Flow and fluctuations in high-energy nuclear collisions

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02.03.2018

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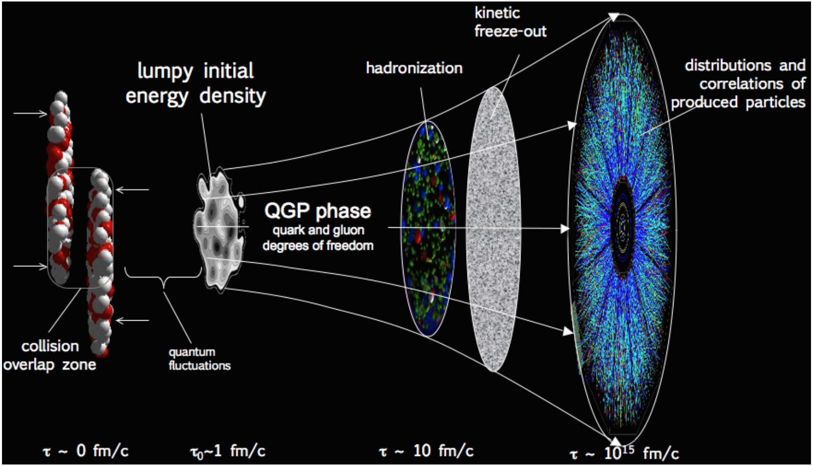
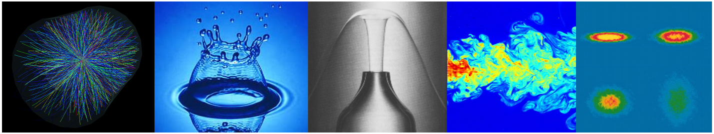
ISOQUANT SFB1225

Little bangs in the laboratory



Fluid dynamics



- long distances, long times or strong enough interactions
- matter or quantum fields form a fluid!
- needs **macroscopic** fluid properties
    - thermodynamic equation of state $p(T, \mu)$
    - shear viscosity $\eta(T, \mu)$
    - bulk viscosity $\zeta(T, \mu)$
    - heat conductivity $\kappa(T, \mu)$
    - relaxation times, ...
- *ab initio* calculation of fluid properties difficult but fixed by **microscopic** properties in $\mathscr{L}_{\text{QCD}}$

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\mathsf{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$

Covariant **conservation laws** $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for **energy density** ϵ

$$u^\mu \partial_\mu \epsilon + (\epsilon + p + \pi_{\mathsf{bulk}})\nabla_\mu u^\mu + \pi^{\mu\nu}\nabla_\mu u_\nu = 0$$

- equation for **fluid velocity** u^μ

$$(\epsilon + p + \pi_{\mathsf{bulk}})u^\mu \nabla_\mu u^\nu + \Delta^{\nu\mu}\partial_\mu(p + \pi_{\mathsf{bulk}}) + \Delta^\nu{}_\alpha \nabla_\mu \pi^{\mu\alpha} = 0$$

- equation for **particle number density** n

$$u^\mu \partial_\mu n + n\nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

Constitutive relations

Second order relativistic fluid dynamics:

- equation for **shear stress** $\pi^{\mu\nu}$

$$\tau_{\text{shear}}\, P^{\rho\sigma}{}_{\alpha\beta}\, u^\mu \nabla_\mu \pi^{\alpha\beta} + \pi^{\rho\sigma} + 2\eta\, P^{\rho\sigma\alpha}{}_{\beta}\, \nabla_\alpha u^\beta + \ldots = 0$$

  with **shear viscosity** $\eta(T,\mu)$

- equation for **bulk viscous pressure** π_{bulk}

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \pi_{\text{bulk}} + \zeta\, \nabla_\mu u^\mu + \ldots = 0$$

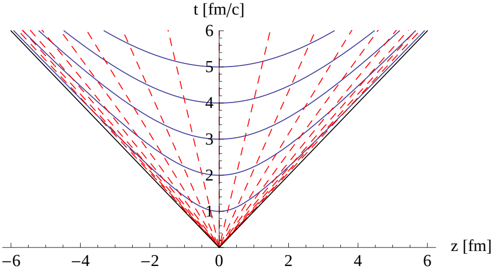
  with **bulk viscosity** $\zeta(T,\mu)$

- equation for **baryon diffusion current** ν^μ

$$\tau_{\text{heat}}\, \Delta^\alpha{}_\beta\, u^\mu \nabla_\mu \nu^\beta + \nu^\alpha + \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \Delta^{\alpha\beta} \partial_\beta \left(\frac{\mu}{T}\right) + \ldots = 0$$

  with **heat conductivity** $\kappa(T,\mu)$

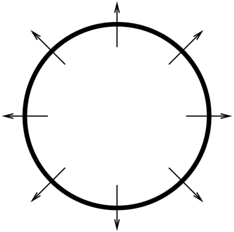
Bjorken boost invariance



How does the fluid velocity look like?

- Bjorkens guess: $v_z(t, x, y, z) = z/t$
- leads to an invariance under Lorentz-boosts in the z-direction
- use coordinates $\tau = \sqrt{t^2 - z^2}$, x, y, $\eta = \text{arctanh}(z/t)$
- Bjorken boost symmetry is reasonably accurate close to mid-rapidity $\eta \approx 0$

Transverse expansion



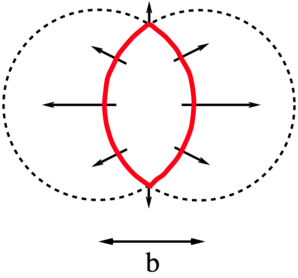
- for central collisions ($r = \sqrt{x^2 + y^2}$)

$$\epsilon = \epsilon(\tau, r)$$

- initial pressure gradient leads to **radial flow**

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \; f(\tau, r)$$

Non-central collisions



- pressure gradients larger in reaction plane
- leads to larger fluid velocity in this direction
- more particles fly in this direction
- can be quantified in terms of elliptic flow v_2
- particle distribution

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left[1 + 2\sum_m v_m \cos\left(m\left(\phi - \psi_R\right)\right)\right]$$

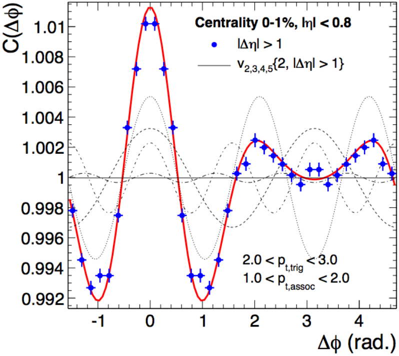
- symmetry $\phi \to \phi + \pi$ implies $v_1 = v_3 = v_5 = \ldots = 0$.

Two-particle correlation function

- normalized two-particle correlation function

$$C(\phi_1, \phi_2) = \frac{\left\langle \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \right\rangle_{\text{events}}}{\left\langle \frac{dN}{d\phi_1} \right\rangle_{\text{events}} \left\langle \frac{dN}{d\phi_2} \right\rangle_{\text{events}}} = 1 + 2 \sum_m v_m^2 \cos(m(\phi_1 - \phi_2))$$

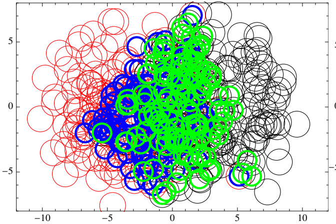
- surprisingly v_2, v_3, v_4, v_5 and v_6 are all non-zero!

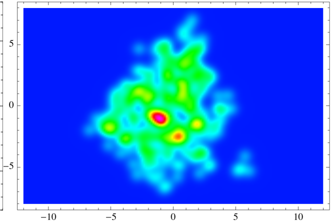


[ALICE 2011, similar results from CMS, ATLAS, Phenix, Star]

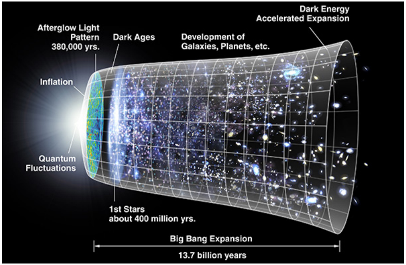
Event-by-event fluctuations

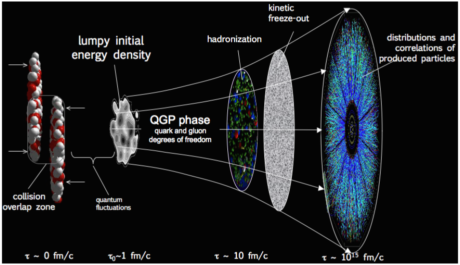
- deviations from symmetric initial energy density distribution from event-by-event fluctuations
- one example is Glauber model





Big bang – little bang analogy



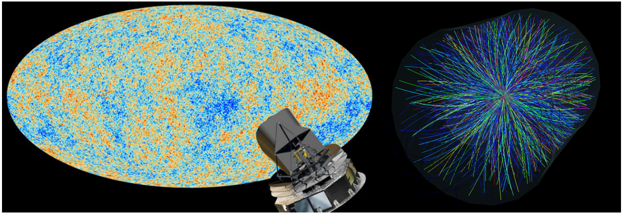


- cosmol. scale: MPc$= 3.1 \times 10^{22}$ m
- Gravity + QED + Dark sector
- one big event

- nuclear scale: fm$= 10^{-15}$ m
- QCD
- very many events

- initial conditions not directly accessible
- all information must be reconstructed from final state
- dynamical description as a fluid
- fluctuating initial state

Similarities to cosmological fluctuation analysis



- fluctuation spectrum contains info from early times
- many numbers can be measured and compared to theory
- can lead to detailed understanding of evolution

The dark matter fluid

- **heavy ion collisions**

$$\mathscr{L}_{\text{QCD}} \quad \to \quad \text{fluid properties}$$

- **late time cosmology**

$$\text{fluid properties} \quad \to \quad \mathscr{L}_{\text{dark matter}}$$

- until direct detection of dark matter it can only be observed via gravity

$$G^{\mu\nu} = 8\pi G_{\text{N}}\, T^{\mu\nu}$$

  so all we can access is

$$T^{\mu\nu}_{\text{dark matter}}$$

- strong motivation to study heavy ion collisions and cosmology together!

What perturbations are interesting and why?

- **Initial fluid perturbations**:
  - energy density ϵ
  - fluid velocity u^μ
  - shear stress $\pi^{\mu\nu}$
  - more general also: baryon number density n, electric charge density, electromagnetic fields, ...
- governed by universal evolution equations
- can be used to constrain **thermodynamic and transport properties**
- contain interesting information from early times

A program to understand fluid perturbations

1. characterize initial perturbations
2. propagated them through fluid dynamic regime
3. determine influence on particle spectra and harmonic flow coefficients
4. take also perturbations from non-hydro sources (jets) into account
   [see work with K. Zapp, EPJC 74 (2014) 12, 3189]

Mode expansion for fluid fields

Bessel-Fourier expansion at initial time

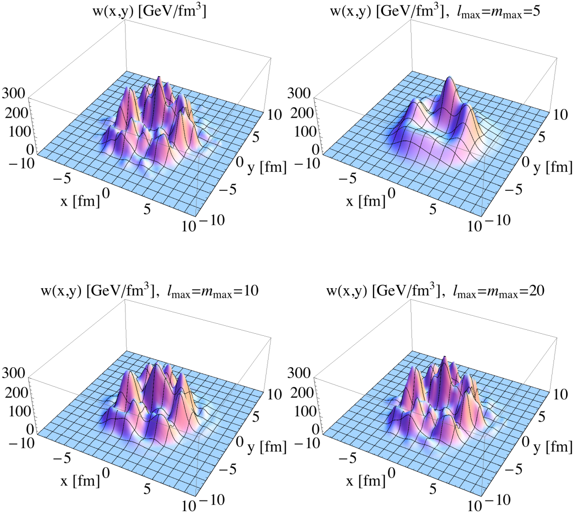
[Floerchinger & Wiedemann 2013, see also Coleman-Smith, Petersen & Wolpert 2012]

- for enthalpy density $w = \epsilon + p$

$$w(r, \phi, \eta) = w_{\mathsf{BG}}(r) \left[1 + \sum_{m,l} \int_k w_l^{(m)}(k)\, e^{im\phi + ik\eta}\; J_m(z_l^{(m)} \rho(r)) \right]$$

- azimuthal wavenumber m, radial wavenumber l, rapidity wavenumber k
- higher m and l correspond to finer spatial resolution
- works similar for vectors (velocity) and tensors (shear stress)

Transverse density from Glauber model



Cosmological perturbation theory

[Lifshitz, Peebles, Bardeen, Kosama, Sasaki, Ehler, Ellis, Hawking, Mukhanov, Weinberg, ...]

- solves evolution equations for fluid + gravity
- expands in perturbations around homogeneous background
- detailed understanding how different modes evolve
- very simple equations of state $p = w\,\epsilon$
- viscosities usually neglected $\eta = \zeta = 0$
- photons and neutrinos are free streaming

Fluid dynamic perturbation theory for heavy ion collisions

[Floerchinger & Wiedemann, PLB 728, 407 (2014)]

- solves evolution equations for relativistic QCD fluid
- expands in perturbations around event-averaged solution
- leads to linear + non-linear response formalism
- good convergence properties
  [Floerchinger *et al.*, PLB 735, 305 (2014), Brouzakis *et al.* PRD 91, 065007 (2015)]
- comparison to cosmology rather direct

Perturbative expansion

write fluid fields $h = (\epsilon, n, u^\mu, \pi^{\mu\nu}, \pi_{\text{Bulk}}, \dots)$

- at initial time τ_0 as

$$h = h_0 + \epsilon\, h_1$$

  background part h_0, fluctuation part $\epsilon\, h_1$

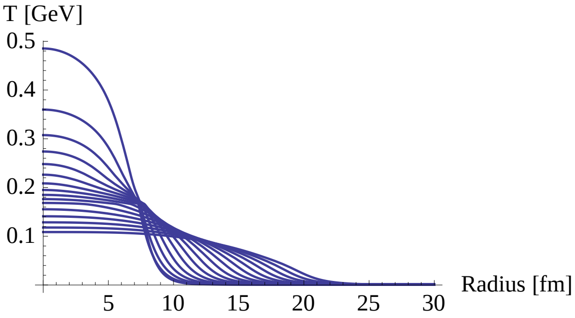
- at later time $\tau > \tau_0$ as

$$h = h_0 + \epsilon\, h_1 + \epsilon^2 h_2 + \epsilon^3 h_3 + \dots$$

- h_0 is solution of full, non-linear hydro equations in symmetric situation: azimuthal rotation and Bjorken boost invariant
- h_1 is solution of linearized hydro equations around h_0, can be solved mode-by-mode
- h_2 can be obtained by from interactions between modes etc.

Background evolution

- coupled $1 + 1$ dimensional partial differential equations

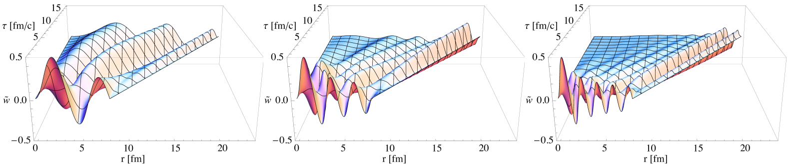


Evolving perturbation modes

- linearized hydro equations
- use Fourier expansion

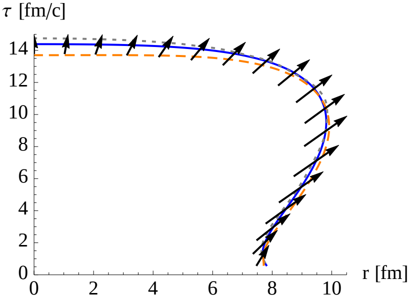
$$h_j(\tau, r, \phi, \eta) = \sum_m \int \frac{dk}{2\pi}\, h_j^{(m)}(\tau, r; k)\; e^{i(m\phi + k\eta)}$$

- reduces problem to $1 + 1$ dimensions



Freeze-out surface

- background and fluctuations are propagated until $T_{\text{fo}} = 120\,\text{MeV}$
- free streaming for later times [Cooper, Frye]
- perturbative expansion also at freeze-out [Floerchinger, Wiedemann 2013]
- resonance decays can be taken into account



Particle distribution

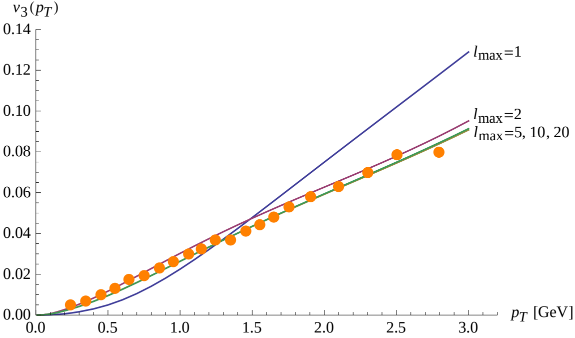
for single event

$$\ln\left(\frac{dN^{\text{single event}}}{p_T dp_T d\phi dy}\right) = \underbrace{\ln S_0(p_T)}_{\text{from background}} + \underbrace{\sum_{m,l} w_l^{(m)}\ e^{im\phi}\ \theta_l^{(m)}(p_T)}_{\text{from fluctuations}}$$

- each mode has an angle $w_l^{(m)} = |w_l^{(m)}|\ e^{-im\psi_l^{(m)}}$
- each mode has its p_T-dependence $\theta_l^{(m)}(p_T)$

Harmonic flow coefficients for central collisions

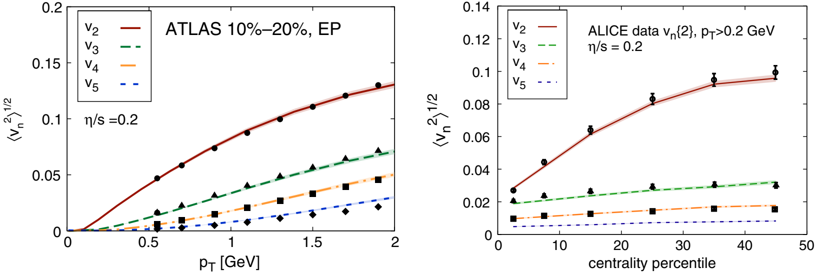
Triangular flow for charged particles



Points: ALICE, 0%-2% most central collisions [PRL 107, 032301 (2011)]
Curves: varying maximal resolution l_{max} [Floerchinger, Wiedemann (2014)]

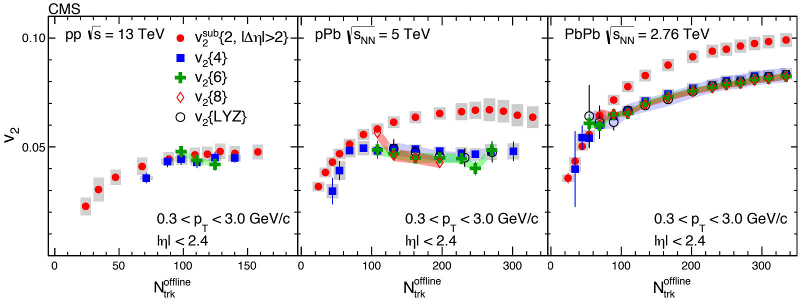
Fluid dynamic simulations

- second order relativistic fluid dynamics simulated numerically
- fluctuating initial conditions
- η/s is varied to find experimentally favored value



[Gale, Jeon, Schenke, Tribedy, Venugopalan (2013)]

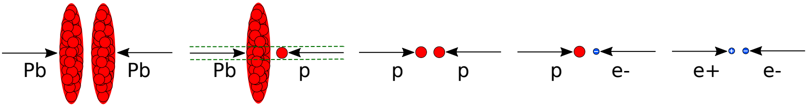
Collective behavior in large and small systems



- flow coefficients from higher order cumulants $v_2\{n\}$ agree:
  $\rightarrow$ collective behavior
- elliptic flow signals also in **pPb** and **pp**!
- can fluid approximation work for pp collisions?

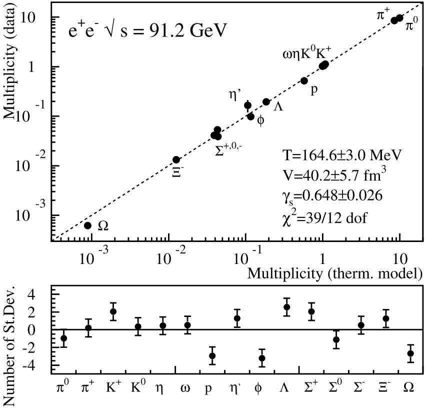
Questions and puzzles

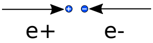
- how universal are collective flow and fluid dynamics?
- what determines density distribution of a proton?
  - constituent quarks?
  - interacting gluon cloud?
  - generalized parton distribution functions?
- do we really understand elementary particle collision physics?
- multi-parton interactions?
- more elementary systems such as ep or e^+e^-?



The thermal model puzzle

- elementary e^+e^- collision experiments show thermal-like features
- particle multiplicities well described by thermal model



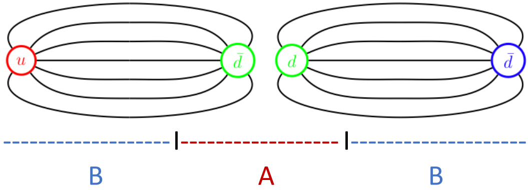


[Becattini, Casterina, Milov & Satz, EPJC 66, 377 (2010)]

- conventional thermalization by final state interactions unlikely
- alternative explanations needed

QCD strings and entanglement

[Berges, Floerchinger, Venugopalan (2017)]



- particle production from QCD strings
- Lund model (Pythia)
- different regions in a string are **entangled**
- subinterval A has reduced density matrix of **mixed form** even if ρ is pure

$$\rho_A = \text{Tr}_B\{\rho\}$$

- characterization by **entanglement entropy**

$$S_A = -\text{Tr}_A\left\{\rho_A \ln(\rho_A)\right\}$$

- could this lead to thermal-like effects?

Schwinger model

- QED in 1+1 dimension

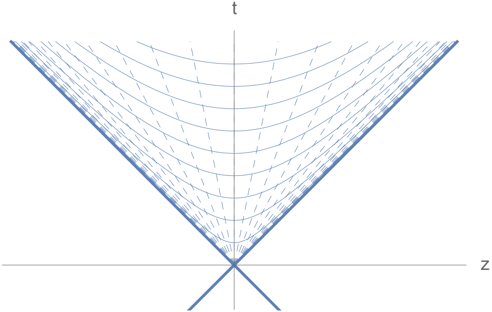
$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - iqA_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- geometric confinement
- U(1) charge related to string tension $q = \sqrt{2\sigma}$
- for single fermion one can **bosonize theory** exactly
  [Coleman, Jackiw, Susskind (1975)]

$$S = \int d^2x \sqrt{g} \Big\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 \phi^2 - \frac{m\, q\, e^\gamma}{2\pi^{3/2}} \cos\left(2\sqrt{\pi}\phi + \theta\right) \Big\}$$

- Schwinger bosons are dipoles $\phi \sim \bar{\psi}\psi$
- mass is related to U(1) charge by $M = q/\sqrt{\pi} = \sqrt{2\sigma/\pi}$
- massless Schwinger model $m = 0$ leads to free bosonic theory

Expanding string solution



- quark-anti-quark pair on trajectories $z = \pm t$
- coordinates: Bjorken time $\tau = \sqrt{t^2 - z^2}$, rapidity $\eta = \text{arctanh}(z/t)$
- Bjorken boost symmetry $\eta \to \eta + \Delta\eta$

Coherent field evolution

- Schwinger boson field expectation value depends only on τ

$$\bar{\phi} = \langle \phi \rangle = \bar{\phi}(\tau)$$

- equation of motion

$$\partial_\tau^2 \bar{\phi} + \frac{1}{\tau} \partial_\tau \bar{\phi} + M^2 \bar{\phi} = 0$$

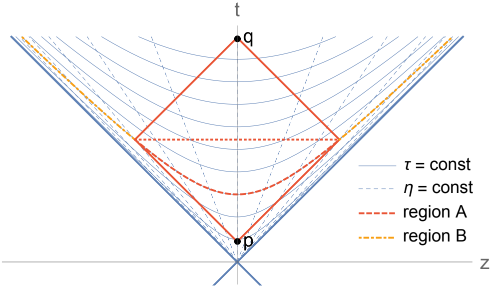
- Gauss law: electric field $E = q\phi/\sqrt{\pi}$ must approach U(1) charge

$$\bar{\phi}(\tau) \to \sqrt{\pi} \qquad (\text{for } \tau \to 0_+)$$

- solution of equation of motion [Loshaj, Kharzeev (2011)]

$$\bar{\phi}(\tau) = \sqrt{\pi} J_0(M\tau)$$

Reduced density matrix and early time limit



- rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$
- excitations around coherent field

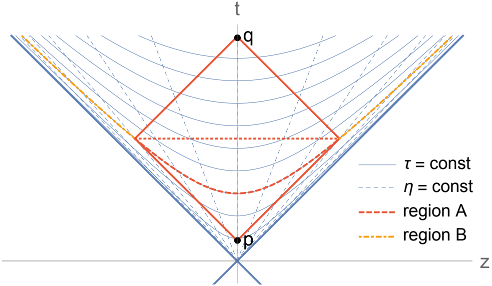
$$\phi(\tau, \eta) = \bar{\phi}(\tau) + \delta\phi(\tau, \eta)$$

- early times: expansion rate dominates

$$\frac{1}{\tau} \gg M = \frac{q}{\sqrt{\pi}}, m$$

- emergent **conformal symmetry**

Modular or entanglement Hamiltonian



- conformal field theory [Casini, Huerta, Myers (2011), Arias, Blanco, Casini, Huerta (2017), see also Candelas, Dowker (1979)]

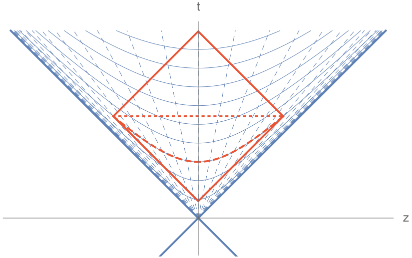
$$\rho_A = \frac{1}{Z_A} e^{-K}, \qquad Z_A = \text{Tr}\, e^{-K}$$

- modular or entanglement Hamiltonian **local expression**

$$K = \int_\Sigma d\Sigma_\mu\, \xi_\nu(x)\, T^{\mu\nu}(x)$$

- energy-momentum of excitations around coherent field $T^{\mu\nu}(x)$

Time-dependent temperature



- combination of fluid velocity and temperature $\xi^\mu(x) = \frac{u^\mu(x)}{T(x)}$
- for $\Delta\eta \to \infty$: fluid velocity in τ-direction & time-dependent temperature [Berges, Floerchinger, Venugopalan (2017)]

$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

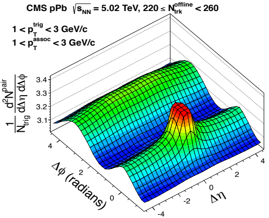
- **Entanglement between rapidity intervals leads to local thermal density matrix at very early times !**
- Hawking-Unruh temperature in Rindler wedge $T(x) = \frac{\hbar c}{2\pi x}$

Conclusions

- high energy nuclear collisions produce a relativistic QCD fluid!
- interesting parallels between cosmology and heavy ion collisions
- similar physics of evolving fluid fluctuations
- experimental hints for collective flow also in pPb and pp collisions
- expanding QCD strings: entanglement between rapidity intervals can lead to thermal-like effects!

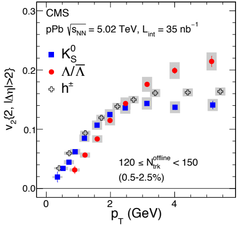
Backup slides

Detailed studies of the ridge in pPb

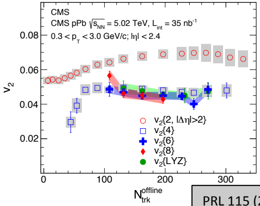


PLB 742 (2015) 200

Mass ordering



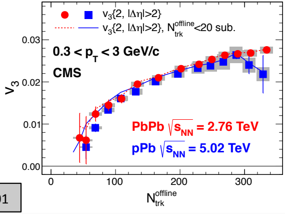
v2{2} > v2{4} ≈ v2{6} ≈ v2{8}



PRL 115 (2015) 012301

PLB 724 (2013) 213

v_3 Similar to Pb-Pb



Mode interactions

- non-linear terms in the evolution equations lead to mode interactions
- quadratic and higher order in initial perturbations
- can be determined from iterative solution
- convergence can be tested with numerical solution of full hydro equations

Statistics of initial density perturbations

Independent point-sources model (IPSM)

$$w(\vec{x}) = \left[\frac{1}{\tau_0}\frac{dW_{\text{BG}}}{d\eta}\right]\frac{1}{N}\sum_{j=1}^{N}\delta^{(2)}(\vec{x}-\vec{x}_j)$$

- random positions $\vec{x}_j$, independent and identically distributed
- probability distribution $p(\vec{x}_j)$ reflects collision geometry
- possible to determine correlation functions analytically for *central* and *non-central* collisions [Floerchinger & Wiedemann (2014)]
- Long-wavelength modes (small m and l) that don't resolve differences between point-like and extended sources have *universal statistics*.

Fluctuations around vanishing baryon number

- Evolution of baryon number density

$$u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

with diffusion current ν^α determined by heat conductivity κ

$$\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon + p} \right]^2 \Delta^{\alpha\beta} \partial_\beta \left(\frac{\mu}{T} \right)$$

- Consider situation with $\langle n(x) \rangle = \langle \mu(x) \rangle = 0$
  but event-by-event fluctuation $\delta n \neq 0$
- Concentrate now on Bjorken flow profile for u^μ

$$\partial_\tau \delta n + \frac{1}{\tau} \delta n - \kappa \left[\frac{nT}{\epsilon + p} \right]^2 \left(\frac{\partial(\mu/T)}{\partial n} \right)_\epsilon \left(\partial_x^2 + \partial_y^2 + \frac{1}{\tau^2} \partial_\eta^2 \right) \delta n = 0$$

- Structures in transverse and rapidity directions are “flattened out” by heat conductive dissipation

Baryon number correlations experimentally

- Two-particle correlation function of baryons minus anti-baryons

$$C_{\text{Baryon}}(\phi_1 - \phi_2, \eta_1 - \eta_2) = \langle n(\phi_1, \eta_1) n(\phi_2, \eta_2) \rangle_c$$

- In Fourier representation

$$C_{\text{Baryon}}(\Delta\phi, \Delta\eta) = \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi} \, \tilde{C}_{\text{Baryon}}(m, q) \, e^{im\Delta\phi + iq\Delta\eta}$$

heat conductivity leads to exponential suppression

$$\tilde{C}_{\text{Baryon}}(m, q) = e^{-m^2 I_1 - q^2 I_2} \; \tilde{C}_{\text{Baryon}}(m, q)\Big|_{\kappa=0}$$

- I_1 and I_2 can be approximated as

$$I_1 \approx \int_{\tau_0}^{\tau_f} d\tau \, \frac{2}{R^2} \, \kappa \left[\frac{nT}{\epsilon + p} \right]^2 \left(\frac{\partial(\mu/T)}{\partial n} \right)_\epsilon$$

$$I_2 \approx \int_{\tau_0}^{\tau_f} d\tau \, \frac{2}{\tau^2} \, \kappa \left[\frac{nT}{\epsilon + p} \right]^2 \left(\frac{\partial(\mu/T)}{\partial n} \right)_\epsilon$$

- $I_2 \gg I_1$ would lead to long-range correlations in rapidity direction ("baryon number ridge")

Remarks on baryon number fluctuations

- Initial ("primordial") baryon number fluctuations are poorly understood so far but presumably non-vanishing.
- Heat conductivity of QCD also poorly understood theoretically so far
  - from perturbation theory [Danielewicz & Gyulassy, PRD 31, 53 (1985)]

$$\kappa \sim \frac{T^4}{\mu^2 \alpha_s^2 \ln \alpha_s} \qquad (\mu \ll T)$$

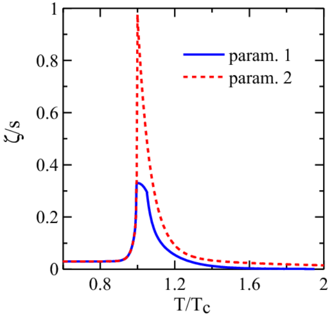
  - from AdS/CFT [Son & Starinets, JHEP 0603 (2006)]

$$\kappa = 8\pi^2 \frac{T}{\mu^2} \eta = 2\pi \frac{sT}{\mu^2} \qquad (\mu \ll T)$$

- More refined study needed to take transverse expansion properly into account.
- Seems to be interesting topic for further experimental and theoretical studies.

Bulk viscosity in heavy ion physics

- In heavy ion physics people start now to consider bulk viscosity.
- Becomes relevant close to chiral crossover

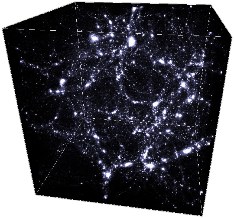


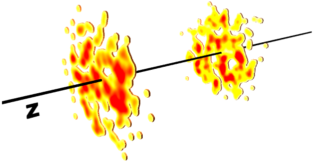
[Denicol, Gale & Jeon (2015)]

- Is there a first-order phase transition triggered by the expansion?
- What is the relation to chemical and kinetic freeze-out?
- More detailed understanding needed, both for heavy ion physics and cosmology

Symmetries in a statistical sense

- concrete realization breaks symmetry
- statistical properties are symmetric





Cosmology

- cosmological principle: universe homogeneous and isotropic
- 3D translation and rotation

→ 3D Fourier expansion

Heavy ion collisions

- 1D azimuthal rotation for central collisions
- 1D Bjorken boost (approximate)

→ Bessel-Fourier expansion [Floerchinger & Wiedemann (2013)]

Initial conditions in cosmology

- Perturbations are classified into scalars, vectors, tensors
- Vector modes are decaying, need not be specified
- Tensor modes are gravitational waves, can be neglected for most purposes
- Decaying scalar modes also not relevant
- Growing scalar modes are further classified by wavelength
- For relevant range of wavelength: close to Gaussian probability distribution
- Almost scale invariant initial spectrum

$$\langle \delta(\mathbf{k})\, \delta(\mathbf{k}') \rangle = P(k)\ \delta^{(3)}(\mathbf{k} + \mathbf{k}')$$

with

$$P(k) \sim k^{n_s - 1} \qquad\qquad n_s = 0.968 \pm 0.006$$ [Planck (2015)]

Gravitational growth of perturbations

- Small initial density perturbations

$$\delta = \frac{\Delta\epsilon}{\bar{\epsilon}} \ll 1$$

- At photon decoupling (CMB)

$$\delta \approx 10^{-5}$$

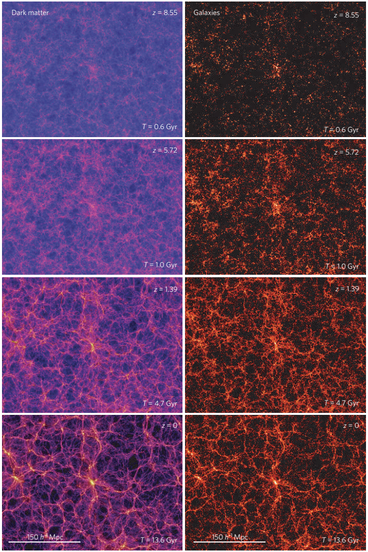
- Structure growth due to attractive gravitational interaction
- Perturbative treatment possible up to

$$\delta \approx 1$$

- For late times and small wavelengths

$$\delta \gg 1$$

Dark matter Visible galaxies



[Springel, Frenk & White, Nature 440, 1137 (2006)]

Ideal fluid versus collision-less gas

- Many codes used in cosmology describe dark matter as **ideal, cold and pressure-less fluid**

$$T^{\mu\nu} = \epsilon \, u^{\mu} u^{\nu}$$

- Equation of state $p = 0$
- No shear stress and bulk viscous pressure $\pi^{\mu\nu} = \pi_{\text{bulk}} = 0$
- Dark matter is also modeled as **collision-less gas** of massive particles, interacting via gravity only
- Two pictures are in general **not consistent**

Dissipative properties

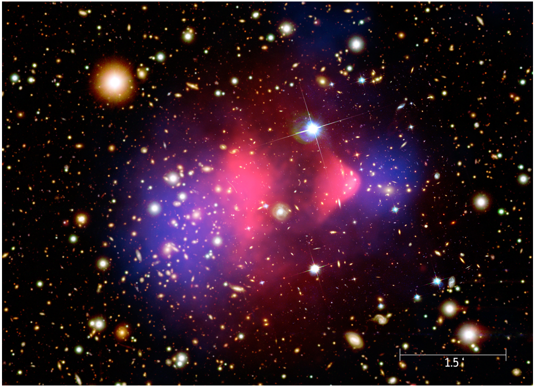
Viscosities

- Diffusive transport of momentum [Maxwell (1860)]
- Depend strongly on interaction properties
- Example: non-relativistic gas of particles with mass m, mean peculiar velocity $\bar{v}$, elastic $2 \to 2$ cross-section σ_{el}

$$\eta = \frac{m\,\bar{v}}{3\,\sigma_{\text{el}}} \qquad\qquad \zeta = 0$$

- Interesting additional **information about dark matter**

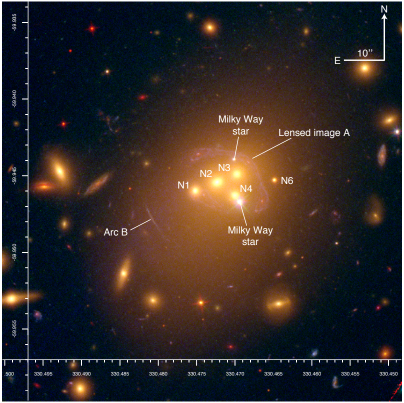
Material properties of dark matter

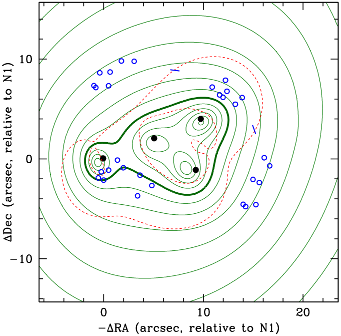


Gravitational lensing and x-ray image of "bullet cluster" 1E0657-56

- so far: dark matter is non-interacting $\rightarrow$ can collide without stopping
- Future decade: analysis of colliding galaxy clusters will refine this picture
- Dark energy self interacting
  - $\rightarrow$ modification of equation of state
  - $\rightarrow$ dissipation

Is dark matter self-interacting?





Galaxy cluster Abell 3827

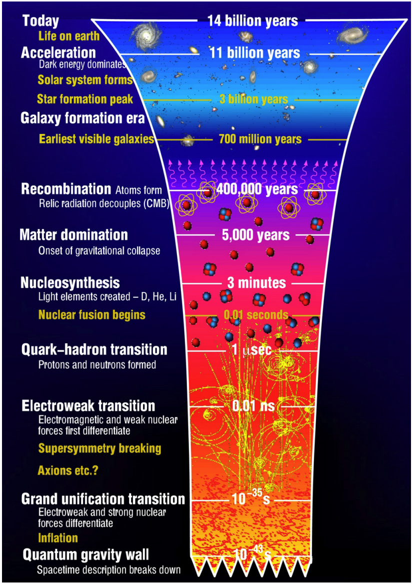
[Massey *et al.*, MNRAS 449, 3393 (2015)]

- Offset between stars and dark matter falling into cluster
- Is this a first indication for a dark matter self interaction?
  [Kahlhoefer, Schmidt-Hoberg, Kummer & Sarkar, MNRAS 452, 1 (2015)]

$$\frac{\sigma}{m_{\mathrm{DM}}} \approx 3\,\frac{\mathrm{cm}^2}{\mathrm{g}} \approx 0.5\,\frac{\mathrm{b}}{\mathrm{GeV}} \qquad \text{(under debate)}$$

The quark gluon plasma in the early universe

- Quark-gluon plasma has filled the universe until $\sim 10^{-6}$ s
- Probably not much information from that era transmitted
- Baryogenesis / Leptogenesis presumably much earlier



QCD in two dimensions

- QCD in 1+1 dimensions described by 't Hooft model

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - ig\mathbf{A}_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{2} \mathrm{tr}\, \mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}$$

- fermionic fields ψ_i with sums over flavor species $i = 1, \ldots, N_f$
- SU(N_c) gauge fields $\mathbf{A}_\mu$ with field strength tensor $\mathbf{F}_{\mu\nu}$
- gluons are not dynamical in two dimensions
- gauge coupling g has dimension of mass
- non-trivial, interacting theory, cannot be solved exactly
- spectrum of excitations known for $N_c \to \infty$ with $g^2 N_c$ fixed
  ['t Hooft (1974)]