Fluctuations in the fluid dynamics of heavy ion collisions

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Functional Methods in Strongly Correlated Systems
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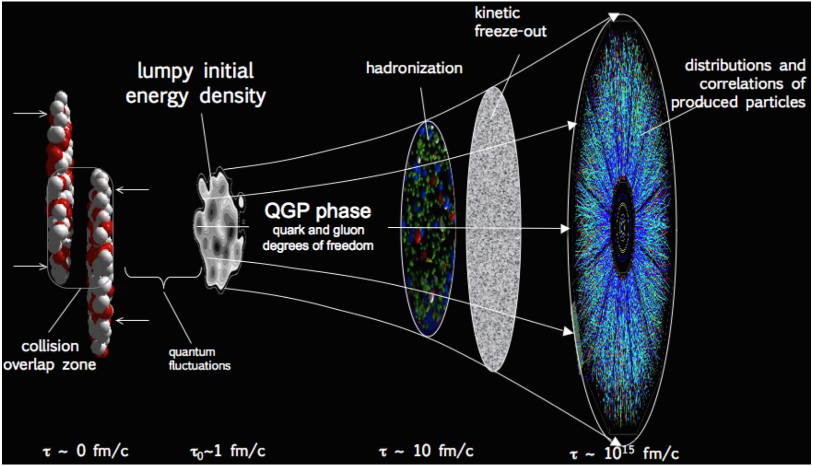
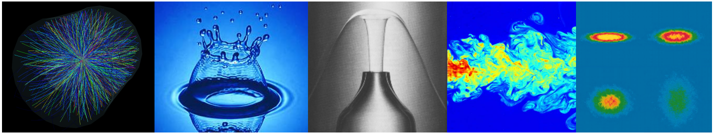
Little bangs in the laboratory



Fluid dynamics



- long distances, long times or strong enough interactions
- matter or quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear viscosity $\eta(T, \mu)$
  - bulk viscosity $\zeta(T, \mu)$
  - heat conductivity $\kappa(T, \mu)$
  - relaxation times, ...
- *ab initio* calculation of fluid properties difficult but fixed by **microscopic** properties in $\mathscr{L}_{\mathsf{QCD}}$

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\mathsf{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$

Covariant **conservation laws** $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for **energy density** ϵ

$$u^\mu \partial_\mu \epsilon + (\epsilon + p + \pi_{\mathsf{bulk}})\nabla_\mu u^\mu + \pi^{\mu\nu}\nabla_\mu u_\nu = 0$$

- equation for **fluid velocity** u^μ

$$(\epsilon + p + \pi_{\mathsf{bulk}})u^\mu \nabla_\mu u^\nu + \Delta^{\nu\mu}\partial_\mu(p + \pi_{\mathsf{bulk}}) + \Delta^\nu{}_\alpha \nabla_\mu \pi^{\mu\alpha} = 0$$

- equation for **particle number density** n

$$u^\mu \partial_\mu n + n\nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

Constitutive relations

Second order relativistic fluid dynamics:

- equation for **shear stress** $\pi^{\mu\nu}$

$$\tau_{\text{shear}}\, P^{\rho\sigma}{}_{\alpha\beta}\, u^\mu \nabla_\mu \pi^{\alpha\beta} + \pi^{\rho\sigma} + 2\eta\, P^{\rho\sigma\alpha}{}_{\beta}\, \nabla_\alpha u^\beta + \ldots = 0$$

  with **shear viscosity** $\eta(T,\mu)$

- equation for **bulk viscous pressure** π_{bulk}

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \pi_{\text{bulk}} + \zeta\, \nabla_\mu u^\mu + \ldots = 0$$

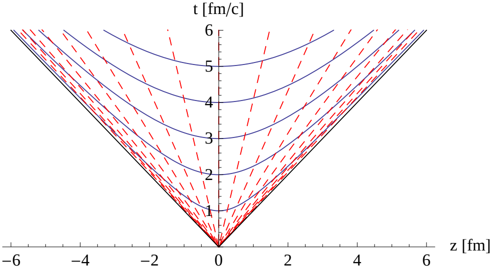
  with **bulk viscosity** $\zeta(T,\mu)$

- equation for **baryon diffusion current** ν^μ

$$\tau_{\text{heat}}\, \Delta^\alpha{}_\beta\, u^\mu \nabla_\mu \nu^\beta + \nu^\alpha + \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \Delta^{\alpha\beta} \partial_\beta \left(\frac{\mu}{T}\right) + \ldots = 0$$

  with **heat conductivity** $\kappa(T,\mu)$

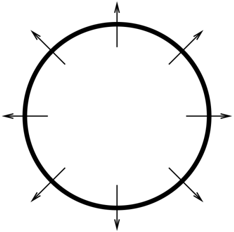
Bjorken boost invariance



How does the fluid velocity look like?

- Bjorkens guess: $v_z(t, x, y, z) = z/t$
- leads to an invariance under Lorentz-boosts in the z-direction
- use coordinates $\tau = \sqrt{t^2 - z^2}$, x, y, $\eta = \text{arctanh}(z/t)$
- Bjorken boost symmetry is reasonably accurate close to mid-rapidity $\eta \approx 0$

Transverse expansion



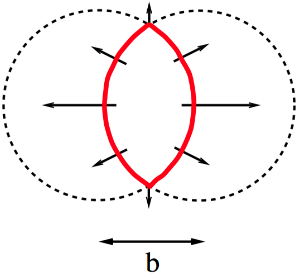
- for central collisions ($r = \sqrt{x^2 + y^2}$)

$$\epsilon = \epsilon(\tau, r)$$

- initial pressure gradient leads to **radial flow**

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \; f(\tau, r)$$

Non-central collisions



- pressure gradients larger in reaction plane
- leads to larger fluid velocity in this direction
- more particles fly in this direction
- can be quantified in terms of elliptic flow v_2
- particle distribution

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left[1 + 2\sum_m v_m \cos\left(m\left(\phi - \psi_R\right)\right)\right]$$

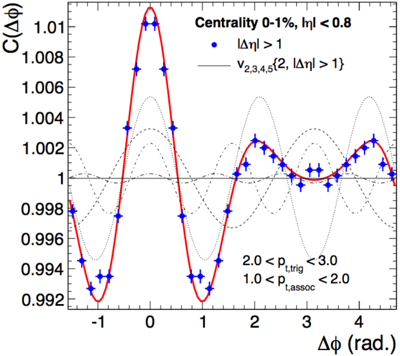
- symmetry $\phi \to \phi + \pi$ implies $v_1 = v_3 = v_5 = \ldots = 0$.

Two-particle correlation function

- normalized two-particle correlation function

$$C(\phi_1, \phi_2) = \frac{\left\langle \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \right\rangle_{\text{events}}}{\left\langle \frac{dN}{d\phi_1} \right\rangle_{\text{events}} \left\langle \frac{dN}{d\phi_2} \right\rangle_{\text{events}}} = 1 + 2 \sum_m v_m^2 \cos(m(\phi_1 - \phi_2))$$

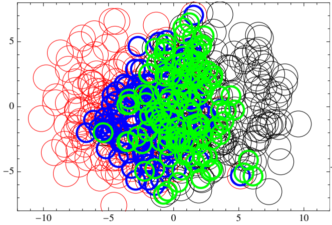
- surprisingly v_2, v_3, v_4, v_5 and v_6 are all non-zero!

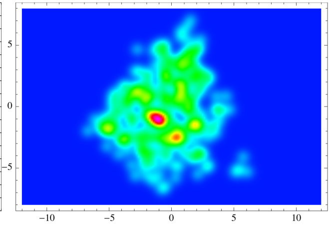


[ALICE 2011, similar results from CMS, ATLAS, Phenix, Star]

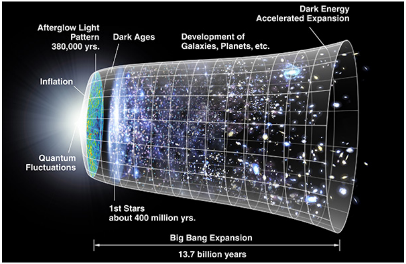
Event-by-event fluctuations

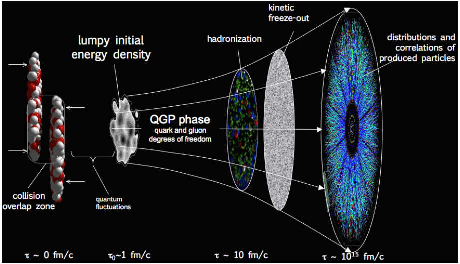
- deviations from symmetric initial energy density distribution from event-by-event fluctuations
- one example is Glauber model





Big bang – little bang analogy



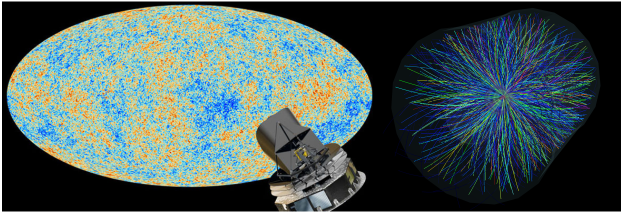


- cosmol. scale: MPc= 3.1×10^{22} m
- Gravity + QED + Dark sector
- one big event

- nuclear scale: fm= 10^{-15} m
- QCD
- very many events

- initial conditions not directly accessible
- all information must be reconstructed from final state
- dynamical description as a fluid
- fluctuating initial state

Similarities to cosmological fluctuation analysis



- fluctuation spectrum contains info from early times
- many numbers can be measured and compared to theory
- can lead to detailed understanding of evolution

What perturbations are interesting and why?

- Fluid fluctuations
  - energy density ϵ
  - fluid velocity u^μ
  - shear stress $\pi^{\mu\nu}$
  - more general also: baryon number density n,
    electric charge density, electromagnetic fields, ...
- **Initial fluid perturbations**
  - governed by universal evolution equations
  - can be used to constrain thermodynamic and transport properties
  - contain interesting information from early times
- **Thermal and quantum fluid fluctuations**
  - needed for more detailed description
  - could grow large close to critical point
- **Non-fluid fluctuations**
  - feed down from energy loss of hard particles or jets

A program to understand initial fluid perturbations

1. characterize initial perturbations
2. propagated them through fluid dynamic regime
3. determine influence on particle spectra and harmonic flow coefficients

Mode expansion for fluid fields

Bessel-Fourier expansion at initial time

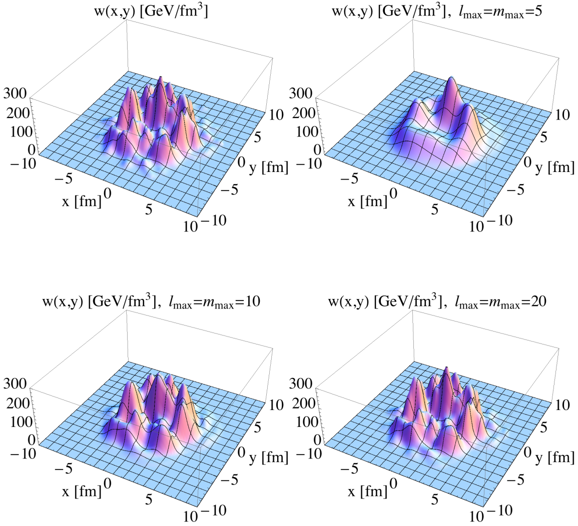
[Floerchinger & Wiedemann 2013, see also Coleman-Smith, Petersen & Wolpert 2012]

- for enthalpy density $w = \epsilon + p$

$$w(r, \phi, \eta) = w_{\mathsf{BG}}(r) \left[1 + \sum_{m,l} \int_k w_l^{(m)}(k) \, e^{im\phi + ik\eta} \; J_m(z_l^{(m)} \rho(r)) \right]$$

- azimuthal wavenumber m, radial wavenumber l, rapidity wavenumber k
- higher m and l correspond to finer spatial resolution
- works similar for vectors (velocity) and tensors (shear stress)

Transverse density from Glauber model



Cosmological perturbation theory

[Lifshitz, Peebles, Bardeen, Kosama, Sasaki, Ehler, Ellis, Hawking, Mukhanov, Weinberg, ...]

- solves evolution equations for fluid + gravity
- expands in perturbations around homogeneous background
- detailed understanding how different modes evolve
- very simple equations of state $p = w\,\epsilon$
- viscosities usually neglected $\eta = \zeta = 0$
- photons and neutrinos are free streaming

Fluid dynamic perturbation theory for heavy ion collisions

[Floerchinger & Wiedemann, PLB 728, 407 (2014)]

- solves evolution equations for relativistic QCD fluid
- expands in perturbations around event-averaged solution
- leads to linear + non-linear response formalism
- good convergence properties
  [Floerchinger *et al.*, PLB 735, 305 (2014), Brouzakis *et al.* PRD 91, 065007 (2015)]

Perturbative expansion

write fluid fields $\chi = (\epsilon, n, u^\mu, \pi^{\mu\nu}, \pi_{\mathsf{Bulk}}, \ldots)$

- at initial time τ_0 as

$$\chi = \chi_0 + \epsilon\, \chi_1$$

  background part h_0, fluctuation part $\epsilon\, h_1$

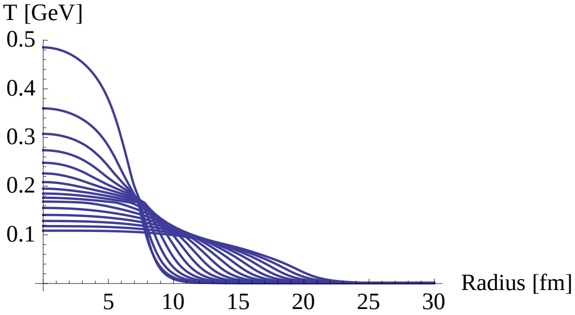
- at later time $\tau > \tau_0$ as

$$\chi = \chi_0 + \epsilon\, \chi_1 + \epsilon^2 \chi_2 + \epsilon^3 \chi_3 + \ldots$$

- χ_0 is solution of full, non-linear hydro equations in symmetric situation: azimuthal rotation and Bjorken boost invariant
- χ_1 is solution of linearized hydro equations around h_0, can be solved mode-by-mode
- χ_2 can be obtained by from interactions between modes etc.

Background evolution

- coupled $1 + 1$ dimensional partial differential equations

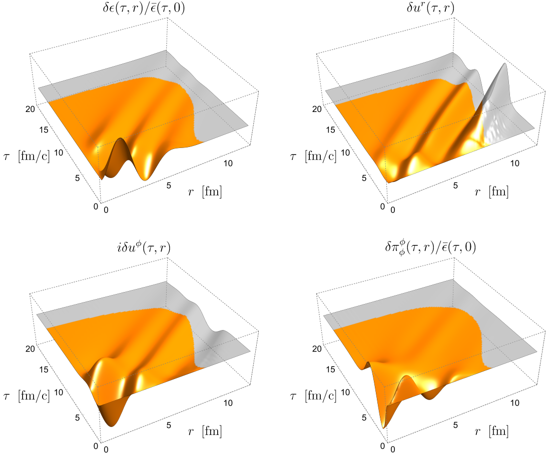


Evolving perturbation modes

- linearized hydro equations, use Fourier expansion

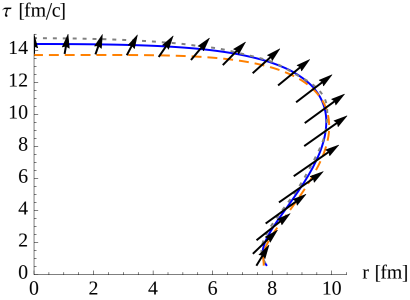
$$\chi_j(\tau, r, \phi, \eta) = \sum_m \int \frac{dk}{2\pi}\, \chi_j^{(m)}(\tau, r; k)\; e^{i(m\phi + k\eta)}$$

- reduces problem to $1+1$ dimensions



Freeze-out surface

- background and fluctuations are propagated until $T_{\text{fo}} = 120\,\text{MeV}$
- free streaming for later times [Cooper, Frye]
- perturbative expansion also at freeze-out [Floerchinger, Wiedemann (2013)]
- resonance decays can be taken into account [Maszeliauskas, Floerchinger, Grossi, Teaney (2018)]



Cooper-Frye freeze out

- single particle momentum distribution from local occupation number

$$E\frac{dN_j}{d^3p} = -\frac{1}{(2\pi)^3}\int_{\Sigma_f} d\Sigma_\mu \, p^\mu f_j(x,p).$$

- distribution function from fluid fields

$$f_j = f_j(p^\mu, u^\mu(x), T(x), \mu_i(x), \pi^{\mu\nu}(x), \pi_{\text{bulk}}(x))$$

- ideal gas approximation

$$f_k(x,p) = \frac{1}{\exp\left[-\beta_\nu(x)p^\nu - \sum_j Q_j^{(k)}\alpha_j(x)\right] \mp 1}$$

- depends on $\beta^\mu(x) = \frac{u^\mu(x)}{T(x)}$ and $\alpha_j(x) = \frac{\mu_j(x)}{T(x)}$

Particle distribution

for single event

$$\ln\left(\frac{dN^{\text{single event}}}{p_T dp_T d\phi dy}\right) = \underbrace{\ln S_0(p_T)}_{\text{from background}} + \underbrace{\sum_{m,l} w_l^{(m)}\, e^{im\phi}\, \theta_l^{(m)}(p_T)}_{\text{from fluctuations}}$$

- each mode has an angle $w_l^{(m)} = |w_l^{(m)}|\, e^{-im\psi_l^{(m)}}$
- each mode has its p_T-dependence $\theta_l^{(m)}(p_T)$

Harmonic flow coefficients

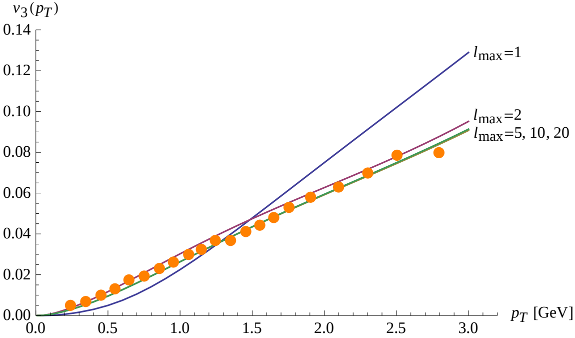
Double differential harmonic flow coefficient to lowest order

$$v_m^2\{2\}(p_T^a, p_T^b) = \sum_{l_1,l_2=1}^{l_{\text{max}}} \theta_{l_1}^{(m)}(p_T^a)\, \theta_{l_2}^{(m)}(p_T^b)\, \langle w_{l_1}^{(m)} w_{l_2}^{(m)*} \rangle$$

- intuite matrix expression
- in general no factorization
- higher order corrections important for non-central collisions

Harmonic flow coefficients for central collisions

Triangular flow for charged particles



Points: ALICE, 0%-2% most central collisions [PRL 107, 032301 (2011)]
Curves: varying maximal resolution l_{max} [Floerchinger, Wiedemann (2014)]

Two-particle correlation function 1

- Generalization of Cooper-Frye for two-particle spectrum (non-identical particles)

$$E_p E_q \frac{dN_{jk}}{d^3p d^3q} = \frac{1}{(2\pi)^6} \int d\Sigma_\mu d\Sigma'_\nu \, p^\mu q^\nu f_j(x,p) f_k(x',q).$$

- expand distribution functions

$$\begin{aligned} f_j(x,p) f_k(x',q) &= f_j f_k\big|_0 \\ &+ \sum_n \left\{ \frac{\partial f_j}{\partial \chi_n} f_k\big|_0 \chi_n(x) + f_j \frac{\partial f_k}{\partial \chi_n}\big|_0 \chi_n(x') \right\} \\ &+ \sum_{m,n} \left\{ \frac{\partial f_j}{\partial \chi_m} \frac{\partial f_k}{\partial \chi_n}\big|_0 \chi_m(x) \chi_n(x') \right\} + \ldots \end{aligned}$$

- take expectation values

$$\begin{aligned} \langle f_j(x,p) f_k(x',q) \rangle &= f_j f_k\big|_0 \\ &+ \sum_{m,n} \left\{ \frac{\partial f_j}{\partial \chi_m} \frac{\partial f_k}{\partial \chi_n}\big|_0 \langle \chi_m(x) \chi_n(x') \rangle_c \right\} + \ldots \end{aligned}$$

Two-particle correlation function 2

- ratio to product of single particle spectra

$$
\begin{aligned}
C_{jk}(p,q) =& \frac{\langle E_p E_q \frac{dN_{jk}}{d^3p d^3q} \rangle}{\langle E_p \frac{dN_j}{d^3p} \rangle \langle E_q \frac{dN_k}{d^3q} \rangle} \\
=& 1 + \frac{\frac{1}{(2\pi)^6} \int_{\Sigma_f} d\Sigma_\mu d\Sigma'_\nu \, p^\mu q^\nu \left\{ \frac{\partial f_j}{\partial \chi_m} \frac{\partial f_k}{\partial \chi_n} \Big|_0 \langle \chi_m(x) \chi_n(x') \rangle_c \right\}}{\langle E_p \frac{dN_j}{d^3p} \rangle \langle E_q \frac{dN_k}{d^3q} \rangle}
\end{aligned}
$$

- depends on two-point correlation function of fluid fields on the freeze-out surface

$$
G_{mn}(x,x') = \langle \chi_m(x) \chi_n(x') \rangle_c
$$

Symmetry implications

- two-particle (momentum) correlation function can be expanded

$$C_{jk}(p_T, q_T; \Delta\phi, \Delta\eta) = 1 + \int \frac{dk}{2\pi} \sum_{m=-\infty}^{\infty} e^{im\Delta\phi + ik\Delta\eta} c_{jk}^{(m)}(p_T, q_T; k)$$

- two-point correlation function on freeze-out surface can be expanded

$$G_{st}(\tau, \tau', r, r'; \Delta\varphi, \Delta y) = \int \frac{dk}{2\pi} \sum_{m=-\infty}^{\infty} e^{im\Delta\varphi + ik\Delta\eta} g_{st}^{(m)}(\tau, \tau', r, r'; k)$$

- diagonal relation in m and k as consequence of symmetry

$$c_{jk}^{(m)}(p_T, q_T; k) \sim \sum_{s,t} \frac{\partial f_j}{\partial \chi_s} \frac{\partial f_k}{\partial \chi_t} \, g_{st}^{(m)}(\tau, \tau', r, r'; k)$$

Thermal fluctuations

- fluid dynamics describes expectation values

$$\bar{T}^{\mu\nu} = \langle T^{\mu\nu} \rangle = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$\bar{N}^{\mu} = \langle N^{\mu} \rangle = n\, u^\mu + \nu^\mu$$

- what about correlation functions ?

$$\langle [T^{\mu\nu}(x) - \bar{T}^{\mu\nu}(x)][T^{\rho\sigma}(y) - \bar{T}^{\rho\sigma}(y)] \rangle$$

- no reason this should vanish, already in thermal equilibrium
- ideal fluid part can be parametrized by

$$\bar{T}(x), \qquad \bar{\mu}(x), \qquad \bar{u}^\rho(x)$$

- Thermodynamic variables (ϵ, n) and (T, μ) related by Legendre transforms
- Legendre transforms of $T^{\mu\nu}$ correlation functions lead to

$$\langle [T(x) - \bar{T}(x)][T(y) - \bar{T}(y)] \rangle, \qquad \text{etc.}$$

Correlation functions on a hypersurface 1

- generalization of "fixed time" is hypersurface Σ
- for example chemical or kinetic freeze-out surface
- coordinate system α^j with $j = 1, 2, 3$, embedding $x^\mu(\alpha)$ and induced metric on hypersurface

$$h_{jk}(\alpha) = g_{\mu\nu} \left[\frac{\partial}{\partial \alpha^j} x^\mu(\alpha) \right] \left[\frac{\partial}{\partial \alpha^k} x^\nu(\alpha) \right]$$

- fluctuation field $\chi_n(\alpha)$, e.g.

$$\chi_1(\alpha) = T(\alpha) - \bar{T}(\alpha), \qquad \chi_2(\alpha) = \mu(\alpha) - \bar{\mu}(\alpha)$$

- probability density of fluctuations on Σ

$$p[\chi] = \frac{1}{Z} e^{-I_\Sigma[\chi]}, \qquad Z = \int D\chi \, e^{-I_\Sigma[\chi]}.$$

Correlation functions on a hypersurface 1

- partition function on hypersurface Σ

$$Z[J] = \int D\chi \ \exp\left[-I_\Sigma[\chi] + \int d^3\alpha\sqrt{h}J_n(\alpha)\chi(\alpha)\right]$$

with $\sqrt{h} = \sqrt{\det(h_{jk})}$ and the invariant volume element $d^3\alpha\sqrt{h}$

- allows to obtain correlation functions

$$\langle\chi_n(\alpha)\chi_m(\beta)\rangle = \frac{1}{Z[J]}\left(\frac{1}{\sqrt{h(\alpha)}}\frac{\delta}{\delta J_n(\alpha)}\right)\left(\frac{1}{\sqrt{h(\beta)}}\frac{\delta}{\delta J_m(\beta)}\right)Z[J]\Big|_{J=0}$$

- but what is the action $I_\Sigma[\chi]$?

Probability of fluctuations 1

- probability for a thermal fluctuation in macroscopic fields

$$p[\chi] = \frac{1}{Z} e^{-I_\Sigma[\chi]}$$

- determined by change in entropy

$$I_\Sigma[\chi] = -\Delta S[\chi] + \text{const.}$$

- differential of entropy (with $\beta_\nu = \frac{u_\nu}{T}$ and $\alpha_j = \frac{\mu_j}{T}$)

$$dS = \beta_\nu dP^\nu + \sum_j \alpha_j dN_j$$

Probability of fluctuations 2

- split into two parts

$$\begin{aligned}
dS =& dS_0 + dS_1 \\
=& \beta_{0,\nu} dP_0^\nu + \beta_{1,\nu} dP_1^\nu + \sum_j \alpha_{j,0} dN_{j,0} + \sum_j \alpha_{j,1} dN_{j,1} \\
=& \Delta\beta_\nu dP^\nu + \sum_j \Delta\alpha_j dN_j
\end{aligned}$$

- used here conservation laws $dP_0^\nu + dP_1^\nu = 0$ and $dN_{j,0} + dN_{j,1} = 0$
- have set $\Delta\beta_\nu = \beta_{\nu,1} - \beta_{\nu,0}$ and $\Delta\alpha_j = \alpha_{j,1} - \alpha_{j,0}$
- abbreviate $dP^\nu = dP_1^\nu$ and $dN_j = dN_{j,1}$
- $\Delta\beta_\nu$ and $\Delta\alpha_j$ are linear in ΔP^ν and ΔN_j to lowest order
- allows to integrate

$$\Delta S = \frac{1}{2}\left(\Delta\beta_\nu \Delta P^\nu + \sum_j \Delta\alpha_j \Delta N_j\right)$$

Probability of fluctuations 3

- change in entropy

$$\Delta S = \frac{1}{2}\left(\Delta\beta_\nu \Delta P^\nu + \sum_j \Delta\alpha_j \Delta N_j\right)$$
$$= -\frac{1}{2}\int d\Sigma_\mu \left\{\Delta\beta_\nu \Delta T^{\mu\nu} + \sum_j \Delta\alpha_j \Delta N_j^\mu\right\}$$

- uses surface element $d\Sigma_\mu d^3\alpha\sqrt{h}\, n_\mu$ with normal vector n_μ
- assume that this works locally in each volume element (strong local equilibrium assumption)
- leads to action on hypersurface

$$I_\Sigma[\chi] = -\Delta S = \frac{1}{2}\int d\Sigma_\mu \left\{\Delta\beta_\nu(x)\Delta T^{\mu\nu}(x) + \sum_j \Delta\alpha_j(x)\Delta N_j^\mu(x)\right\}$$

- ultralocal = no derivatives of fluctuating fields

Fluctuations on hypersurface

- use ideal fluid expressions

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p\Delta^{\mu\nu}, \qquad N_j^\mu = n_j u^\mu,$$

- obtain "action" for fluctuations ΔT, $\Delta\mu_j$ and Δu^μ

$$I_\Sigma = \frac{1}{2}\int d\Sigma_\mu \Bigg\{ \frac{u^\mu}{T}\left(\frac{\partial^2 p}{\partial T^2}\Delta T^2 + 2\sum_j \frac{\partial^2 p}{\partial T \partial \mu_j}\Delta T \Delta\mu_j + \sum_{i,j} \frac{\partial^2 p}{\partial \mu_i \partial \mu_j}\Delta\mu_i \Delta\mu_j \right) + 2\frac{\Delta u^\mu}{T}\left(\frac{\partial p}{\partial T}\Delta T + \sum_j \frac{\partial p}{\partial \mu_j}\Delta\mu_j \right) + \frac{u^\mu}{T}(\epsilon + p)\,\Delta_{\rho\sigma}\Delta u^\rho \Delta u^\sigma \Bigg\}$$

- short range correlations

$$\langle \chi_n(\alpha)\chi_m(\alpha')\rangle_c = \frac{1}{\sqrt{h(\alpha)}}\delta^{(3)}(\alpha - \alpha')\sigma_{nm}(\alpha)$$

Thermal correlation matrix

- for stationary fluid $n^\mu = u^\mu$

$$\sigma_{TT} = \frac{T\frac{\partial^2 p}{\partial \mu^2}}{\frac{\partial^2 p}{\partial T^2}\frac{\partial^2 p}{\partial \mu^2} - \left(\frac{\partial^2 p}{\partial T \partial \mu}\right)^2} = \frac{T^2}{c_V},$$

$$\sigma_{\mu\mu} = \frac{T\frac{\partial^2 p}{\partial T^2}}{\frac{\partial^2 p}{\partial T^2}\frac{\partial^2 p}{\partial \mu^2} - \left(\frac{\partial^2 p}{\partial T \partial \mu}\right)^2} = \frac{T^2}{c_V}\frac{\frac{\partial^2 p}{\partial T^2}}{\frac{\partial^2 p}{\partial \mu^2}},$$

$$\sigma_{T\mu} = \sigma_{\mu T} = \frac{-T\frac{\partial^2 p}{\partial T \partial \mu}}{\frac{\partial^2 p}{\partial T^2}\frac{\partial^2 p}{\partial \mu^2} - \left(\frac{\partial^2 p}{\partial T \partial \mu}\right)^2} = -\frac{T^2}{c_V}\frac{\frac{\partial^2 p}{\partial T \partial \mu}}{\frac{\partial^2 p}{\partial \mu^2}},$$

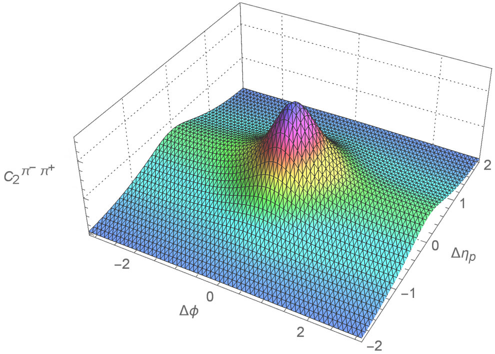
$$\sigma_{u^i u^j} = \frac{T}{\epsilon + p}\Delta^{ij}$$

- depends on thermodynamic equation of state $p(T, \mu_j)$
- for $T \to 0$ fluctuations vanish $\sigma_{nm} \to 0$

Two-particle correlation

[D. Guenduez]

- two-particle pion correlation from thermal fluctuations



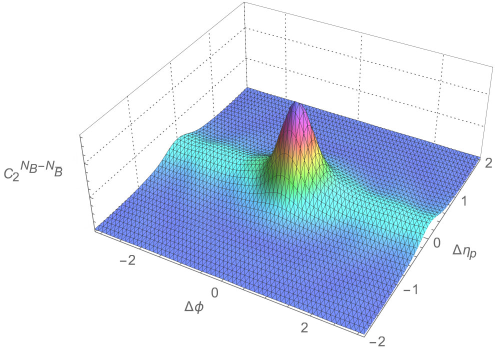
(preliminary)

- strong local equilibrium approximation
- equation of state $p(T, \mu_j)$ from hadron resonance gas

Net baryon number correlations

[D. Guenduez]

- net baryon number correlations in momentum space from local thermal fluctuations on the freeze-out surface



(preliminary)

- strong local equilibrium approximation
- equation of state $p(T, \mu_j)$ from hadron resonance gas

Conclusions

- fluid dynamics of heavy ion collisions can be analyzed in "functional manner" using a mode expansion
- fluctuations from initial state
- thermal fluctuations
- characterization of fluctuations on hypersurfaces
- relation to experimentally accessible two-particle correlation functions
- more information about two-point correlation functions needed
- better understanding of close-to-equilibrium dynamics of QCD

Backup

An effective action for the ideal fluid

- consider effective action

$$\Gamma[g_{\mu\nu}, \beta^\mu] = \Gamma_R[g_{\mu\nu}, \beta^\mu] = \int d^d x \sqrt{g}\, U(T)$$

with effective potential $U(T) = -p(T)$ and temperature

$$T = \frac{1}{\sqrt{-g_{\mu\nu}\beta^\mu\beta^\nu}}$$

- energy-momentum tensor from effective action

$$\frac{\delta\Gamma[g_{\mu\nu}, \beta^\mu]}{\delta g_{\mu\nu}(x)} = -\frac{1}{2}\sqrt{g}\, \langle T^{\mu\nu}(x)\rangle$$

- variation at fixed β^μ lead to ideal fluid form

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p g^{\mu\nu}$$

where $\epsilon + p = Ts = T\frac{\partial}{\partial T}p$ is the enthalpy density

- general covariance or covariant conservation $\nabla_\mu T^{\mu\nu} = 0$ leads to

$$u^\mu \partial_\mu \epsilon + (\epsilon + p)\nabla_\mu u^\mu = 0,$$
$$(\epsilon + p)u^\mu \nabla_\mu u^\nu + \Delta^{\nu\mu}\partial_\mu p = 0.$$