Entanglement at collider energies

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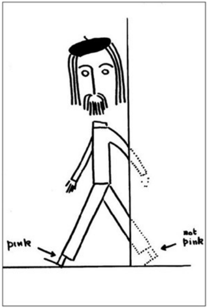
What is entanglement and why is it interesting?

- *Can quantum-mechanical description of physical reality be considered complete?* [Einstein, Podolsky, Rosen (1935), Bohm (1951)]

$$\psi = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B\right)$$
$$= \frac{1}{\sqrt{2}}\left(|\rightarrow\rangle_A|\leftarrow\rangle_B - |\leftarrow\rangle_A|\rightarrow\rangle_B\right)$$

- *Bertlemann's socks and the nature of reality* [Bell (1980)]



Bell's inequalities and Bell tests

[John Stewart Bell (1966)]

- most popular version [Clauser, Horne, Shimony, Holt (1969)]

$$S = |E(a,b) - E(a,b') + E(a',b) + E(a',b')| \leq 2$$

  holds for local hidden variable theories
- expectation value of product of two observables

$$E(a,b) = \langle A(a)B(b) \rangle$$

  with possible values $A = \pm 1$, $B = \pm 1$.
- depending on measurement settings a, a' and b, b' respectively
- quantum mechanical bound is $S \leq 2\sqrt{2}$
- experimental values $2 < S \leq 2\sqrt{2}$ rule out local hidden variables
- one measurement setting but at different times [Leggett, Garg (1985)]

Entanglement at collider energies

[. . ., Elze (1996), Kovner, Lublinsky (2015), Kharzeev & Levin (2017), Berges, Floerchinger & Venugopalan (2017), Shuryak & Zahed (2017), Kovner, Lublinsky, Serino (2018), Baker & Kharzeev (2018), Tu, Kharzeev & Ullrich (2019), Armesto, Dominguez, Kovner, Lublinsky, Skokov (2019), . . .]

- entanglement of *quantum fields* instead of *particles*
- entanglement on sub-nucleonic scales
- entanglement in non-Abelian gauge theory / color / confinement
- discussions in mathematical physics [e. g. Witten (2018)]
- connections to black holes and holography [Ryu & Takayanagi (2006)]
- thermalization in closed quantum systems

Entropy in quantum theory

[John von Neumann (1932)]

$$S = -\mathsf{Tr}\{\rho \ln \rho\}$$

- based on the quantum density operator ρ
- for pure states $\rho = |\psi\rangle\langle\psi|$ one has $S = 0$
- for mixed states $\rho = \sum_j p_j |j\rangle\langle j|$ one has $S = -\sum_j p_j \ln p_j > 0$
- unitary time evolution conserves entropy

$$-\mathsf{Tr}\{(U\rho U^\dagger)\ln(U\rho U^\dagger)\} = -\mathsf{Tr}\{\rho \ln \rho\} \qquad \rightarrow \qquad S = \text{const.}$$

- global characterization of quantum state

Local dissipation, entropy and entanglement

- local dissipation = local entropy production

$$-\nabla_\mu s^\mu(x) \geq 0$$

- relativistic fluid dynamics in Navier-Stokes approximation

$$-\nabla_\mu s^\mu = \frac{1}{T}\left[2\eta\sigma_{\mu\nu}\sigma^{\mu\nu} + \zeta(\nabla_\rho u^\rho)^2\right]$$

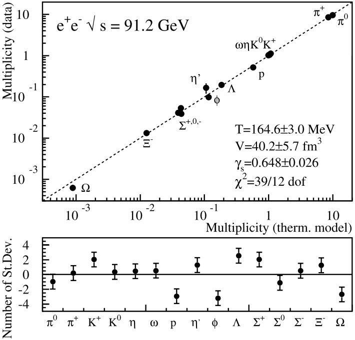
- *can not* be density of global von-Neumann entropy for closed system
- kinetic theory for weakly coupled (quasi-) particles [Boltzmann (1890)]

$$s^\mu(x) = -\int \frac{d^3p}{p^0}\left\{p^\mu f(x,p)\ln f(x,p)\right\}$$

- how to go beyond weak coupling / quasiparticles?
- local dissipation = *entanglement generation*
- $s^\mu(x)$ must (most likely) be seen as entanglement entropy current

The thermal model puzzle

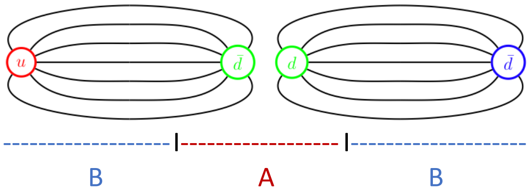
- elementary particle collision experiments such as $e^+ \, e^-$ collisions show some thermal-like features
- particle multiplicities well described by thermal model



[Becattini, Casterina, Milov & Satz, EPJC 66, 377 (2010)]

- conventional thermalization by collisions unlikely
- more thermal-like features difficult to understand in PYTHIA [Fischer, Sjöstrand (2017)]
- alternative explanations needed

QCD strings



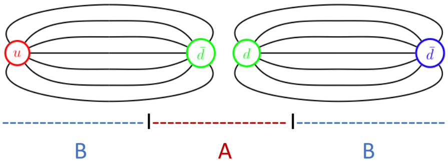
- particle production from QCD strings
- Lund string model (e. g. PYTHIA)
- different regions in a string are entangled
- subinterval A is described by reduced density matrix

$$\rho_A = \mathrm{Tr}_B\{\rho\}$$

- reduced density matrix is of mixed state form
- could this lead to thermal-like effects?

Entropy and entanglement

- consider a split of a quantum system into two $A + B$



- reduced density operator for system A

$$\rho_A = \mathrm{Tr}_B\{\rho\}$$

- entropy associated with subsystem A: **entanglement entropy**

$$S_A = -\mathrm{Tr}_A\{\rho_A \ln \rho_A\}$$

- **globally pure** state $S = 0$ can be **locally mixed** $S_A > 0$
- **coherent information** $I_{B\rangle A} = S_A - S$ can be **positive**

Microscopic model

- QCD in 1+1 dimensions described by 't Hooft model

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - ig\mathbf{A}_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{2}\mathrm{tr}\,\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}$$

- fermionic fields ψ_i with sums over flavor species $i = 1, \ldots, N_f$
- SU(N_c) gauge fields $\mathbf{A}_\mu$ with field strength tensor $\mathbf{F}_{\mu\nu}$
- gluons are not dynamical in two dimensions
- gauge coupling g has dimension of mass
- non-trivial, interacting theory, cannot be solved exactly
- spectrum of excitations known for $N_c \to \infty$ with $g^2 N_c$ fixed
  ['t Hooft (1974)]

Schwinger model

- QED in 1+1 dimension

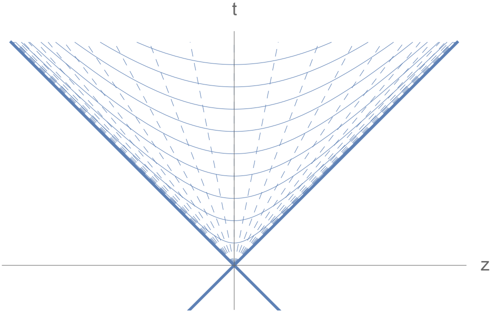
$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - iqA_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- geometric confinement
- U(1) charge related to string tension $q = \sqrt{2\sigma}$
- for single fermion one can **bosonize theory** exactly
  [Coleman, Jackiw, Susskind (1975)]

$$S = \int d^2x \sqrt{g} \Big\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 \phi^2 - \frac{m\, q\, e^\gamma}{2\pi^{3/2}} \cos\left(2\sqrt{\pi}\phi + \theta\right) \Big\}$$

- Schwinger bosons are dipoles $\phi \sim \bar{\psi}\psi$
- scalar mass related to U(1) charge by $M = q/\sqrt{\pi} = \sqrt{2\sigma/\pi}$
- massless Schwinger model $m = 0$ leads to free bosonic theory

Expanding string solution 1



- external quark-anti-quark pair on trajectories $z = \pm t$
- coordinates: Bjorken time $\tau = \sqrt{t^2 - z^2}$, rapidity $\eta = \text{arctanh}(z/t)$
- metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$
- symmetry with respect to longitudinal boosts $\eta \to \eta + \Delta\eta$

Expanding string solution 2

- Schwinger boson field depends only on τ

$$\bar{\phi} = \bar{\phi}(\tau)$$

- equation of motion

$$\partial_\tau^2 \bar{\phi} + \frac{1}{\tau} \partial_\tau \bar{\phi} + M^2 \bar{\phi} = 0.$$

- Gauss law: electric field $E = q\phi/\sqrt{\pi}$ must approach the U(1) charge of the external quarks $E \to q_{\mathsf{e}}$ for $\tau \to 0_+$

$$\bar{\phi}(\tau) \to \frac{\sqrt{\pi} q_{\mathsf{e}}}{q} \qquad (\tau \to 0_+)$$

- solution of equation of motion [Loshaj, Kharzeev (2011)]

$$\bar{\phi}(\tau) = \frac{\sqrt{\pi} q_{\mathsf{e}}}{q} J_0(M\tau)$$

Gaussian states

- theories with quadratic action often have Gaussian density matrix
- fully characterized by field expectation values

$$\bar{\phi}(x) = \langle \phi(x) \rangle, \qquad \bar{\pi}(x) = \langle \pi(x) \rangle$$

  and connected two-point correlation functions, e. g.

$$\langle \phi(x)\phi(y) \rangle_c = \langle \phi(x)\phi(y) \rangle - \bar{\phi}(x)\bar{\phi}(y)$$

- if ρ is Gaussian, also reduced density matrix ρ_A is Gaussian

Entanglement entropy for Gaussian state

- entanglement entropy of Gaussian state in region A
  [Berges, Floerchinger, Venugopalan, JHEP 1804 (2018) 145]

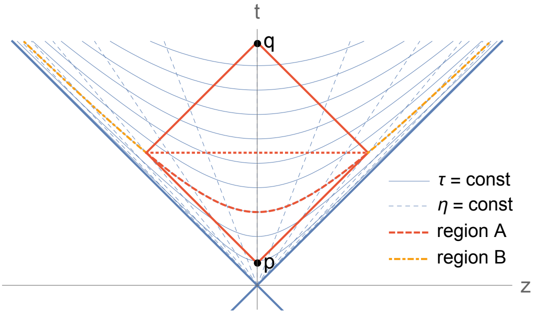
$$S_A = \frac{1}{2}\mathsf{Tr}_A\left\{D\ln(D^2)\right\}$$

- operator trace over region A only
- matrix of correlation functions

$$D(x,y) = \begin{pmatrix} -i\langle\phi(x)\pi(y)\rangle_c & i\langle\phi(x)\phi(y)\rangle_c \\ -i\langle\pi(x)\pi(y)\rangle_c & i\langle\pi(x)\phi(y)\rangle_c \end{pmatrix}$$

- involves connected correlation functions of field $\phi(x)$ and canonically conjugate momentum field $\pi(x)$
- expectation value $\bar{\phi}$ does not appear explicitly
- coherent states and vacuum have equal entanglement entropy S_A

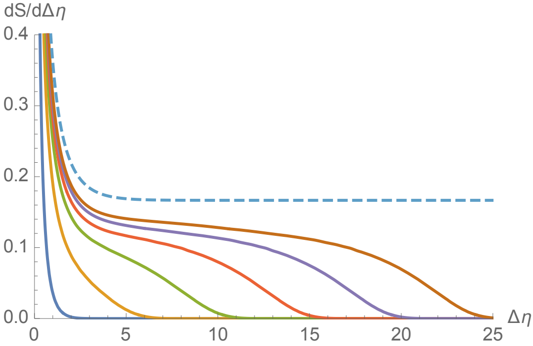
Rapidity interval



- consider rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ at fixed Bjorken time τ
- entanglement entropy does not change by unitary time evolution with endpoints kept fixed
- can be evaluated equivalently in interval $\Delta z = 2\tau \sinh(\Delta\eta/2)$ at fixed time $t = \tau \cosh(\Delta\eta/2)$
- need to solve eigenvalue problem with correct **boundary conditions**

Bosonized massless Schwinger model

- entanglement entropy understood numerically for free massive scalars [Casini, Huerta (2009)]
- entanglement entropy density $dS/d\Delta\eta$ for bosonized massless Schwinger model ($M = \frac{q}{\sqrt{\pi}}$)



$M\tau = 1$, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5}

Conformal limit

- For $M\tau \to 0$ one has conformal field theory limit
[Holzhey, Larsen, Wilczek (1994)]

$$S(\Delta z) = \frac{c}{3} \ln\left(\Delta z/\epsilon\right) + \text{constant}$$

with small length ϵ acting as UV cutoff.

- Here this implies

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln\left(2\tau \sinh(\Delta\eta/2)/\epsilon\right) + \text{constant}$$

- Conformal charge $c = 1$ for free massless scalars or Dirac fermions.
- Additive constant not universal but entropy density is

$$\begin{aligned} \frac{\partial}{\partial \Delta\eta} S(\tau, \Delta\eta) =& \frac{c}{6} \coth(\Delta\eta/2) \\ \to& \frac{c}{6} \qquad\qquad (\Delta\eta \gg 1) \end{aligned}$$

- Entropy becomes extensive in $\Delta\eta$!

Universal entanglement entropy density

- for very early times "Hubble" expansion rate dominates over masses and interactions

$$H = \frac{1}{\tau} \gg M = \frac{q}{\sqrt{\pi}}, m$$

- theory dominated by free, massless fermions
- universal entanglement entropy density

$$\frac{dS}{d\Delta\eta} = \frac{c}{6}$$

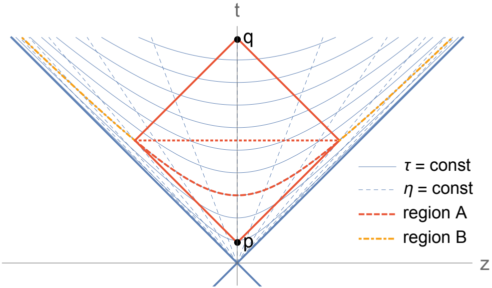
  with conformal charge c
- for QCD in 1+1 D (gluons not dynamical, no transverse excitations)

$$c = N_c \times N_f$$

- from fluctuating transverse coordinates (Nambu-Goto action)

$$c = N_c \times N_f + 2 \approx 9 + 2 = 11$$

Modular or entanglement Hamiltonian 1



- conformal field theory
- hypersurface Σ with boundary on the intersection of two light cones
- reduced density matrix [Casini, Huerta, Myers (2011), Arias, Blanco, Casini, Huerta (2017), see also Candelas, Dowker (1979)]

$$\rho_A = \frac{1}{Z_A} e^{-K}, \qquad Z_A = \mathrm{Tr}\, e^{-K}$$

- modular or entanglement Hamiltonian K

Modular or entanglement Hamiltonian 2

- modular or entanglement Hamiltonian is **local expression**

$$K = \int_\Sigma d\Sigma_\mu \, \xi_\nu(x) \, T^{\mu\nu}(x).$$

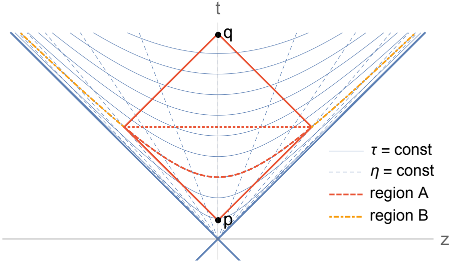
- energy-momentum tensor $T^{\mu\nu}(x)$ of excitations
- vector field

$$\xi^\mu(x) = \tfrac{2\pi}{(q-p)^2}[(q-x)^\mu(x-p)(q-p) + (x-p)^\mu(q-x)(q-p) - (q-p)^\mu(x-p)(q-x)]$$

  end point of future light cone q, starting point of past light cone p
- inverse temperature and fluid velocity

$$\xi^\mu(x) = \beta^\mu(x) = \frac{u^\mu(x)}{T(x)}$$

Modular or entanglement Hamiltonian 3



- for $\Delta\eta \to \infty$: fluid velocity in τ-direction, τ-dependent temperature

$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

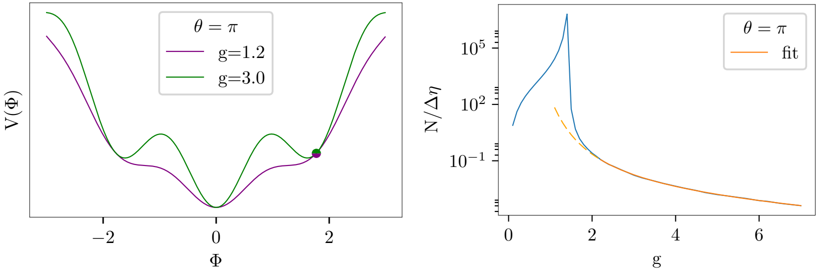
- **Entanglement between different rapidity intervals alone leads to local thermal density matrix at very early times !**
- Hawking-Unruh temperature in Rindler wedge $T(x) = \hbar c/(2\pi x)$

Physics picture

- coherent state vacuum at early time contains entangled pairs of quasi-particles with opposite wave numbers
- on finite rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ in- and out-flux of quasi-particles with thermal distribution via boundaries
- technically limits $\Delta\eta \to \infty$ and $M\tau \to 0$ do not commute
  - $\Delta\eta \to \infty$ for any finite $M\tau$ gives pure state
  - $M\tau \to 0$ for any finite $\Delta\eta$ gives thermal state with $T = 1/(2\pi\tau)$

Particle production in massive Schwinger model

[ongoing work with Lara Kuhn, Jürgen Berges]



- for expanding strings
- asymptotic particle number depends on $g \sim m/q$
- exponential suppression for large fermion mass $g \gg 1$

$$\frac{N}{\Delta\eta} \sim e^{-0.55\frac{m}{q}+7.48\frac{q}{m}+\ldots} = e^{-0.55\frac{m}{\sqrt{2\sigma}}+7.48\frac{\sqrt{2\sigma}}{m}+\ldots}$$

Conclusions

- entanglement at colliders is fascinating emerging topic of research
- experimental proof for entanglement needs Bell test
- entanglement entropy useful to describe local thermalization
- rapidity intervals in an expanding string are entangled
- at very early times theory effectively conformal

$$\frac{1}{\tau} \gg m, q$$

  reduced density matrix is of locally thermal form with temperature

$$T = \frac{\hbar}{2\pi\tau}$$

- entanglement important to understand early thermalization

Backup

Wigner distribution and entanglement

- Classical field approximation usually based on non-negative Wigner representation of density matrix
- leads for many observables to classical statistical description
- can nevertheless show entanglement and pass Bell test for "improper" variables where Weyl transform of operator has values outside of its spectrum [Revzen, Mello, Mann, Johansen (2005)]
- Bell test violation also possible for negative Wigner distribution [Bell (1986)]

Transverse coordinates

- so far dynamics strictly confined to 1+1 dimensions
- transverse coordinates may fluctuate, can be described by Nambu-Goto action ($h_{\mu\nu} = \partial_\mu X^m \partial_\nu X_m$)

$$S_{\mathsf{NG}} = \int d^2x \sqrt{-\mathsf{det}\, h_{\mu\nu}}\, \{-\sigma + \dots\}$$
$$\approx \int d^2x \sqrt{g} \left\{-\sigma - \frac{\sigma}{2} g^{\mu\nu} \partial_\mu X^i \partial_\nu X^i + \dots\right\}$$

- two additional, massless, bosonic degrees of freedom corresponding to transverse coordinates X^i with $i = 1, 2$

Temperature and entanglement entropy

- for conformal fields, entanglement entropy has also been calculated at non-zero temperature.
- for static interval of length L [Korepin (2004); Calabrese, Cardy (2004)]

$$S(T,l) = \frac{c}{3} \ln \left(\frac{1}{\pi T \epsilon} \sinh(\pi L T) \right) + \text{const}$$

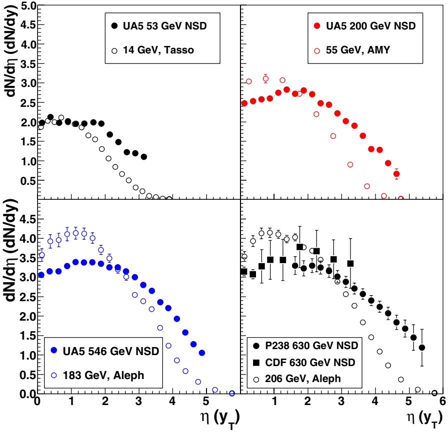
- compare this to our result in expanding geometry

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln \left(\frac{2\tau}{\epsilon} \sinh(\Delta\eta/2) \right) + \text{const}$$

- expressions agree for $L = \tau \Delta\eta$ (with metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$) and time-dependent temperature

$$T = \frac{1}{2\pi\tau}$$

Rapidity distribution



[open (filled) symbols: e^+e^- (pp), Grosse-Oetringhaus & Reygers (2010)]

- rapidity distribution $dN/d\eta$ has plateau around midrapidity
- only logarithmic dependence on collision energy

Experimental access to entanglement ?

- could longitudinal entanglement be tested experimentally?
- unfortunately entropy density $dS/d\eta$ not straight-forward to access
- measured in e^+e^- is the number of charged particles per unit rapidity $dN_{\text{ch}}/d\eta$ (rapidity defined with respect to the thrust axis)
- typical values for collision energies $\sqrt{s} = 14 - 206$ GeV in the range

$$dN_{\text{ch}}/d\eta \approx 2 - 4$$

- entropy per particle S/N can be estimated for a hadron resonance gas in thermal equilibrium $S/N_{\text{ch}} = 7.2$ would give

$$dS/d\eta \approx 14 - 28$$

- this is an upper bound: correlations beyond one-particle functions would lead to reduced entropy

Entanglement and QCD physics

- how strongly entangled is the nuclear wave function?
- what is the entropy of quasi-free partons and can it be understood as a result of entanglement? [Kharzeev, Levin (2017)]
- does saturation at small Bjorken-x have an entropic meaning?
- entanglement entropy and entropy production in the color glass condensate [Kovner, Lublinsky (2015); Kovner, Lublinsky, Serino (2018)]
- could entanglement entropy help for a non-perturbative extension of the parton model?
- entropy of perturbative and non-perturbative Pomeron descriptions [Shuryak, Zahed (2017)]