,\text{sinh}(\Delta\eta/2)/\epsilon\right) + \text{constant}$$

- Conformal charge $c = 1$ for free massless scalars or Dirac fermions.
- Additive constant not universal but entropy density is

$$\begin{aligned} \frac{\partial}{\partial \Delta\eta} S(\tau, \Delta\eta) =& \frac{c}{6} \text{coth}(\Delta\eta/2) \\ \to& \frac{c}{6} \qquad\qquad (\Delta\eta \gg 1) \end{aligned}$$

- Entropy becomes extensive in $\Delta\eta$!

Temperature and entanglement entropy

- for conformal fields, entanglement entropy has also been calculated at non-zero temperature.
- for static interval of length L [Korepin (2004); Calabrese, Cardy (2004)]

$$S(T,l) = \frac{c}{3}\ln\left(\frac{1}{\pi T\epsilon}\sinh(\pi LT)\right) + \text{const}$$

- compare this to our result in expanding geometry

$$S(\tau,\Delta\eta) = \frac{c}{3}\ln\left(\frac{2\tau}{\epsilon}\,\text{sinh}(\Delta\eta/2)\right) + \text{const}$$

- expressions agree for $L = \tau\Delta\eta$ (with metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$) and time-dependent temperature

$$T = \frac{1}{2\pi\tau}$$

Universal entanglement entropy density

- for very early times "Hubble" expansion rate dominates over masses and interactions

$$H = \frac{1}{\tau} \gg M = \frac{q}{\sqrt{\pi}}, m$$

- theory dominated by free, massless fermions
- universal entanglement entropy density

$$\frac{dS}{d\Delta\eta} = \frac{c}{6}$$

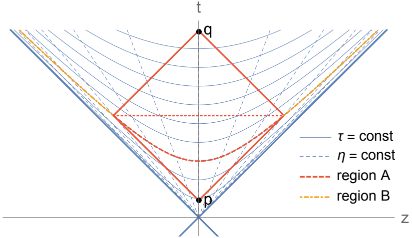
  with conformal charge c
- for QCD in 1+1 D (gluons not dynamical, no transverse excitations)

$$c = N_c \times N_f$$

- from fluctuating transverse coordinates (Nambu-Goto action)

$$c = N_c \times N_f + 2 \approx 9 + 2 = 11$$

Local density matrix and temperature in expanding string



- Bjorken time $\tau = \sqrt{t^2 - z^2}$, rapidity $\eta = \text{arctanh}(z/t)$
- **local density matrix thermal at early times as result of entanglement**
  [Berges, Floerchinger, Venugopalan, PLB778, 442 (2018); JHEP 1804 (2018) 145]

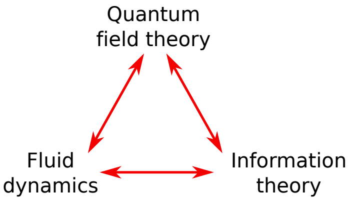
$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

- Hawking-Unruh temperature in Rindler space $T(x) = \frac{\hbar c}{2\pi x}$

Physics picture

- coherent state at early time contains entangled pairs of quasi-particles with opposite wave numbers
- on finite rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ in- and out-flux of quasi-particles with thermal distribution via boundaries
- technically limits $\Delta\eta \to \infty$ and $M\tau \to 0$ do not commute
  - $\Delta\eta \to \infty$ for any finite $M\tau$ gives pure state
  - $M\tau \to 0$ for any finite $\Delta\eta$ gives thermal state with $T = 1/(2\pi\tau)$

Quantum field dynamics

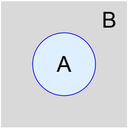


- new hypothesis

local dissipation = *quantum entanglement generation*

- quantum information is spread
- locally, quantum state approaches mixed state form
- full loss of *local* quantum information = *local* thermalization

Entanglement entropy in quantum field theory



- entanglement entropy of region A is a local notion of entropy

$$S_A = -\text{tr}_A \{\rho_A \ln \rho_A\} \qquad \rho_A = \text{tr}_B \{\rho\}$$

- however, it is infinite already in vacuum state

$$S_A = \frac{\text{const}}{\epsilon^{d-2}} \int_{\partial A} d^{d-2}\sigma \sqrt{h} \; + \; \text{subleading divergences} \; + \; \text{finite}$$

- UV divergence proportional to entangling surface
- quantum fields are very strongly entangled already in vacuum
- Theorem [Reeh & Schlieder (1961)]: local operators in region A can create all particle states

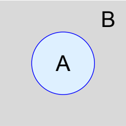
Relative entropy

- **relative entropy** of two density matrices

$$S(\rho|\sigma) = \mathrm{tr}\,\{\rho\,(\ln\rho - \ln\sigma)\}$$

- measures how well state ρ can be distinguished from a model σ
- Gibbs inequality: $S(\rho|\sigma) \geq 0$
- $S(\rho|\sigma) = 0$ if and only if $\rho = \sigma$
- quantum generalization of Kullback-Leibler divergence

Relative entanglement entropy



- consider now reduced density matrices

$$\rho_A = \mathsf{Tr}_B\{\rho\}, \qquad \sigma_A = \mathsf{Tr}_B\{\sigma\}$$

- define **relative entanglement entropy**

$$S_A(\rho|\sigma) = \mathsf{Tr}\left\{\rho_A\left(\ln \rho_A - \ln \sigma_A\right)\right\} = -\mathsf{Tr}\left\{\rho_A \ln \Delta_A\right\}$$

  with relative modular operator Δ_A

- measures how well ρ is represented by σ locally in region A
- UV divergences cancel: contains real physics information
- well defined in algebraic quantum field theory [Araki (1977)]
  [see also works by Casini, Myers, Lashkari, Witten, Liu, ...]

An approximate local description

[Dowling, Floerchinger & Haas, PRD 102 (2020) 10, 105002]

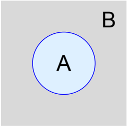
- consider non-equilibrium situation with
  - true density matrix ρ
  - local equilibrium approximation

$$\sigma = \frac{1}{Z} e^{-\int d\Sigma_\mu \{\beta_\nu(x) T^{\mu\nu} + \alpha(x) N^\mu\}}$$

- reduced density matrices $\rho_A = \text{Tr}_B\{\rho\}$ and $\sigma_A = \text{Tr}_B\{\sigma\}$
- σ is very good model for ρ in region A when

$$S_A = \text{Tr}_A\{\rho_A(\ln \rho_A - \ln \sigma_A)\} \to 0$$

- does *not* imply that globally $\rho = \sigma$



Monotonicity of relative entropy

- monotonicity of relative entropy [Lindblad (1975)]

$$S(\mathcal{N}(\rho)|\mathcal{N}(\sigma)) \leq S(\rho|\sigma)$$

  with $\mathcal{N}$ completely positive, trace-preserving map

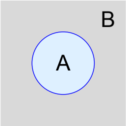
- $\mathcal{N}$ unitary evolution

$$S(\mathcal{N}(\rho)|\mathcal{N}(\sigma)) = S(\rho|\sigma)$$

- $\mathcal{N}$ open system evolution with generation of entanglement to environment

$$S(\mathcal{N}(\rho)|\mathcal{N}(\sigma)) < S(\rho|\sigma)$$

- leads to local, second law type relation
  [Dowling, Floerchinger & Haas, PRD 102 (2020) 10, 105002]



Entropy production

[Floerchinger, JHEP 1609, 099 (2016)]

- variational principle with effective dissipation from analytic continuation
- analysis of general covariance leads to entropy current and local entropy production

$$\nabla_\mu s^\mu = \frac{1}{\sqrt{g}} \frac{\delta \Gamma_D}{\delta \Phi_a}\Big|_{\text{ret}} \beta^\lambda \partial_\lambda \Phi_a + \beta_\mu \nabla_\nu \left(- \frac{2}{\sqrt{g}} \frac{\delta \Gamma_D}{\delta g_{\mu\nu}}\Big|_{\text{ret}} \right)$$

- can likely be understood as entanglement generation

Entropic uncertainty relations

Heisenberg / Robertson uncertainty relation [Robertson (1929)]

$$\sigma(X)\sigma(Z) \geq \frac{1}{2}|\langle\psi|[X,Z]|\psi\rangle|$$

Entropic uncertainty relations [Maassen & Uffink (1988), Frank & Lieb (2012)]

$$H(X) + H(Z) \geq \ln\frac{1}{c} + S(\rho)$$

- Shannon information entropy for measurement outcome

$$H(X) = -\sum_x p(x)\ln p(x)$$

- von-Neumann entropy

$$S(\rho) = -\mathrm{Tr}\{\rho\ln\rho\}$$

- maximal overlap between basis states

$$c = \max_{x,z}|\langle x|z\rangle|^2$$

- formulation in terms of relative entropy [Floerchinger, Haas & Hoeber, 2012.10080]

Entanglement and entropic uncertainty relations

[Berta et al. (2010)]

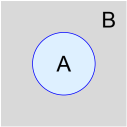
- side information from entanglement with system B

$$H(X_A|X_B) + H(Z_A|Z_B) \geq \ln \frac{1}{c} + S(A|B)$$

- use measurement on B to infer outcome on A
- quantum conditional entropy can be negative for positive coherent information

$$S(A|B) = S(\rho) - S(\rho_B) = -I_{A\rangle B}$$

- experiments with cold atoms [with M. Gärttner and M. Oberthaler]
- towards test of *local dissipation = quantum entanglement generation*
- more applications in nuclear and high energy physics to be explored

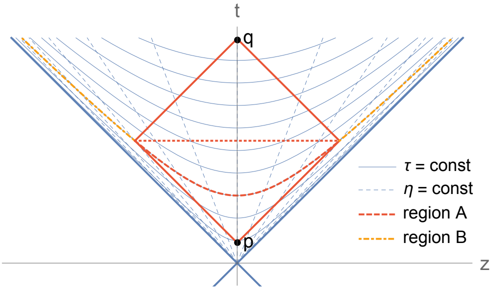


Conclusions

- relativistic fluid dynamics has a foundation in quantum information theory
- proper description of local thermalization in terms of relative entanglement
- quantum field theoretic description with two density matrices:
  - true density matrix ρ evolves unitary
  - fluid model σ agrees locally but evolves non-unitary
- local "thermalization" without collisions possible
- need to test the picture with more calculations and experiments
- entropic uncertainty relations may allow to access entanglement entropies

Backup

Modular or entanglement Hamiltonian 1



- conformal field theory
- hypersurface Σ with boundary on the intersection of two light cones
- reduced density matrix [Casini, Huerta, Myers (2011), Arias, Blanco, Casini, Huerta (2017), see also Candelas, Dowker (1979)]

$$\rho_A = \frac{1}{Z_A} e^{-K}, \qquad Z_A = \mathrm{Tr}\, e^{-K}$$

- modular or entanglement Hamiltonian K

Modular or entanglement Hamiltonian 2

- modular or entanglement Hamiltonian is **local expression**

$$K = \int_\Sigma d\Sigma_\mu \, \xi_\nu(x) \, T^{\mu\nu}(x).$$

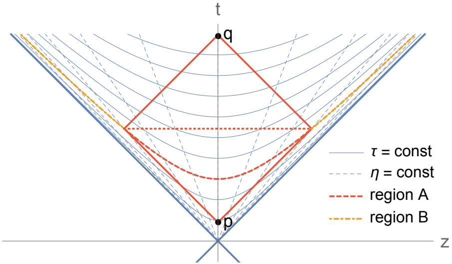
- energy-momentum tensor $T^{\mu\nu}(x)$ of excitations
- vector field

$$\xi^\mu(x) = \tfrac{2\pi}{(q-p)^2} [(q-x)^\mu (x-p)(q-p) \\ + (x-p)^\mu (q-x)(q-p) - (q-p)^\mu (x-p)(q-x)]$$

  end point of future light cone q, starting point of past light cone p
- inverse temperature and fluid velocity

$$\xi^\mu(x) = \beta^\mu(x) = \frac{u^\mu(x)}{T(x)}$$

Modular or entanglement Hamiltonian 3



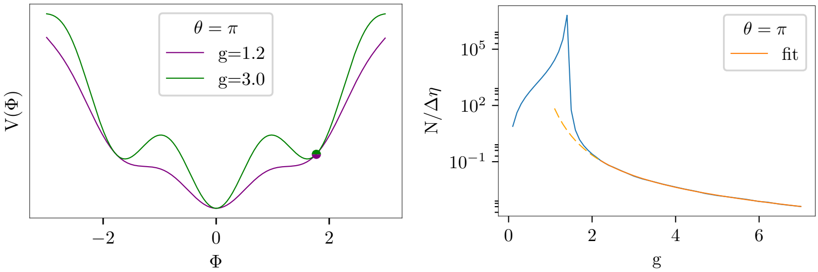
- for $\Delta\eta \to \infty$: fluid velocity in τ-direction, τ-dependent temperature

$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

- **Entanglement between different rapidity intervals alone leads to local thermal density matrix at very early times !**
- Hawking-Unruh temperature in Rindler wedge $T(x) = \hbar c/(2\pi x)$

Particle production in massive Schwinger model

[ongoing work with Lara Kuhn, Jürgen Berges]



- for expanding strings
- asymptotic particle number depends on $g \sim m/q$
- exponential suppression for large fermion mass $g \gg 1$

$$\frac{N}{\Delta\eta} \sim e^{-0.55\frac{m}{q}+7.48\frac{q}{m}+\dots} = e^{-0.55\frac{m}{\sqrt{2\sigma}}+7.48\frac{\sqrt{2\sigma}}{m}+\dots}$$

Wigner distribution and entanglement

- Classical field approximation usually based on non-negative Wigner representation of density matrix
- leads for many observables to classical statistical description
- can nevertheless show entanglement and pass Bell test for "improper" variables where Weyl transform of operator has values outside of its spectrum [Revzen, Mello, Mann, Johansen (2005)]
- Bell test violation also possible for negative Wigner distribution [Bell (1986)]

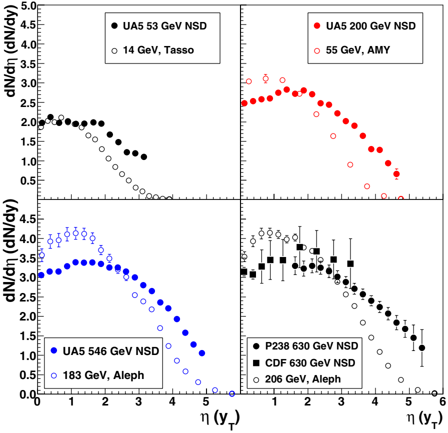
Transverse coordinates

- so far dynamics strictly confined to 1+1 dimensions
- transverse coordinates may fluctuate, can be described by Nambu-Goto action ($h_{\mu\nu} = \partial_\mu X^m \partial_\nu X_m$)

$$
\begin{aligned}
S_{\mathsf{NG}} &= \int d^2x \sqrt{-\mathsf{det}\, h_{\mu\nu}}\, \{-\sigma + \ldots\} \\
&\approx \int d^2x \sqrt{g} \left\{-\sigma - \frac{\sigma}{2} g^{\mu\nu} \partial_\mu X^i \partial_\nu X^i + \ldots \right\}
\end{aligned}
$$

- two additional, massless, bosonic degrees of freedom corresponding to transverse coordinates X^i with $i = 1, 2$

Rapidity distribution



[open (filled) symbols: e^+e^- (pp), Grosse-Oetringhaus & Reygers (2010)]

- rapidity distribution $dN/d\eta$ has plateau around midrapidity
- only logarithmic dependence on collision energy

Experimental access to entanglement ?

- could longitudinal entanglement be tested experimentally?
- unfortunately entropy density $dS/d\eta$ not straight-forward to access
- measured in e^+e^- is the number of charged particles per unit rapidity $dN_{\mathsf{ch}}/d\eta$ (rapidity defined with respect to the thrust axis)
- typical values for collision energies $\sqrt{s} = 14 - 206$ GeV in the range

$$dN_{\mathsf{ch}}/d\eta \approx 2 - 4$$

- entropy per particle S/N can be estimated for a hadron resonance gas in thermal equilibrium $S/N_{\mathsf{ch}} = 7.2$ would give

$$dS/d\eta \approx 14 - 28$$

- this is an upper bound: correlations beyond one-particle functions would lead to reduced entropy

Entanglement and QCD physics

- how strongly entangled is the nuclear wave function?
- what is the entropy of quasi-free partons and can it be understood as a result of entanglement? [Kharzeev, Levin (2017)]
- does saturation at small Bjorken-x have an entropic meaning?
- entanglement entropy and entropy production in the color glass condensate [Kovner, Lublinsky (2015); Kovner, Lublinsky, Serino (2018)]
- could entanglement entropy help for a non-perturbative extension of the parton model?
- entropy of perturbative and non-perturbative Pomeron descriptions [Shuryak, Zahed (2017)]