Thermal excitation spectrum from entanglement in an expanding QCD string

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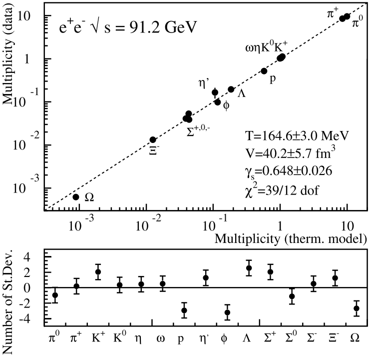
Non-Equilibrium, Mazara, 19/09/2017

based on

- J. Berges, S. Floerchinger & R. Venugopalan, *Thermal excitation spectrum from entanglement in an expanding QCD string* [arxiv:1707.05338]
- J. Berges, S. Floerchinger & R. Venugopalan [to appear]

Motivation

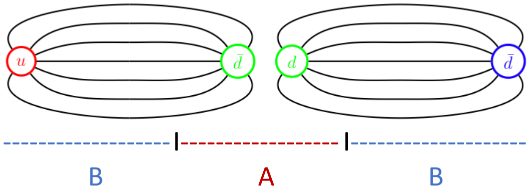
- Elementary particle collision experiments such as $e^+ \, e^-$ collisions show thermal-like features.
- Example: particle multiplicities



[Becattini, Casterina, Milov & Satz, EPJC 66, 377 (2010)]

- Conventional thermalization by collisions unlikely.
- Alternative explanations needed.

QCD strings



- Particle production from QCD strings.
- e. g. Lund model (Pythia).
- Different regions in a string are entangled.
- Subinterval A is described by reduced density matrix

$$\rho_A = \mathrm{Tr}_B \rho.$$

- Reduced density matrix is of mixed state form.
- Could this lead to thermal-like effects?

Microscopic model

- QCD in 1+1 dimensions described by 't Hooft model

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - ig\mathbf{A}_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{2} \mathrm{tr}\, \mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu}$$

- Fermionic fields ψ_i with sums over flavor species $i = 1, \ldots, N_f$
- SU(N_c) gauge fields $\mathbf{A}_\mu$ with field strength tensor $\mathbf{F}_{\mu\nu}$
- Gluons are not dynamical in two dimensions
- Gauge coupling g has dimension of mass
- Non-trivial, interacting theory, cannot be solved exactly
- Spectrum of excitations known for $N_c \to \infty$ with $g^2 N_c$ fixed

Schwinger model

- QED in 1+1 dimension

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - iqA_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- Geometric confinement
- U(1) charge related to string tension $q = \sqrt{2\sigma}$
- For single massless fermion one can bosonize theory exactly

$$S = \int d^2x \sqrt{g} \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 \phi^2 \right\}$$

- Schwinger bosons are dipoles $\phi \sim \bar{\psi}\psi$
- Mass is related to U(1) charge by $M = q/\sqrt{\pi}$

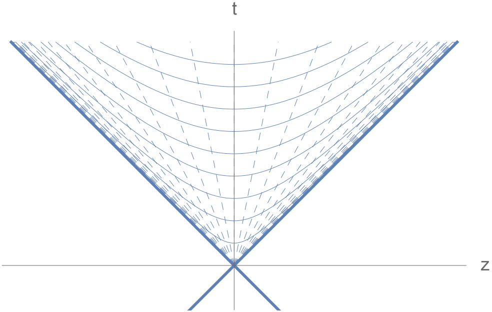
Transverse coordinates

- So far dynamics strictly confined to 1+1 dimensions
- Transverse coordinates may fluctuate, can be described by Nambu-Goto action ($h_{\mu\nu} = \partial_\mu X^m \partial_\nu X_m$)

$$
\begin{aligned}
S_{\mathsf{NG}} &= \int d^2x \sqrt{-\mathsf{det}\, h_{\mu\nu}}\, \{-\sigma + \ldots\} \\
&\approx \int d^2x \sqrt{g} \left\{ -\sigma - \frac{\sigma}{2} g^{\mu\nu} \partial_\mu X^i \partial_\nu X^i + \ldots \right\}
\end{aligned}
$$

- Two additional, massless, bosonic degrees of freedom corresponding to transverse coordinates X^i with $i = 1, 2$.

Expanding string solution 1



- Consider string formed between (external) quark-anti-quark pair on trajectories

$$z = \pm t$$

- Coordinate system with Bjorken time $\tau = \sqrt{t^2 - z^2}$ and rapidity $\eta = \text{arctanh}(z/t)$
- Symmetry with respect to longitudinal boosts $\eta \to \eta + \Delta\eta$

Expanding string solution 2

- Schwinger boson field depends only on τ

$$\bar{\phi} = \bar{\phi}(\tau)$$

- Equation of motion

$$\partial_\tau^2 \bar{\phi} + \frac{1}{\tau} \partial_\tau \bar{\phi} + M^2 \bar{\phi} = 0.$$

- Gauss law: electric field $E = q\phi/\sqrt{\pi}$ must approach the U(1) charge of the external quarks $E \to q_{\mathsf{e}}$ for $\tau \to 0_+$

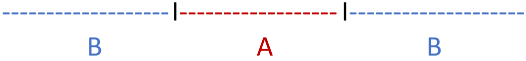
$$\bar{\phi}(\tau) \to \frac{\sqrt{\pi} q_{\mathsf{e}}}{q} \qquad (\tau \to 0_+)$$

- Solution of equation of motion

$$\bar{\phi}(\tau) = \frac{\sqrt{\pi} q_{\mathsf{e}}}{q} J_0(M\tau)$$

Reduced density matrix

- Consider now physical processes such as hadron formation
- Assume that these are local processes in some space region A



- Reduced density matrix, trace over complement region B

$$\rho_A = \mathrm{Tr}_B\, \rho$$

- In general ρ_A mixed state density matrix even if ρ is pure
- Reason: entanglement between regions A and B
- Characterization by entanglement entropy

$$S_A = -\mathrm{Tr}\left\{\rho_A \ln(\rho_A)\right\}$$

Gaussian states

- Theories with quadratic action typically have Gaussian density matrix
- Fully characterized by field expectation values

$$\bar{\phi}(x) = \langle \phi(x) \rangle, \qquad \bar{\pi}(x) = \langle \pi(x) \rangle$$

and connected two-point correlation functions, e. g.

$$\langle \phi(x)\phi(y) \rangle_c = \langle \phi(x)\phi(y) \rangle - \bar{\phi}(x)\bar{\phi}(y)$$

- If ρ is Gaussian, also reduced density matrix ρ_A is Gaussian

Entanglement entropy for Gaussian state

- Entanglement entropy of Gaussian state in region A
  [Berges, Floerchinger, Venugopalan, to appear]

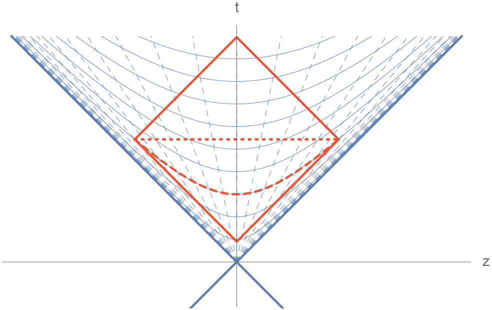
$$S_A = \frac{1}{2}\mathsf{Tr}_A \left\{ D \ln(D^2) \right\},$$

- Operator trace over region A only
- Matrix of correlation functions

$$D(x,y) = \begin{pmatrix} -i\langle\phi(x)\pi(y)\rangle_c & i\langle\phi(x)\phi(y)\rangle_c \\ -i\langle\pi(x)\pi(y)\rangle_c & i\langle\pi(x)\phi(y)\rangle_c \end{pmatrix}.$$

- Involves connected correlation functions of field $\phi(x)$ and canonically conjugate momentum field $\pi(x)$
- Expectation value $\bar{\phi}$ does not appear explicitly
- Coherent states and vacuum have equal entanglement entropy S_A

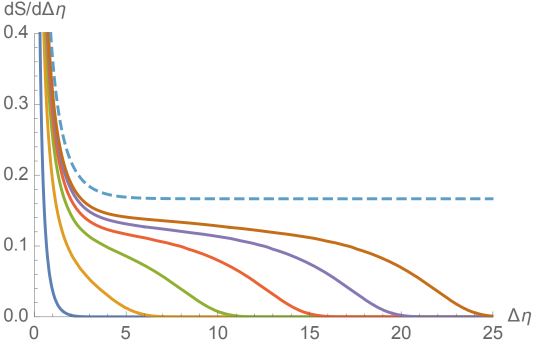
Rapidity interval



- Consider rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ at fixed Bjorken time τ
- Entanglement entropy does not change by unitary time evolution with endpoints kept fixed
- Can be evaluated equivalently in interval $\Delta z = 2\tau \sinh(\Delta\eta/2)$ at fixed time $t = \tau \cosh(\Delta\eta/2)$
- Need to solve eigenvalue problem with correct **boundary conditions**

Bosonized massless Schwinger model

- Entanglement entropy understood numerically for free massive scalars [Casini, Huerta (2009)]
- Entanglement entropy density $dS/d\Delta\eta$ for bosonized massless Schwinger model ($M = \frac{q}{\sqrt{\pi}}$)



$M\tau = 1$, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5}

Conformal limit

- For $M\tau \to 0$ one has conformal field theory limit [Holzhey, Larsen, Wilczek (1994); Calabrese, Cardy (2004)]

$$S(\Delta z) = \frac{c}{3} \ln \left(\Delta z / \epsilon \right) + \text{constant}$$

with small length ϵ acting as UV cutoff.

- Here this implies

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln \left(2\tau \, \text{sinh}(\Delta\eta/2)/\epsilon \right) + \text{constant}$$

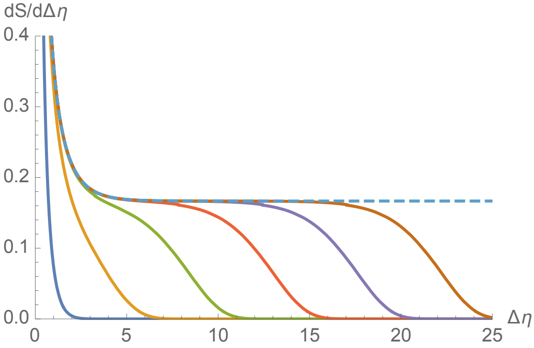
- Conformal charge $c = 1$ for free massless scalars or Dirac fermions.
- Additive constant not universal but entropy density is

$$\begin{aligned} \frac{\partial}{\partial \Delta\eta} S(\tau, \Delta\eta) =& \frac{c}{6} \text{coth}(\Delta\eta/2) \\ \to& \frac{c}{6} \qquad\qquad (\Delta\eta \gg 1) \end{aligned}$$

- Entropy becomes extensive in $\Delta\eta$!

Free massive fermions

- Entanglement entropy can also be calculated for free Dirac fermions of mass m



$m\tau = 1$, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5}

- Same universal plateau $c/6$ with $c = 1$ at early time
- Conformal limit corresponds to non-interacting fermions
- Consistent with or without bosonization

Universal entanglement entropy density

- For very early times "Hubble" expansion rate dominates over masses and interactions

$$H = \frac{1}{\tau} \gg M = \frac{q}{\sqrt{\pi}}, m$$

- Theory dominated by free, massless fermions
- Universal entanglement entropy density

$$\frac{dS}{d\Delta\eta} = \frac{c}{6}$$

  with conformal charge c
- For QCD in 1+1 dimensions (gluons not dynamical)

$$c = N_c \times N_f$$

- From fluctuating transverse coordinates (Nambu-Goto action)

$$c = N_c \times N_f + 2 \approx 9 + 2 = 11$$

Experimental access to entanglement ?

- Could longitudinal entanglement be tested experimentally?
- Unfortunately entropy density $dS/d\eta$ not straight-forward to access.
- Measured in e^+e^- is the number of charged particles per unit rapidity $dN_{\text{ch}}/d\eta$ (rapidity defined with respect to the thrust axis)
- Around mid-rapidity logarithmic dependence on the collision energy.
- Typical values for collision energies $\sqrt{s} = 14 - 206$ GeV in the range

$$dN_{\text{ch}}/d\eta \approx 2 - 4$$

- Entropy per particle S/N can be estimated for a hadron resonance gas in thermal equilibrium $S/N_{\text{ch}} = 7.2$ would give

$$dS/d\eta \approx 14 - 28$$

- This is an upper bound: correlations beyond one-particle functions would lead to reduced entropy.

Temperature and entanglement entropy

- For conformal fields, entanglement entropy has also been calculated at non-zero temperature.
- For static interval of length l [Calabrese, Cardy (2004)]

$$S(T,l) = \frac{c}{3}\ln\left(\frac{1}{\pi T\epsilon}\sinh(\pi l T)\right) + \text{const}$$

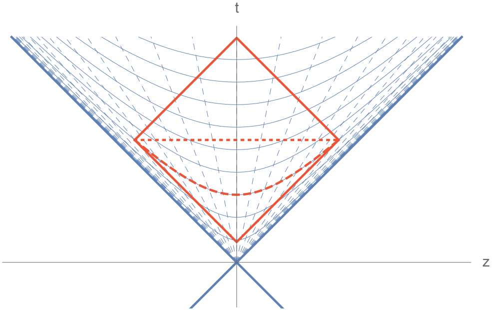
- Compare this to our result in expanding geometry

$$S(\tau,\Delta\eta) = \frac{c}{3}\ln\left(\frac{2\tau}{\epsilon}\,\mathsf{sinh}(\Delta\eta/2)\right) + \text{constant}$$

- Expressions agree for $l = \tau\Delta\eta$ (with metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$) and time-dependent temperature

$$T = \frac{1}{2\pi\tau}$$

Modular or entanglement Hamiltonian 1



- Conformal field theory
- Hypersurface Σ with boundary on the intersection of two light cones
- Reduced density matrix
  [Casini, Huerta, Myers (2011), Arias, Blanco, Casini, Huerta (2017)]

$$\rho_A = \frac{1}{Z_A} e^{-K}, \qquad Z_A = \mathrm{Tr}\, e^{-K},$$

- Modular or entanglement Hamiltonian K.

Modular or entanglement Hamiltonian 2

- Modular or entanglement Hamiltonian is **local expression**

$$K = \int_\Sigma d\Sigma^\mu \, \xi^\nu(x) \, T_{\mu\nu}(x).$$

- Energy-momentum tensor $T_{\mu\nu}(x)$ and $\xi^\nu(x)$ is a vector field

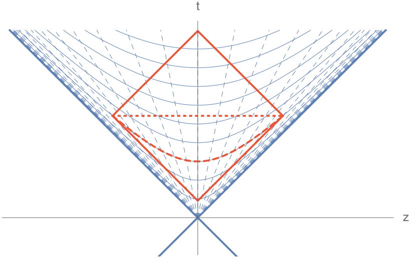
$$\xi^\mu(x) = \tfrac{2\pi}{(k-p)^2} [(k-x)^\mu (x-p)(k-p) + (x-p)^\mu \\ \times (k-x)(k-p) - (k-p)^\mu (x-p)(k-x)]$$

  with end point of the future light cone k and starting point of the past light cone p.

- Inverse temperature and fluid velocity

$$\xi^\mu(x) = \beta^\mu(x) = \frac{u^\mu(x)}{T(x)}$$

Modular or entanglement Hamiltonian 3



- For k very far in the future $\xi^\mu(x) \to 2\pi\, x^\mu$
- Fluid velocity in τ-direction & time-dependent temperature

$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

- **Entanglement between different rapidity intervals alone leads to local thermal density matrix at very early times !**
- Hawking-Unruh temperature in Rindler wedge $T(x) = \hbar c/(2\pi x)$

Conclusions

- Rapidity intervals in an expanding string are entangled
- Entanglement comes in via boundary terms
- At very early times theory effectively conformal

$$\frac{1}{\tau} \gg m, g$$

- Entanglement entropy extensive in rapidity $\frac{dS}{d\Delta\eta} = \frac{c}{6}$
- Determined by conformal charge $c = N_c \times N_f + 2$
- Reduced density matrix for conformal field theory is of locally thermal form with temperature

$$T = \frac{\hbar}{2\pi\tau}$$

- Entanglement could be important ingredient to understand apparent "thermal effects" in e^+e^- and other collider experiments