Physics of net-charge fluctuations: theory and phenomenology

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Heavy-ion meeting, CEA Saclay, December 19, 2019.

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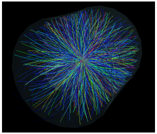
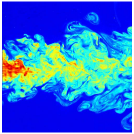
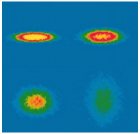
STRUCTURES
CLUSTER OF EXCELLENCE

Content

- Fluid dynamics
- Thermodynamics
- Moments and cumulants
- Differential correlation functions
- Freeze-out with correlation functions
- Transport of conserved charges
- Fluid dynamics with Mode expansion (FLUIDUM)

Fluid dynamics

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - equation of state $p(T, \mu)$
  - shear + bulk viscosity
  - heat conductivity / baryon diffusion constant, ...
- fixed by **microscopic** properties of QCD encoded in Lagrangian
- old dream of condensed matter physics: understand the fluid properties!

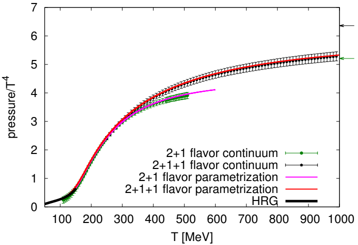
Thermodynamic equation of state

- describes volume V with temperature T and chemical potentials μ_B, μ_C and μ_S associated with conserved baryon, charge and strangeness numbers
- exchange of energy and particles with heat bath
- can be simulated with Lattice QCD
- all thermodynamic properties follow from

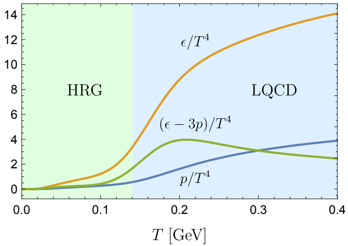
$$p(T, \mu_B, \mu_Q, \mu_S)$$

- chemical potentials
    - μ_B for (net) baryon number
    - μ_Q for (net) electric charge
    - μ_S for (net) strangeness

Thermodynamics of QCD

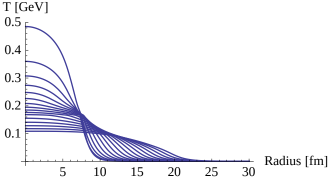


[Borsányi *et al.* (2016)], similar Bazavov *et al.* (2014)



[Floerchinger, Grossi & Lion (2018)]

- thermodynamic equation of state $p(T)$ rather well understood now
- used for fluid dynamics at LHC energies



Moments and cumulants at equilibrium

- mean value of net baryon number

$$\bar{N}_B = \langle N_B \rangle = V \frac{\partial}{\partial \mu_B} p(T, \mu_B, \mu_Q, \mu_S)$$

- variance in terms of $\delta N_B = N_B - \bar{N}_B$

$$\sigma_B^2 = \langle \delta N_B^2 \rangle = TV \frac{\partial^2}{\partial \mu_B^2} p(T, \mu_B, \mu_Q, \mu_S)$$

- skewness

$$S_B = \frac{\langle \delta N_B^3 \rangle}{\sigma_B^3} = \frac{1}{\sigma_B^3} T^2 V \frac{\partial^3}{\partial \mu_B^3} p(T, \mu_B, \mu_Q, \mu_S)$$

- kurtosis

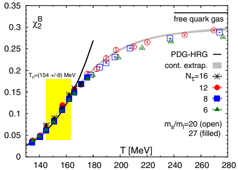
$$\kappa_B = \frac{\langle \delta N_B^4 \rangle - 3\langle \delta N_B^2 \rangle^2}{\sigma_B^4} = \frac{1}{\sigma_B^4} T^3 V \frac{\partial^4}{\partial \mu_B^4} p(T, \mu_B, \mu_Q, \mu_S)$$

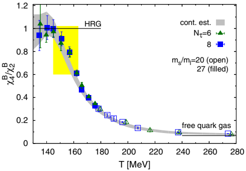
- similar for mixed derivatives

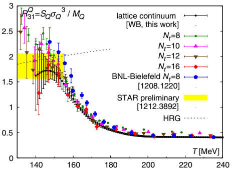
Lattice QCD results for cumulants

- Lattice QCD results for

$$\chi_2^B = \frac{\sigma_B^2}{VT^3} = \frac{\langle \delta N_B^2 \rangle}{VT^3} \qquad \frac{\chi_4^B}{\chi_2^B} = \frac{\langle \delta N_B^4 \rangle}{\langle \delta N_B^2 \rangle} \qquad R_{31}^Q = \frac{\chi_3^Q}{\chi_1^Q} = \frac{\langle \delta N_Q^3 \rangle}{\langle \delta N_Q \rangle}$$







[Bazavov *et al.* (2017), similar Bellwied *et al.* (2015)] [Borsányi *et al.* (2013)]

- Hadron resonance gas (HRG) approximation works at small temperatures

Hadron resonance gas

- pressure for free hadrons and resonances with vacuum masses

$$p = \frac{T^2}{\pi^2} \sum_i d_i m_i^2 K_2 \left(\frac{m_i}{T} \right) \cosh \left(\frac{B_i \mu_B + Q_i \mu_Q + S_i \mu_S}{T} \right)$$

- implies relations like

$$\kappa_B \sigma_B^2 = \frac{T^2 \frac{\partial^4}{\partial \mu_B^4} p}{\frac{\partial^2}{\partial \mu_B^2} p} = \frac{\langle B_i^4 \rangle}{\langle B_i^2 \rangle} = 1, \qquad \kappa_B M_B = S_B \sigma_B ,$$

when only baryons with $B_i = \pm 1$ contribute

- and for $\mu_S = \mu_Q = 0$ one has relations like

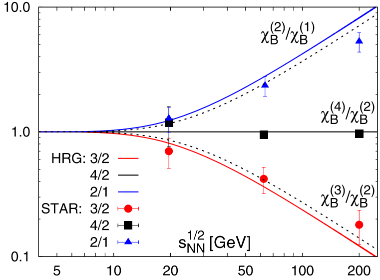
$$S_B \sigma_B = \frac{T \frac{\partial^3}{\partial \mu_B^3} p}{\frac{\partial^2}{\partial \mu_B^2} p} = \tanh \left(\frac{\mu_B}{T} \right)$$

Hadron resonance gas versus experiment

- ratios of cumulants are independent of volume V and less sensitive to kinematic cuts

$$\frac{\chi_B^{(2)}}{\chi_B^{(1)}} = \frac{\sigma_q^2}{M_q}, \qquad \frac{\chi_B^{(3)}}{\chi_B^{(2)}} = S_q\sigma_q, \qquad \frac{\chi_B^{(4)}}{\chi_B^{(2)}} = \kappa_q\sigma_q^2$$

- particularly well suited to compare to experiment



Data: STAR, Lines: HRG. [F. Karsch, K. Redlich, PLB 695, 136 (2011)]

Moments versus differential correlation functions

- problem 1: what is optimal range of acceptance?
- full coverage for ^{208}Pb - ^{208}Pb: no fluctuations at all

$$N_B = 2 \times 208 = 416, \qquad N_Q = 2 \times 82 = 164, \qquad N_S = 0.$$

- too small coverage: Poisson statistics
- problem 2: fireball is not in thermal equilibrium
- approximate local equilibrium $\hat{=}$ viscous fluid dynamics
- need **more differential description** including dependence on rapidity, azimuthal angle and transverse momentum

Correlation functions as generalized moments / cumulants

- correlation function of baryon number density

$$C_2^{(B,B)}(t,\vec{x};t',\vec{x}') = \langle n_B(t,\vec{x})\, n_B(t',x')\rangle - \langle n_B(t,\vec{x})\rangle\langle n_B(t',\vec{x}')\,\rangle$$

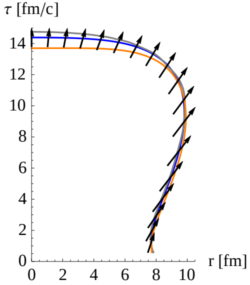
- integral over equal time correlation gives variance

$$\sigma_B^2 = \langle \delta N_B^2\rangle = \int_V d^3x \int_V d^3x'\; C_2^{(B,B)}(t,\vec{x};t,\vec{x}')$$

- similar for higher order correlation functions
- thermodynamic variables can be traded

$$(\epsilon, n_B, n_Q, n_S) \quad \leftrightarrow \quad (T, \mu_B, \mu_Q, \mu_S)$$

Cooper-Frye freeze-out



- single particle distribution [Cooper & Frye (1974)]

$$E\frac{dN_i}{d^3p} = -p^\mu \int_{\Sigma_f} \frac{d\Sigma_\mu}{(2\pi)^3}\, f_i(p;x)$$

  with close-to equilibrium distribution

$$f_i(p;x) = f_i(p;T(x),\mu_i(x),u^\mu(x),\pi^{\mu\nu}(x),\varphi(x),\ldots)$$

- precise position of freeze-out surface is unknown, usual assumption

$$\langle T(x)\rangle = T_{\mathrm{fo}} = \mathrm{const}$$

Particle correlations from freeze-out

[Floerchinger & Guenduez, work in progress]

- can be used for expectation values...

$$\left\langle E \frac{dN_i}{d^3p} \right\rangle = \left\langle -p_\mu \int_{\Sigma_f} \frac{d\Sigma^\mu}{(2\pi)^3} \, f_i(p;x) \right\rangle$$

- ... but also for correlation functions

$$\left\langle E \frac{dN_i}{d^3p} E' \frac{dN_j}{d^3p'} \right\rangle = p_\mu p'_\nu \int_{\Sigma_f} \frac{d\Sigma^\mu}{(2\pi)^3} \frac{d\Sigma'^\nu}{(2\pi)^3} \left\langle f_i(p;x) \, f_j(p';x') \right\rangle$$

- the right hand side involves correlation functions

$$\left\langle f_i(p;x) \, f_j(p';x') \right\rangle$$

between different points x and x' on the freeze-out surface.

- works similar for higher order correlation functions.
- thermal fluctuations and initial state fluctuations contribute to correlations

Particle correlations from field correlation functions

[Floerchinger & Guenduez, work in progress]

- one can decompose

$$T(x) = \bar{T}(x) + \delta T(x), \qquad \mu(x) = \bar{\mu}(x) + \delta\mu(x)$$

and expand the distribution functions

$$\begin{aligned} f_i(p;x) =& f_i(p;\bar{T}(x), \bar{\mu}_i(x), \ldots) \\ &+ \delta T(x) \frac{\partial}{\partial T} f_i(p;\bar{T}(x), \bar{\mu}(x), \ldots) \\ &+ \delta\mu(x) \frac{\partial}{\partial \mu} f_i(p;\bar{T}(x), \bar{\mu}(x), \ldots) + \ldots \end{aligned}$$

- two-particle correlation function governed by integral over

$$\begin{aligned} \langle f_i(p;x)\, f_j(p';x') \rangle =& f_i(p;\bar{T}(x), \ldots)\, f_j(p';\bar{T}(x'), \ldots) \\ &+ \langle \delta T(x) \delta T(x') \rangle \frac{\partial}{\partial T} f_i(p;\bar{T}(x), \ldots) \frac{\partial}{\partial T} f_j(p;\bar{T}(x'), \ldots) \\ &+ \langle \delta\mu(x) \delta\mu(x') \rangle \frac{\partial}{\partial \mu} f_i(p;\bar{T}(x), \ldots) \frac{\partial}{\partial \mu} f_j(p;\bar{T}(x'), \ldots) \\ &+ \langle \delta\varphi(x) \delta\varphi(x') \rangle \frac{\partial}{\partial \varphi} f_i(p;\bar{T}(x), \ldots) \frac{\partial}{\partial \varphi} f_j(p;\bar{T}(x'), \ldots) \\ &+ \ldots \end{aligned}$$

Critical physics

- critical physics shows up in correlation functions
- in homogeneous space

$$\langle \varphi(\vec{x})\varphi(\vec{x}+\vec{r})\rangle \sim \frac{1}{r^{d-2+\eta}} \exp\left(-\frac{r}{\xi}\right)$$

with correlation length

$$\xi \sim \frac{1}{|T-T_c|^{\nu}}$$

- critical slowing down triggers drop out of equilibrium

Relativistic fluid dynamics

- evolution of baryon number density from conservation law $\nabla_\mu N^\mu = 0$

$$u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

- diffusion current ν^α determined by heat conductivity κ

$$\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon + p} \right]^2 \Delta^{\alpha\beta} \partial_\beta \left(\frac{\mu}{T} \right)$$

- can be extended to second order in gradients
- similar for net strangeness, charm and beauty currents
- evolution of electric current needs also electro-magnetic fields $F_{\mu\nu}$

Evolution of baryon number in fluid dynamics

- small perturbation in static medium with $u^\mu = (1,0,0,0)$

$$\frac{\partial}{\partial t}\delta n(t,\vec{x}) = D\vec{\nabla}^2\delta n(t,\vec{x})$$

- baryon number diffusion constant

$$D = \kappa\left[\frac{nT}{\epsilon+p}\right]^2\left(\frac{\partial(\mu/T)}{\partial n}\right)_\epsilon$$

- heat capacity κ appears here because

baryon diffusion in Landau frame	$\hat{=}$	heat conduction in Eckart frame

- is D finite for $n \to 0$?

Heat conductivity

- heat conductivity of QCD rather poorly understood theoretically so far.
- from perturbation theory [Danielewicz & Gyulassy, PRD 31, 53 (1985)]

$$\kappa \sim \frac{T^4}{\mu^2 \alpha_s^2 \ln \alpha_s} \qquad (\mu \ll T)$$

- from AdS/CFT [Son & Starinets, JHEP 0603 (2006)]

$$\kappa = 8\pi^2 \frac{T}{\mu^2}\eta = 2\pi \frac{sT}{\mu^2} \qquad (\mu \ll T)$$

- baryon diffusion constant D finite for $\mu \to 0$!

Bjorken expansion

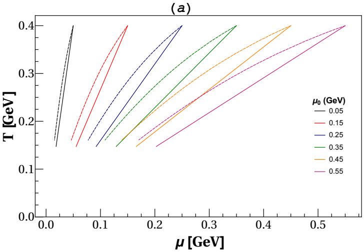
[Floerchinger & Martinez, PRC 92, 064906 (2015)]

- consider Bjorken type expansion

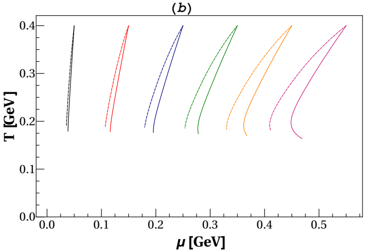
$$\partial_\tau \epsilon + (\epsilon + p)\frac{1}{\tau} - \left(\tfrac{4}{3}\eta + \zeta\right)\frac{1}{\tau^2} = 0$$

$$\partial_\tau n + n\frac{1}{\tau} = 0$$

- heat conductivity κ does not enter by symmetry argument
- compare ideal gas to lattice QCD equation of state



ideal gas EOS



lattice QCD EOS [Borsanyi et al. (2012)]

Perturbations around Bjorken expansion

[Floerchinger & Martinez, PRC 92, 064906 (2015)]

- consider situation with $\langle n(x) \rangle = \langle \mu(x) \rangle = 0$
- local event-by-event fluctuation $\delta n \neq 0$
- concentrate now on Bjorken flow profile for u^μ
- consider perturbation δn

$$\partial_\tau \delta n + \frac{1}{\tau} \delta n - D(\tau) \left(\partial_x^2 + \partial_y^2 + \frac{1}{\tau^2} \partial_\eta^2 \right) \delta n = 0$$

- structures in transverse and rapidity directions are "flattened out" by heat conductive dissipation

Solution by Bessel-Fourier expansion

[Floerchinger & Martinez, PRC 92, 064906 (2015)]

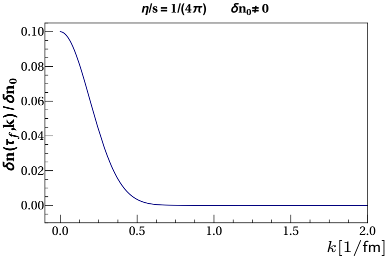
- expand perturbations like

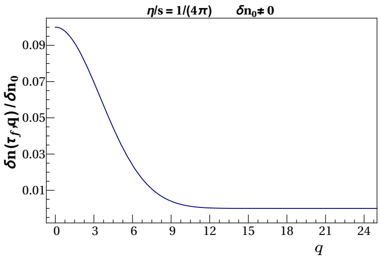
$$\delta n(\tau, r, \phi, \eta) = \int_0^\infty dk\, k \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi}\, \delta n(\tau, k, m, q)\, e^{i(m\phi + q\eta)} J_m(kr)$$

- leads to

$$\partial_\tau \delta n + \frac{1}{\tau}\delta n + D(\tau)\left(k^2 + \frac{q^2}{\tau^2}\right)\delta n = 0.$$

- ratio of final to initial fluctuations



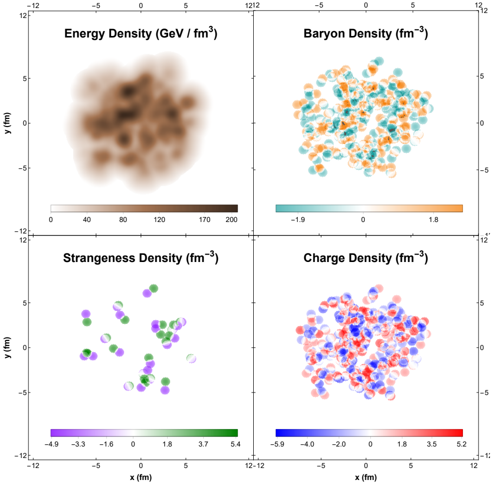


- only long-range fluctuations survive diffusive damping

Initial transverse densities for conserved charge fluctuations

[Martinez, Sievert, Wertepny & Noronha-Hostler, 1911.10272]

- conserved charge distribution from gluon to quark-anti-quark splitting
- Monte-Carlo implementation



Fluctuations at freeze-out

- background-perturbation splitting can also be used at freeze-out
- interesting observable is net baryon number

$$\frac{dN_B}{d\phi d\eta} = \frac{dN_{\text{baryon}}}{d\phi d\eta} - \frac{dN_{\text{anti-baryon}}}{d\phi d\eta}$$

- correlation functions and distributions contain information about baryon number fluctuations
- two-particle correlation function of net baryon number

$$C_{\text{Baryon}}(\phi_1 - \phi_2, \eta_1 - \eta_2) = \left\langle \frac{dN_B}{d\phi_1 d\eta_1} \, \frac{dN_B}{d\phi_2 d\eta_2} \right\rangle_c$$

Baryon number correlation function

- in Fourier representation

$$C_{\text{Baryon}}(\Delta\phi, \Delta\eta) = \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi}\, \tilde{C}_{\text{Baryon}}(m,q)\, e^{im\Delta\phi + iq\Delta\eta}$$

  heat conductivity leads to exponential suppression

$$\tilde{C}_{\text{Baryon}}(m,q) = e^{-m^2 I_1 - q^2 I_2}\; \tilde{C}_{\text{Baryon}}(m,q)\big|_{\kappa=0}$$

- I_1 and I_2 can be approximated as

$$I_1 \approx \int_{\tau_0}^{\tau_f} d\tau\, \frac{2}{R^2}\, \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_\epsilon$$

$$I_2 \approx \int_{\tau_0}^{\tau_f} d\tau\, \frac{2}{\tau^2}\, \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_\epsilon$$

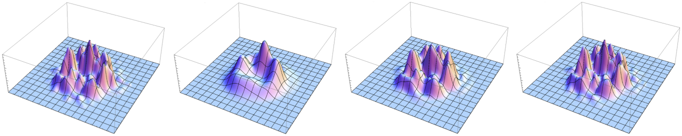
- $I_2 \gg I_1$ would lead to long-range correlations in rapidity direction ("baryon number ridge")

More detailed theory: mode expansion

- Bessel-Fourier expansion of initial transverse density
  [Floerchinger & Wiedemann (2013), see also Coleman-Smith, Petersen & Wolpert (2012)]

$$\boxed{\epsilon(r,\phi,\eta) = \bar{\epsilon}(r)\Big[1 + \sum_{m,l} \int_k w_l^{(m)}(k)\, e^{im\phi + ik\eta}\, J_m(z_l^{(m)} \rho(r))\Big]}$$

- azimuthal wavenumber m, radial wavenumber l, rapidity wavenumber k
- can also be used for conserved charges
- fast convergence



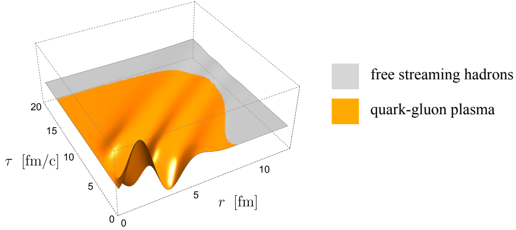
(original) $(m, l \leq 5)$ $(m, l \leq 10)$ $(m, l \leq 20)$

Fluid dynamic response

Fluid dynamics of heavy ion collisions with Mode expansion (FluiduM)

[Floerchinger & Wiedemann (2014), Floerchinger, Grossi & Lion (2019)]

- evolution of perturbations mode-by-mode
- e. g. energy density, $m = 2$, $l = 3$



- can also be used for conserved charges
- particle distribution through response functions

$$\frac{dN}{p_T dp_T d\phi dy} = \underbrace{S_0(p_T)}_{\text{from background}} \left[1 + \underbrace{\sum_{m,l} w_l^{(m)} \, e^{im\phi} \, \theta_l^{(m)}(p_T) + \ldots}_{\text{from fluctuations}} \right]$$

- resonance decays [Mazeliauskas, Floerchinger, Grossi & Teaney (2019)]

Conclusions

- differential correlation functions of conserved quantum numbers
  - net baryon number
  - electric charge
  - strangeness
  - charm, beauty

  contain very interesting physics information
- sensitive to QGP transport properties
  - heat conductivity $\hat{=}$ baryon diffusion
  - electric conductivity
  - heavy quark diffusion
- need theoretical and experimental effort to understand this in detail