Collectivity in small systems and in heavy-ion collisions: theoretical overview

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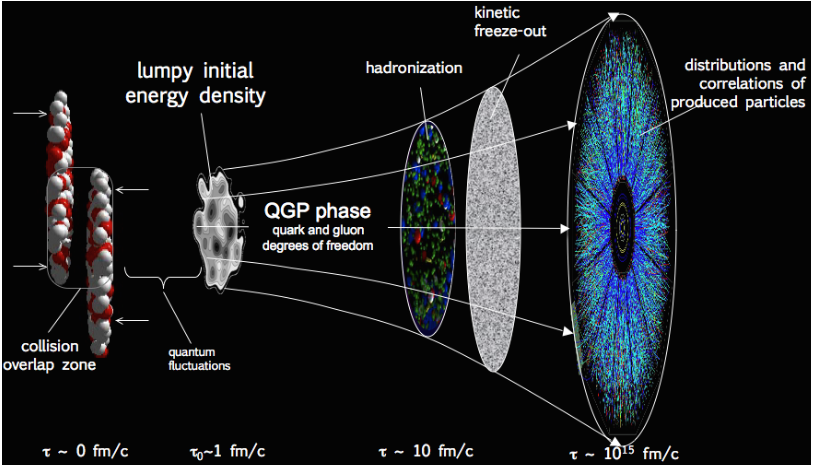
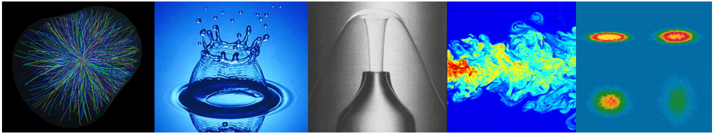
Little bangs in the laboratory

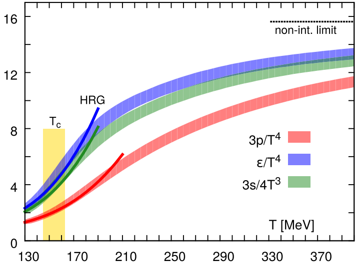


Fluid dynamics

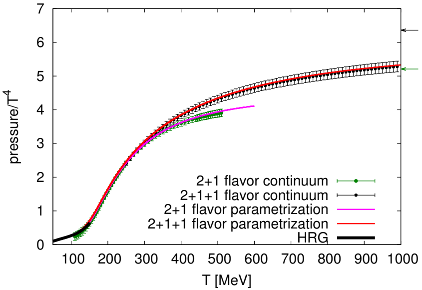


- long distances, long times or strong enough interactions
- matter or quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear viscosity $\eta(T, \mu)$
  - bulk viscosity $\zeta(T, \mu)$
  - heat conductivity $\kappa(T, \mu)$
  - relaxation times, ...
- *ab initio* calculation of fluid properties difficult but fixed by **microscopic** properties in $\mathscr{L}_{\text{QCD}}$

from lattice gauge theory



[Bazavov *et al.* (HotQCD) (2014)]



[Borsányi *et al.* (2016)]

- thermodynamic equation of state $p(T)$ rather well understood now
- also $\mu \neq 0$ is being explored
- progress in computing power

Transport coefficients

- from perturbation theory / effective kinetic theory at leading order
[Arnold, Moore, Yaffe (2003)]

$$\eta(T) = k \frac{T^3}{g^4 \log(1/g)} \,,$$

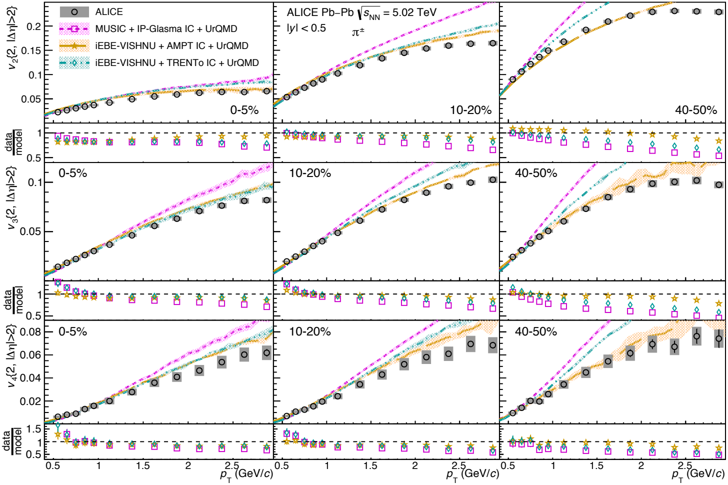
- next-to-leading order also understood now
[Ghiglieri, Moore, Teaney (2015-2018)]
- form AdS/CFT correspondence (very strong coupling)
[Kovtun, Son, Starinets (2003)]

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$$

- more transport properties and intermediate coupling regime to be understood

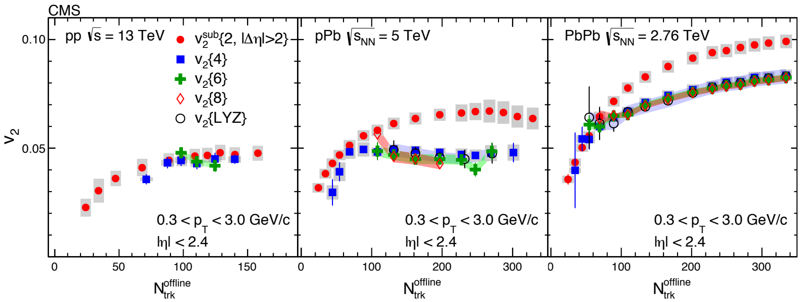
Fluid dynamics in heavy ion collisions

[ALICE, 1805.04390 (2018)]



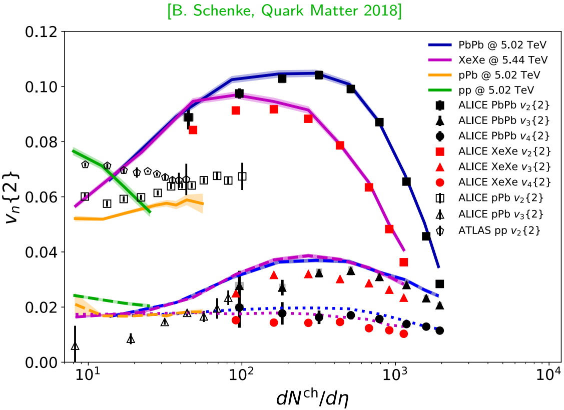
- $v_n(p_T)$ for pions in PbPb collisions well described by fluid dynamics
- initial conditions matter

Fluid dynamics for smaller systems 1



- flow coefficients from higher order cumulants $v_2\{n\}$ agree:
  $\rightarrow$ collective behavior
- elliptic flow signals also in **pPb** and **pp**!
- can fluid approximation work for pp collisions?

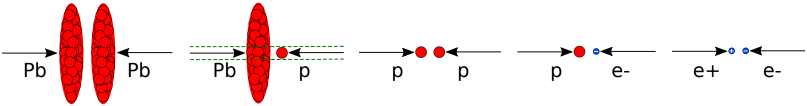
Fluid dynamics for smaller systems 2



- rather good agreement between data and theory for large multiplicity
- fluid approximation + initial state model works best for PbPb but still reasonable for pPb and pp

Questions and puzzles

- how universal are collective flow and fluid dynamics?
  or: when does it break down and how?
- what determines density distribution in a proton?
- role of multi-parton interactions
- more elementary systems such as ep or e^+e^- [News at Quark Matter 2018!]



Idea behind relativistic fluid dynamics

- General principle: macroscopic physics governed by conservation laws
- **Energy-momentum tensor** and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$
- thermal equilibrium = ideal fluid approximation

$$\pi_{\text{bulk}} = \pi^{\mu\nu} = \nu^\mu = 0.$$

Conservation laws

Covariant conservation laws $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for **energy density** ϵ

$$u^\mu \partial_\mu \epsilon + (\epsilon + p + \pi_{\text{bulk}}) \nabla_\mu u^\mu + \pi^{\mu\nu} \nabla_\mu u_\nu = 0$$

- equation for **fluid velocity** u^μ

$$(\epsilon + p + \pi_{\text{bulk}}) u^\mu \nabla_\mu u^\nu + \Delta^{\nu\mu} \partial_\mu (p + \pi_{\text{bulk}}) + \Delta^\nu{}_\alpha \nabla_\mu \pi^{\mu\alpha} = 0$$

- equation for **particle number density** n

$$u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

Relativistic dynamics

- covariance
- causality

Standard derivative or Chapman-Enskog expansion

- take fluid velocity u^μ and thermodynamic fields T, μ as degrees of freedom
- express "viscous stresses" in terms of derivatives
- bulk viscous pressure

$$\pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu + \ldots$$

- shear stress

$$\pi^{\mu\nu} = -\eta \left[\Delta^{\mu\alpha} \nabla_\alpha u^\nu + \Delta^{\nu\alpha} \nabla_\alpha u^\mu - \frac{2}{3} \Delta^{\mu\nu} \nabla_\alpha u^\alpha \right] + \ldots$$

- diffusion current

$$\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon + p} \right]^2 \Delta^{\alpha\beta} \partial_\beta \left(\frac{\mu}{T} \right) + \ldots$$

- restricted to small gradients (large systems)
- does *not* lead to relativistically causal evolution equations

Israel-Stewart type theories

Evolution equations instead of constraints

- equation for **shear stress** $\pi^{\mu\nu}$

$$\tau_{\text{shear}}\, P^{\rho\sigma}{}_{\alpha\beta}\, u^\mu \nabla_\mu \pi^{\alpha\beta} + \pi^{\rho\sigma} + 2\eta\, P^{\rho\sigma\alpha}{}_{\beta}\, \nabla_\alpha u^\beta + \ldots = 0$$

  with **shear viscosity** $\eta(T,\mu)$

- equation for **bulk viscous pressure** π_{bulk}

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \pi_{\text{bulk}} + \zeta\, \nabla_\mu u^\mu + \ldots = 0$$

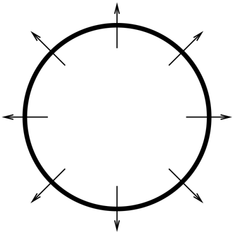
  with **bulk viscosity** $\zeta(T,\mu)$

- equation for **baryon diffusion current** ν^μ

$$\tau_{\text{heat}}\, \Delta^{\alpha}{}_{\beta}\, u^\mu \nabla_\mu \nu^\beta + \nu^\alpha + \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \Delta^{\alpha\beta} \partial_\beta \left(\frac{\mu}{T}\right) + \ldots = 0$$

  with **heat conductivity** $\kappa(T,\mu)$

Transverse expansion



- for central collisions $\epsilon = \epsilon(\tau, r)$
- initial pressure gradient leads to radial flow
- fluid evolution equations for Israel-Stewart type theories

$$A_{ij}(\Phi, \tau, r)\frac{\partial}{\partial \tau}\Phi_j + B_{ij}(\Phi, \tau, r)\frac{\partial}{\partial r}\Phi_j + C_i(\Phi, \tau, r) = 0.$$

- mathematically set of *quasi-linear, first order* partial differential equations

Characteristic velocities

[Floerchinger, Grossi (2017)]

- characteristic velocities $\lambda^{(n)}$ follow from $\det\left(B - \lambda^{(n)} A\right) = 0$ as

$$\lambda^{(1)} = \frac{v + \tilde{c}}{1 + \tilde{c}v}, \qquad \lambda^{(2)} = \frac{v - \tilde{c}}{1 - \tilde{c}v}, \qquad \lambda^{(3)} = \lambda^{(4)} = \lambda^{(5)} = v$$

- equations hyperbolic if $\lambda^{(n)} \in \mathbb{R}$
- causal signal propagation for $\tilde{c} \leq 1$
- modified velocity of sound

$$\tilde{c} = \sqrt{c_s^2 + d}$$

- ideal fluid velocity of sound

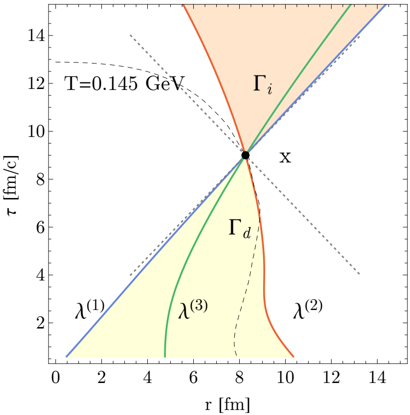
$$c_s^2 = \frac{\partial p}{\partial \epsilon}$$

- viscous correction

$$d = \frac{\frac{4\eta}{3\tau_{\text{shear}}} + \frac{\zeta}{\tau_{\text{bulk}}} + \dots}{\epsilon + p + \pi_{\text{bulk}} - \pi_\phi^\phi - \pi_\eta^\eta}$$

Domains of influence and dependence

[Floerchinger, Grossi (2017)]

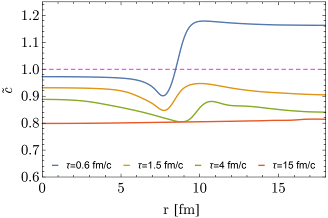


- causality cones are space-, time- and state dependent !
- causality poses a bound to applicability of relativistic fluid dynamics

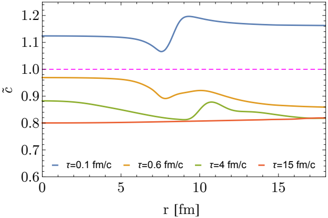
Causality as bound to fluid approximation

[Floerchinger, Grossi (2017)]

- Navier-Stokes initial conditions at $\tau_{\text{initial}} = 0.6$ fm/c



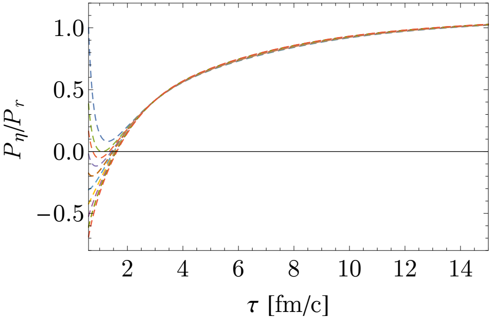
- Navier-Stokes initial conditions at $\tau_{\text{initial}} = 0.1$ fm/c



- causality violations for too large gradients!

The hydro "attractor"

- ratio of longitudinal to transverse "pressure" in Israel-Stewart theory

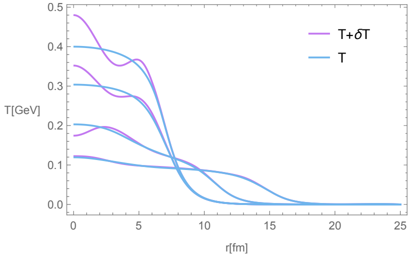


- approach to attractor governed by τ_{shear}
- causality: non-hydrodynamic modes needed!
- also negative longitudinal "pressure" allowed by causailty constraint

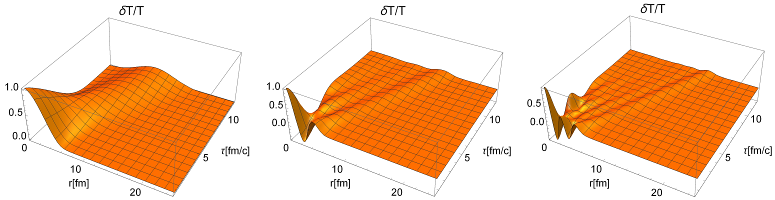
Mode-by-mode fluid dynamics

[Floerchinger, Wiedemann (2014), work in progress with E. Grossi, J. Lion]

- evolution of background & perturbations



- detailed understanding of perturbations



Conclusions

- high energy nuclear collisions produce a relativistic QCD fluid!
- fluid dynamics seems surprisingly universal
- experimental hints for collective flow also in pPb and pp collisions
- improved understanding of relativistic fluid dynamics
- causality: one must go beyond strict derivative expansion
- non-hydrodynamic modes needed
- bound on applicability posed by causality