Renormalization-group flow of the effective action of cosmological large-scale structures

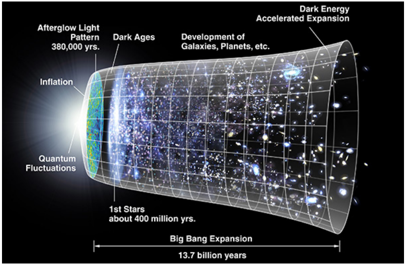
Stefan Flörchinger

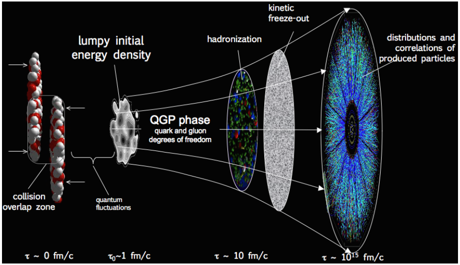
Brookhaven National Laboratory, December 2, 2016.

UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP 1511, 049 (2015)]
[Floerchinger, Garny, Tetradis & Wiedemann, 1607.03453]

Big bang – little bang analogy





Cosmology

- cosmol. scale: MPc$= 3.1 \times 10^{22}$ m
- Gravity + QED + Dark sector
- one big event

Heavy ion collisions

- nuclear scale: fm$= 10^{-15}$ m
- QCD
- very many events

- initial conditions not directly accessible
- all information must be reconstructed from final state
- **dynamical description as a fluid on large scales**

The dark matter fluid

- Heavy ion collisions

$$\mathscr{L}_{\mathsf{QCD}} \quad \rightarrow \quad \text{fluid properties}$$

- Late time cosmology

$$\text{fluid properties} \quad \rightarrow \quad \mathscr{L}_{\text{dark matter}}$$

- Until direct detection of dark matter, it can only be observed via

$$T^{\mu\nu}_{\text{dark matter}}$$

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = (\epsilon + p + \pi_{\text{bulk}})u^\mu u^\nu + (p + \pi_{\text{bulk}})g^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

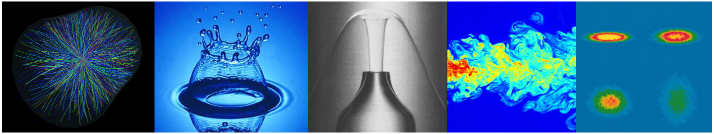
- tensor decomposition w. r. t. fluid velocity u^μ
- pressure $p = p(\epsilon, n)$
- **close-to-equilibrium**: constitutive relations from derivative expansion
  - bulk viscous pressure $\pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu + \ldots$
  - shear stress $\pi^{\mu\nu} = -\eta \left[\Delta^{\mu\alpha}\nabla_\alpha u^\nu + \Delta^{\nu\alpha}\nabla_\alpha u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\right] + \ldots$
  - diffusion current $\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon+p}\right]^2 \Delta^{\alpha\beta}\partial_\beta \left(\frac{\mu}{T}\right) + \ldots$
- more general: **dynamical equations** for π_{bulk}, $\pi^{\mu\nu}$ and ν^μ

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu + \ldots$$

Fluid dynamic equations for ϵ, n and u^μ from covariant **conservation laws**

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \nabla_\mu N^\mu = 0.$$

Fluid dynamics



- Long distances, long times or strong enough interactions
- Needs **macroscopic** fluid properties
  - equation of state $p(\epsilon, n)$
  - shear viscosity $\eta(\epsilon, n)$
  - bulk viscosity $\zeta(\epsilon, n)$
  - heat conductivity $\kappa(\epsilon, n)$
  - relaxation times, ...
- For QCD no full *ab initio* calculation of transport properties possible yet but in principle fixed by **microscopic** properties encoded in $\mathscr{L}_{\text{QCD}}$
- Ongoing experimental and theoretical effort to understand this in detail

Ideal fluid versus collision-less gas

- Many codes used in cosmology describe dark matter as **ideal, cold and pressure-less fluid**

$$T^{\mu\nu} = \epsilon \, u^\mu u^\nu$$

- Equation of state $p = 0$
- No shear stress and bulk viscous pressure $\pi^{\mu\nu} = \pi_{\text{bulk}} = 0$
- Dark matter is also modeled as **collision-less gas** of massive particles, interacting via gravity only
- Two pictures are in general **not consistent**

Collision-less, cold dark matter

- Collision-less gas in curved space

$$p^\mu \frac{\partial}{\partial x^\mu} f(x,\vec{p}) - \Gamma^j_{\mu\nu} p^\mu p^\nu \frac{\partial}{\partial p^j} f(x,\vec{p}) = 0$$

- For massive particles in non-relativistic limit, Vlasov equation

$$\left(\partial_t + \frac{p_j}{m}\frac{\partial}{\partial x_j}\right) f(x,\vec{p}) - \left(m \frac{\partial}{\partial x_j}\phi(x)\right) \frac{\partial}{\partial p_j} f(x,\vec{p}) = 0$$

with

$$\Delta\phi(x) = 4\pi G_\mathsf{N} m \int_{\vec{p}} f(x,\vec{p})$$

Moments of distribution function

- mass density

$$\rho(x) = \int_{\vec{p}} m\, f(x,\vec{p})$$

- momentum density

$$\rho(x) v_j(x) = \int_{\vec{p}} p_j\, f(x,\vec{p})$$

- energy density

$$\varepsilon(x) = \int_{\vec{p}} \frac{m}{2} \left(\frac{\vec{p}}{m} - \vec{v}(x) \right)^2 f(x,\vec{p})$$

- heat current

$$q_j(x) = \int_{\vec{p}} \frac{m}{2} \left(\frac{\vec{p}}{m} - \vec{v}(x) \right)^2 \frac{p_j}{m}\, f(x,\vec{p})$$

- shear stress

$$T_{jk}(x) = m \int_{\vec{p}} \left(\frac{p_j}{m} - v_j(x) \right) \left(\frac{p_k}{m} - v_k(x) \right) f(x,\vec{p})$$

- higher order moments

$$T_{jkl}(x) = m \int_{\vec{p}} \left(\frac{p_j}{m} - v_j(x) \right) \left(\frac{p_k}{m} - v_k(x) \right) \left(\frac{p_l}{m} - v_l(x) \right) f(x,\vec{p})$$

Moments of Vlasov's equation

- Conservation of mass

$$\partial_t \rho + v_j \partial_j \rho + \rho \partial_j v_j = 0,$$

- Conservation of momentum

$$\rho \left(\partial_t + v_j \partial_j\right) v_k + \partial_j T_{jk} = -\rho\, \partial_k \phi$$

- Conservation of internal energy

$$\left(\partial_t + v_j \partial_j\right) \varepsilon + \varepsilon\, \partial_j v_j + \left(\partial_j v_k\right) T_{jk} + \partial_j q_j = 0$$

- Evolution of shear stress

$$\partial_t T_{jk} + \frac{\partial}{\partial x_l} T_{jkl} + \left(\frac{\partial}{\partial x_l} v_j\right) T_{kl} + \left(\frac{\partial}{\partial x_l} v_k\right) T_{jl} + \left(\frac{\partial}{\partial x_l} v_l\right) T_{jk} + v_l \frac{\partial}{\partial x_l} T_{jk} = 0$$

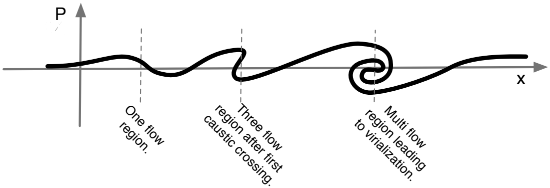
- If $\varepsilon = q_j = T_{jk} = T_{jkl} = \ldots = 0$ at some time t_0, the moment equations imply $\varepsilon = q_j = T_{jk} = T_{jkl} = 0$ also at later times $t > t_0$

Shell crossing

- The approximation $\varepsilon = q_j = T_{jk} = T_{jkl} = \ldots = 0$ is called single stream approximation
- Corresponds to a phase space density

$$f(x, \vec{p}) = \frac{\rho(x)}{m} \delta^{(3)}(\vec{p} - m\vec{v}(x))$$

- However, phase space manifolds can can wind up



- For multi-stream flow

$$f(x, \vec{p}) = \sum_{n=1}^{N_{\text{streams}}(x)} \frac{\rho_n(x)}{m} \delta^{(3)}(\vec{p}_n - m\vec{v}(x))$$

all moments are in general non-vanishing

Dissipative properties

Viscosities

- Diffusive transport of momentum [Maxwell (1860)]
- Depend strongly on interaction properties
- Example: non-relativistic gas of particles with mass m, mean peculiar velocity $\bar{v}$, elastic $2 \to 2$ cross-section σ_{el}

$$\eta = \frac{m\,\bar{v}}{3\,\sigma_{\text{el}}} \qquad\qquad \zeta = 0$$

- Formally, for cold, collision-less dark matter, this is $0/0$...

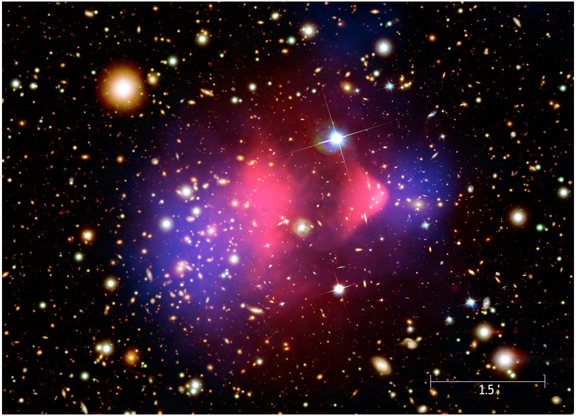
Entropy production

- For dilute gases, described by kinetic theory, Boltzmann's H-theorem:

$$\partial_t H \sim \sigma_{\text{el}}.$$

- In general, dissipation could contain interesting additional **information about dark matter**

Self-interaction of dark matter

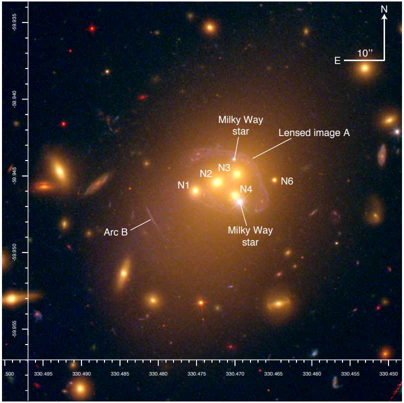


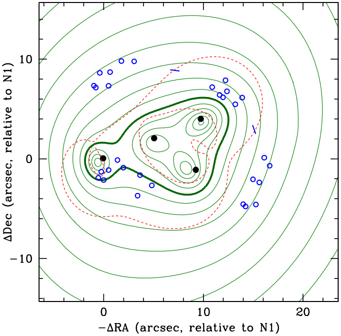
Gravitational lensing and x-ray image of “bullet cluster” 1E0657-56

- so far: dark matter is non-interacting $\rightarrow$ can collide without stopping

$$\frac{\sigma_{\text{el}}}{m} \lesssim 1.2\,\frac{\text{cm}^2}{\text{g}}$$

Is dark matter self-interacting?





Galaxy cluster Abell 3827

[Massey *et al.*, MNRAS 449, 3393 (2015)]

- Offset between stars and dark matter falling into cluster
- Is this a first indication for a dark matter self-interaction?

$$\frac{\sigma_{\text{el}}}{m} \approx 3\,\frac{\text{cm}^2}{\text{g}} \approx 0.5\,\frac{\text{b}}{\text{GeV}} \qquad \text{(under debate)}$$

[Kahlhoefer, Schmidt-Hoberg, Kummer & Sarkar, MNRAS 452, 1 (2015)]

Cosmological inhomogeneities

Cosmological perturbation theory

[Lifshitz, Peebles, Bardeen, Kosama, Sasaki, Ehler, Ellis, Hawking, Mukhanov, Weinberg, ...]

- Solves evolution equations for fluid + gravity
- Expands in perturbations around homogeneous background
- Detailed understanding how different modes evolve
- Diagramatic formalism for non-linear mode-mode interactions

Cosmological fluid

- Very simple equations of state $p = w\,\epsilon$
- Viscosities usually neglected $\eta = \zeta = 0$
- Photons and neutrinos are free streaming

Initial conditions

- Perturbations are classified into scalars, vectors, tensors
- Vector modes are decaying, need not be specified
- Tensor modes are gravitational waves, can be neglected for most purposes
- Decaying scalar modes also not relevant
- Growing scalar modes are further classified by wavelength
- For relevant range of wavelength: close to Gaussian probability distribution
- Almost scale invariant initial spectrum of density contrast $\delta(\mathbf{k}) = \delta\epsilon(\mathbf{k})/\bar{\epsilon}$

$$\langle \delta(\mathbf{k})\, \delta(\mathbf{k}') \rangle = P(k)\; \delta^{(3)}(\mathbf{k} + \mathbf{k}')$$

with

$$P(k) \sim k^{n_s - 1} \qquad\qquad n_s = 0.968 \pm 0.006$$ [Planck (2015)]

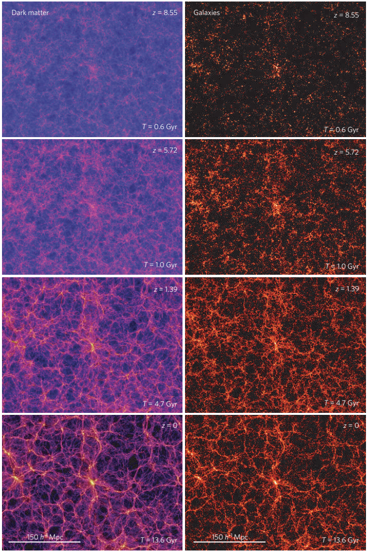
Cosmological structure formation

- Formation of large scale structure
  - tests physics of dark matter
  - tests physics of dark energy
  - gets tested by missions like Euclid
- Cosmological perturbation theory breaks down when density contrast

$$\delta(\mathbf{k}) = \frac{\delta\epsilon(\mathbf{k})}{\bar{\epsilon}} \gg 1$$

  grows large at late times and for small scales
- Numerical simulations (N-body) are expensive and time-consuming
- One would like to have better analytical understanding

Dark matter　　Visible galaxies



[Springel, Frenk & White, Nature 440, 1137 (2006)]

Approximations for large scale cosmology

- cold dark matter, single-stream approximation: ideal, pressure-less fluid

$$T^{\mu\nu} = \epsilon \, u^\mu u^\nu$$

Covariant conservation law

$$u^\mu \nabla_\mu \epsilon + \epsilon \nabla_\mu u^\mu = 0, \qquad \epsilon \, u^\mu \nabla_\mu u^\nu = 0.$$

- only scalar perturbations: no vorticity, fluid velocity characterized by velocity divergence

$$\theta = \vec{\nabla} \cdot \vec{v}$$

- metric with scale factor and Newtonian potential ϕ

$$ds^2 = a^2(\tau) \left[-\left(1 + 2\Phi(\tau, \vec{x})\right) d\tau^2 + \left(1 - 2\Phi(\tau, \vec{x})\right) d\vec{x} d\vec{x} \right]$$

where

$$\Delta\Phi(\tau, \vec{x}) = 4\pi G_{\mathsf{N}} \, \delta\epsilon(\tau, \vec{x})$$

- dynamical fields can be taken as

$$\phi_1(\tau, \vec{k}) = \delta(\tau, \vec{k}) = \frac{\delta\epsilon(\tau, \vec{k})}{\bar{\epsilon}(\tau)}, \qquad \phi_2(\tau, \vec{k}) = -\frac{\theta(\tau, \vec{k})}{\mathcal{H}(\tau)}$$

Equations of motion and action

- Evolution equations for fluid + gravity

$$\partial_\eta \phi_a(\vec{k}) = -\Omega_{ab}(\eta)\phi_b(\vec{k}) + \int d^3p\, d^3q\, \delta^{(3)}(\vec{k}-\vec{p}-\vec{q})\gamma_{abc}(\vec{p},\vec{q},\eta)\, \phi_b(\vec{p})\, \phi_c(\vec{q})$$

with $\eta = \ln a(\tau)$,

$$\Omega(\eta) = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\Omega_m & 1+\frac{\mathcal{H}'}{\mathcal{H}} \end{pmatrix}$$

and vertices

$$\gamma_{121}(\vec{q},\vec{p},\eta) = \gamma_{112}(\vec{p},\vec{q},\eta) = \frac{(\vec{p}+\vec{q})\vec{q}}{2\vec{q}^2}$$

$$\gamma_{222}(\vec{p},\vec{q},\eta) = \frac{(\vec{p}+\vec{q})^2\vec{p}\cdot\vec{q}}{2\vec{p}^2\vec{q}^2}$$

- Can also be obtained from variation of the action

$$S[\phi,\chi] = \int d\eta \Bigg[\int d^3k\, \chi_a(-\vec{k},\eta)\, (\delta_{ab}\partial_\eta + \Omega_{ab})\, \phi_b(\vec{k},\eta)$$

$$- \int d^3k\, d^3p\, d^3q\, \delta^{(3)}(\vec{k}-\vec{p}-\vec{q})\gamma_{abc}(\vec{k},\vec{p},\vec{q})\chi_a(-\vec{k},\eta)\phi_b(\vec{p},\eta)\phi_c(\vec{q},\eta) \Bigg]$$

Partition function

- introduce sources
- integrate over Gaussian distributed initial conditions
- leads to the partition function

$$Z[J,\, K;\, P^0] = \int \mathcal{D}\phi \mathcal{D}\chi \exp\Big\{ -\frac{1}{2}\, \chi_a(\eta_0) P^0_{ab} \chi_b(\eta_0) + i \int d\eta \left[\chi_a (\delta_{ab}\partial_\eta + \Omega_{ab})\phi_b - \gamma_{abc}\chi_a\phi_b\phi_c + J_a\phi_a + K_b\chi_b \right] \Big\}$$

- correlation functions can be obtained by functional derivatives

Schwinger functional

- Schwinger functional is defined as

$$W[J, K; P^0] = -i \log Z[J, K; P^0].$$

- Propagators and correlation functions follow as functional derivatives

$$\begin{aligned}
\left.\frac{\delta^2 W}{\delta J_a(-\vec{k}, \eta)\, \delta J_b(\vec{k}', \eta')}\right|_{J,\, K=0} &= i\delta(\vec{k} - \vec{k}')\; P_{ab}(\vec{k}, \eta, \eta') \\
\left.\frac{\delta^2 W}{\delta J_a(-\vec{k}, \eta)\, \delta K_b(\vec{k}', \eta')}\right|_{J,\, K=0} &= -\delta(\vec{k} - \vec{k}')\; G^R_{ab}(\vec{k}, \eta, \eta') \\
\left.\frac{\delta^2 W}{\delta K_a(-\vec{k}, \eta)\, \delta J_b(\vec{k}', \eta')}\right|_{J,\, K=0} &= -\delta(\vec{k} - \vec{k}')\; G^A_{ab}(\vec{k}, \eta, \eta') \\
\left.\frac{\delta^2 W}{\delta K_a(-\vec{k}, \eta)\, \delta K_b(\vec{k}', \eta')}\right|_{J,\, K=0} &= 0
\end{aligned}$$

Effective action and renormalized field equations

- Effective action is defined as Legendre transform

$$\Gamma[\phi, \chi; P^0] = \int d\eta d^3x \left\{ J_a \phi_a + K_b \chi_b \right\} - W[J, K; P^0],$$

with expectation values

$$\phi_a(\vec{k}, \eta) = \frac{\delta}{\delta J_a(\vec{k}, \eta)} W, \qquad \chi_b(\vec{k}, \eta) = \frac{\delta}{\delta K_b(\vec{k}, \eta)} W$$

- Renormalized field equations

$$\begin{aligned} \frac{\delta}{\delta \phi_a(\vec{x}, \eta)} \Gamma[\phi, \chi] &= J_a(\vec{x}, \eta) \\ \frac{\delta}{\delta \chi_a(\vec{x}, \eta)} \Gamma[\phi, \chi] &= K_a(\vec{x}, \eta) \end{aligned}$$

Coarse-grained effective action and functional RG equation

- Effective action $\Gamma[\phi,\chi]$ contains effect of initial state fluctuations
- Idea: take them into account gradually - organized by scale
- Modify initial spectrum by cutting off the IR

$$P_k^0(\vec{q}) = P^0(\vec{q})\Theta(|\vec{q}|-k)$$

  lowering k takes more and more IR modes into account

- k-dependent Schwinger functional

$$W_k[J,K] \equiv W[J,K;P_k^0]$$

  satisfies exact flow equation [Polchinski 1984]

$$\partial_k W_k[J,K] = \frac{i}{2}\int_q \partial_k (P_k^0(\vec{q}))_{ab} \left\{ \chi_a(\vec{q},0)\chi_b(-\vec{q},0) - i\frac{\delta^2 W_k[J,K]}{\delta K_a(\vec{q},0)\delta K_b(-\vec{q},0)} \right\}$$

- Modified effective action satisfies exact flow equation [Wetterich 1993]

$$\partial_k \Gamma_k[\phi,\chi] = \frac{1}{2}\,\mathrm{Tr}\left\{ \left(\Gamma_k^{(2)}[\phi,\chi] - i\left(P_k^0 - P^0\right)\right)^{-1} \partial_k P_k^0 \right\}$$

Boundary values

- For large regulator scale k, fluctuations are suppressed

$$\lim_{k\to\infty} \Gamma_k[\phi,\chi] = -S[\phi,\chi] - \frac{i}{2}\int_{\vec{q}} \chi_a(-\vec{q},0) P^0_{ab}(\vec{q}) \chi_b(\vec{q},0)$$

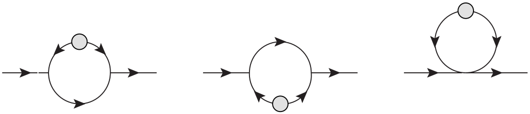
- For small regulator scale k, all fluctuations are included

$$\lim_{k\to 0} \Gamma_k[\phi,\chi] = \Gamma[\phi,\chi]$$

Flow equations for correlation functions

- Flow equations for correlation functions have one-loop form, for example inverse retarded propagator

$$\begin{aligned}\partial_k D^R_{ab,k}(\vec{q},\eta,\eta') = &\frac{1}{2}\,\mathrm{Tr}\left\{ W_k^{(2)} \frac{\delta\Gamma_k^{(2)}}{\delta\chi_a(-\vec{q},\eta)} W_k^{(2)} \frac{\delta\Gamma_k^{(2)}}{\delta\phi_b(\vec{q}',\eta')} W_k^{(2)} \partial_k P_k^0 \right\} \\ &+ \frac{1}{2}\,\mathrm{Tr}\left\{ W_k^{(2)} \frac{\delta\Gamma_k^{(2)}}{\delta\phi_b(\vec{q}',\eta')} W_k^{(2)} \frac{\delta\Gamma_k^{(2)}}{\delta\chi_a(-\vec{q},\eta)} W_k^{(2)} \partial_k P_k^0 \right\} \\ &- \frac{1}{2}\,\mathrm{Tr}\left\{ W_k^{(2)} \frac{\delta^2\Gamma_k^{(2)}}{\delta\chi_a(-\vec{q},\eta)\,\delta\phi_b(\vec{q}',\eta')} W_k^{(2)} \partial_k P_k^0 \right\}\end{aligned}$$



Solving the flow equation for Γ_k

- Flow equation for $\Gamma_k[\phi, \chi]$ can be solved by iteration $\rightarrow$ **Standard perturbation theory**

$$\Gamma_k[\phi, \chi] = -S[\phi, \chi] + \frac{i}{2}\mathsf{Tr}\left\{\ln\left(-S^{(2)}[\phi, \chi] - i(P_k^0 - P_0)\right)\right\} + \ldots$$

- Within **truncations**, one can also find approximate, **non-perturbative** solutions. Make ansatz

$$\Gamma_k[\phi, \chi] = \int d\eta\, d^3x \sum_{j=1}^{N} \alpha_j(k)\mathcal{O}_j[\phi, \chi]$$

and derive flow equations for the coefficients $\alpha_j(k)$

$$k\frac{\partial}{\partial k}\alpha_j = \beta_j(\alpha_1, \ldots, \alpha_N)$$

Renormalization of effective viscosity and pressure

[Floerchinger, Garny, Tetradis & Wiedemann, 1607.03453]

- Effective theory at scale k has additional terms in equations of motion
- Order them by fluid dynamic derivative expansion
- Lowest order: ideal fluid
- Next-to-lowest order: effective sound velocity parameter

$$\gamma_s = \frac{c_s^2}{\mathcal{H}^2} = \frac{dp/d\epsilon}{\mathcal{H}^2}.$$

  and effective viscosity parameter (for compressional modes)

$$\gamma_\nu = \frac{4\eta/3 + \zeta}{(\epsilon + p)\mathcal{H}a}.$$

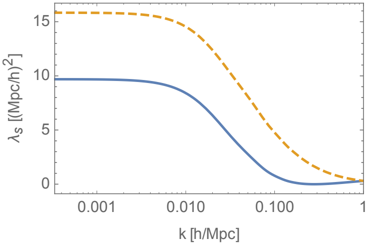
- Both depend on flow parameter k and scale factor a

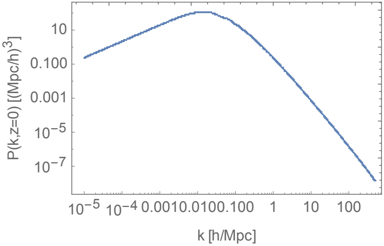
$$\gamma_s = \lambda_s\, a^\kappa, \qquad \gamma_\nu = \lambda_\nu\, a^\kappa$$

  with k-dependent parameters $\lambda_s(k)$, $\lambda_\nu(k)$ and $\kappa(k)$.

RG flow of effective sound velocity parameter

[Floerchinger, Garny, Tetradis & Wiedemann, 1607.03453]





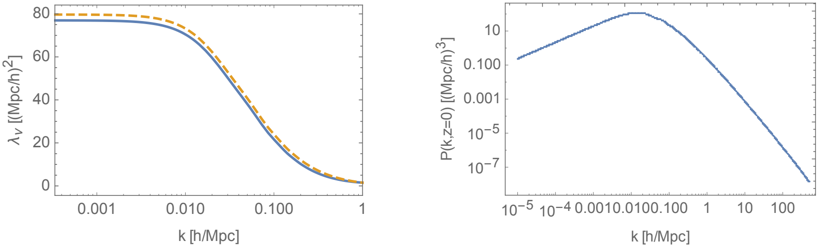
- left: RG flow of effective sound velocity today $\lambda_s = \frac{c_s^2}{H_0^2} = \frac{dp/d\rho}{H_0^2}$
  - dashed line: one-loop approximation

$$\partial_k \lambda_s = -\frac{4\pi}{3}\frac{31}{70}P^0(k)$$

  - solid line: functional RG result
- right: linear density power spectrum

RG flow of effective viscosity parameter

[Floerchinger, Garny, Tetradis & Wiedemann, 1607.03453]



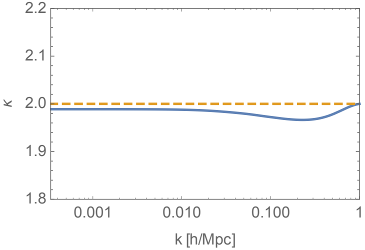
- left: RG flow of effective viscosity today $\lambda_\nu = \frac{4\eta/3+\zeta}{(\rho+p)_0 H_0}$
  - dashed line: one-loop approximation

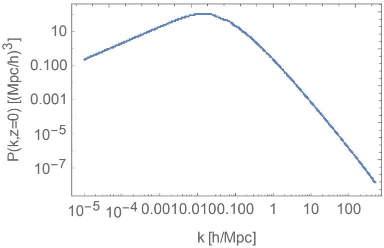
$$\partial_k \lambda_\nu = -\frac{4\pi}{3}\frac{78}{35}P^0(k)$$

  - solid line: functional RG
- right: linear density power spectrum

RG flow of exponent κ

[Floerchinger, Garny, Tetradis & Wiedemann, 1607.03453]



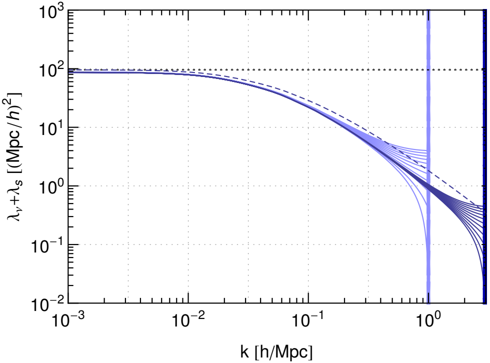


- left: RG flow of exponent κ
  - dashed line: one-loop approximation

$$\partial_k \kappa = \frac{4\pi}{3} P^0(k) \frac{78(\kappa - 2)}{35\tilde{\lambda}_\nu}$$

  - solid line: functional RG
- right: linear density power spectrum

Fixed point behavior



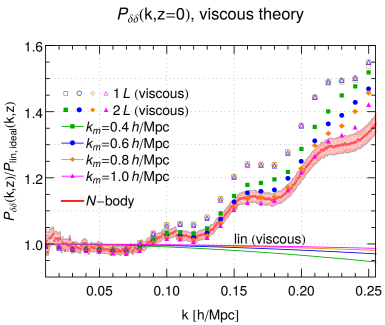
- growing mode is sensitive to $\lambda_s + \lambda_\nu$
- functional RG has IR fixed points

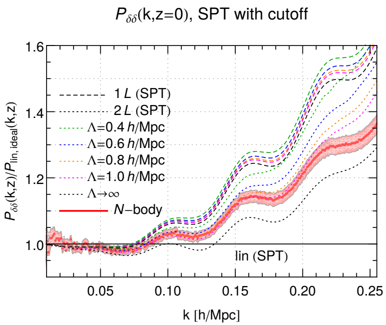
Functional RG + perturbation theory

[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP 1511, 049 (2015)]

[Floerchinger, Garny, Tetradis & Wiedemann, 1607.03453]

- RG evolution to determine effective viscosity and sound velocity at intermediate scale k_m
- Perturbation theory for power spectrum for scales $0 < |\mathbf{q}| < k_m$
- Theory with effective viscosity and sound velocity parameters
- Background + linear + one-loop + two-loop





Conclusions

- Structure formation in the cosmological fluid is dominated by dark matter.
- Would be highly interesting to constrain different dark matter models via the fluid properties.
- A better understanding of non-linear structure formation is needed.
- Field theory and renormalization group techniques can help here.
- Taking into account initial state fluctuations leads to an effective theory, which, to leading order in a derivative expansion, can be characterized by effective viscosity and sound velocity terms.
- First numerical results look promising and agree well with N-body simulations.

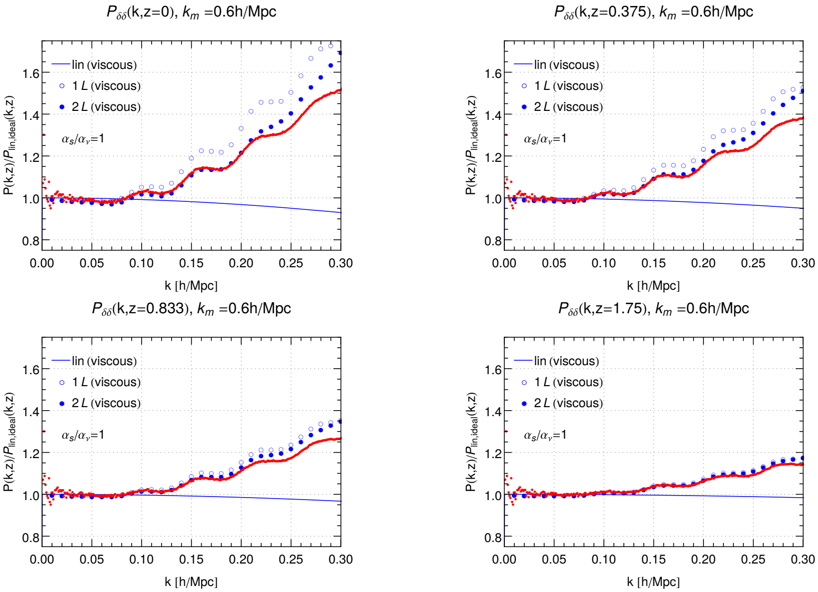
Backup slides

"Fundamental" and "effective" viscosity

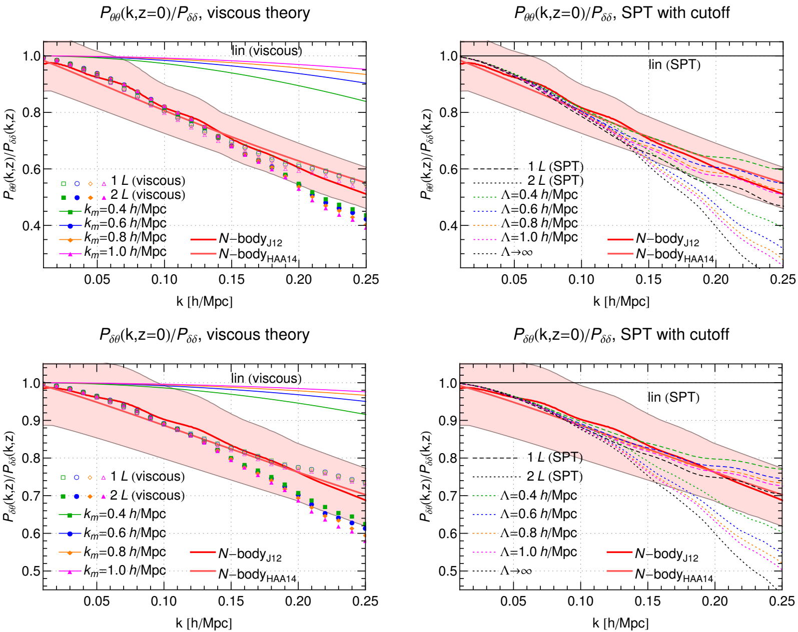
Two types of viscosities for cosmological fluid

1. Momentum transport by particles or radiation
   - governed by interactions
   - from linear response theory [Green (1954), Kubo (1957)]
   - close to equilibrium
2. Momentum transport in the inhomogeneous, coarse-grained fluid
   - governed by non-linear fluid mode couplings
   - determined perturbatively [Blas, Floerchinger, Garny, Tetradis & Wiedemann]
   - non-equilibrium
   - heavy ions: anomalous plasma viscosity [Asakawa, Bass & Müller (2006)]
     eddy viscosity [Romatschke (2008)]

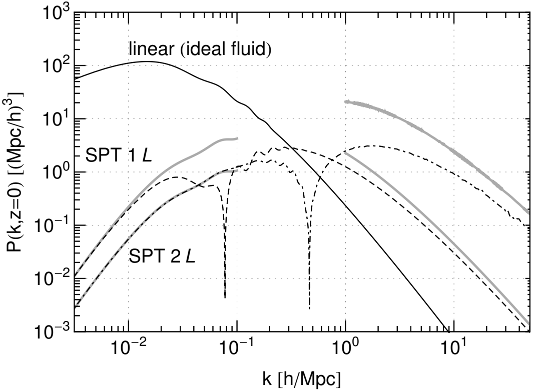
Power spectrum at different redshifts



Velocity spectra



Power spectrum, standard perturbation theory



[D. Blas, M. Garny and T. Konstandin, JCAP 1309 (2013) 024]

Could dissipation affect the overall cosmological expansion ?

Bulk viscosity

- Bulk viscous pressure is negative for expanding universe

$$\pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu = -\zeta\, 3H < 0$$

- Negative effective pressure

$$p_{\text{eff}} = p + \pi_{\text{bulk}} < 0$$

would act similar to dark energy in Friedmann's equations
[Murphy (1973), Padmanabhan & Chitre (1987), Fabris, Goncalves & de Sa Ribeiro (2006), Li & Barrow (2009), Velten & Schwarz (2011), Gagnon & Lesgourgues (2011), ...]

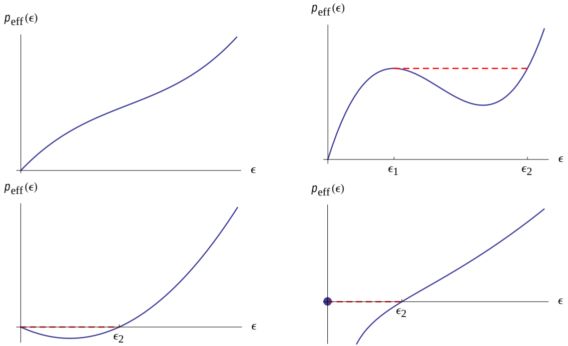
- **Is negative effective pressure physical?**
- In context of heavy ion physics: instability for $p_{\text{eff}} < 0$ ("cavitation")
[Torrieri & Mishustin (2008), Rajagopal & Tripuraneni (2010), Buchel, Camanho & Edelstein (2014), Habich & Romatschke (2015), Denicol, Gale & Jeon (2015)]
- What precisely happens at the instability?

Is negative effective pressure physical?

- Kinetic theory

$$p_{\text{eff}}(x) = \int \frac{d^3p}{(2\pi)^3}\,\frac{\vec{p}^2}{3E_{\vec{p}}}\,f(x,\vec{p}) \geq 0$$

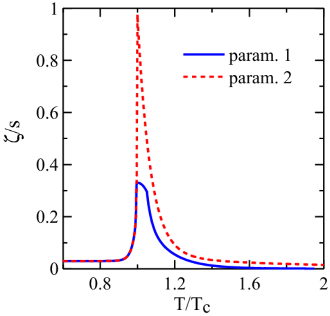
- Stability argument



If there is a vacuum with $\epsilon = p_{\text{eff}} = 0$, phases with $p_{\text{eff}} < 0$ cannot be mechanically stable. (But could be metastable.)

Bulk viscosity in heavy ion physics

- In heavy ion physics people start now to consider bulk viscosity.
- Becomes relevant close to chiral crossover



[Denicol, Gale & Jeon (2015)]

- Is there a first-order phase transition triggered by the expansion?
- What is the relation to chemical and kinetic freeze-out?
- More detailed understanding needed, both for heavy ion physics and cosmology

Backreaction: General idea

- for $0+1$ dimensional, non-linear dynamics

$$\dot{\varphi} = f(\varphi) = f_0 + f_1\,\varphi + \tfrac{1}{2} f_2\,\varphi^2 + \ldots$$

- one has for expectation values $\bar{\varphi} = \langle \varphi \rangle$

$$\dot{\bar{\varphi}} = f_0 + f_1\,\bar{\varphi} + \tfrac{1}{2} f_2\,\bar{\varphi}^2 + \tfrac{1}{2} f_2\,\langle (\varphi - \bar{\varphi})^2 \rangle + \ldots$$

- evolution equation for expectation value $\bar{\varphi}$ depends on two-point correlation function or spectrum $P_2 = \langle (\varphi - \bar{\varphi})^2 \rangle$
- evolution equation for spectrum depends on bispectrum and so on
- more complicated for higher dimensional theories
- more complicated for gauge theories such as gravity

Backreaction in gravity

- Einstein's equations are non-linear.
- Important question [G. F. R. Ellis (1984)]: If Einstein's field equations describe small scales, including inhomogeneities, do they also hold on large scales?
- Is there a sizable backreaction from inhomogeneities to the cosmological expansion?
- Difficult question, has been studied by many people
  [Ellis & Stoeger (1987); Mukhanov, Abramo & Brandenberger (1997); Unruh (1998); Buchert (2000); Geshnzjani & Brandenberger (2002); Schwarz (2002); Wetterich (2003); Räsänen (2004); Kolb, Matarrese & Riotto (2006); Brown, Behrend, Malik (2009); Gasperini, Marozzi & Veneziano (2009); Clarkson & Umeh (2011); Green & Wald (2011); ...]
- Recent reviews: [Buchert & Räsänen, Ann. Rev. Nucl. Part. Sci. 62, 57 (2012); Green & Wald, Class. Quant. Grav. 31, 234003 (2014)]
- No general consensus but most people believe now that **gravitational backreaction is rather small**.
- In the following we look at a new **backreaction on the matter side** of Einstein's equations.

Fluid equation for energy density

First order viscous fluid dynamics

$$u^\mu \partial_\mu \epsilon + (\epsilon + p)\nabla_\mu u^\mu - \zeta\Theta^2 - 2\eta\sigma^{\mu\nu}\sigma_{\mu\nu} = 0$$

For $\vec{v}^2 \ll c^2$ and Newtonian potentials $\Phi, \Psi \ll 1$

$$\begin{aligned} &\dot{\epsilon} + \vec{v} \cdot \vec{\nabla}\epsilon + (\epsilon + p)\left(3\tfrac{\dot{a}}{a} + \vec{\nabla} \cdot \vec{v}\right) \\ &= \tfrac{\zeta}{a}\left[3\tfrac{\dot{a}}{a} + \vec{\nabla} \cdot \vec{v}\right]^2 + \tfrac{\eta}{a}\left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \tfrac{2}{3}(\vec{\nabla} \cdot \vec{v})^2\right] \end{aligned}$$

Fluid dynamic backreaction in Cosmology

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

Expectation value of energy density $\bar{\epsilon} = \langle \epsilon \rangle$

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p} - 3\bar{\zeta}H\right) = D$$

with dissipative backreaction term

$$D = \frac{1}{a^2}\langle \eta \left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}\partial_i v_i \partial_j v_j\right]\rangle \\ + \frac{1}{a^2}\langle \zeta [\vec{\nabla} \cdot \vec{v}]^2 \rangle + \frac{1}{a}\langle \vec{v} \cdot \vec{\nabla}\left(p - 6\zeta H\right)\rangle$$

- D vanishes for unperturbed homogeneous and isotropic universe
- D has contribution from shear & bulk viscous dissipation and thermodynamic work done by contraction against pressure gradients
- dissipative terms in D are positive semi-definite
- for spatially constant viscosities and scalar perturbations only

$$D = \frac{\bar{\zeta} + \frac{4}{3}\bar{\eta}}{a^2} \int d^3q \; P_{\theta\theta}(q)$$

Dissipation of perturbations

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

- Dissipative backreaction does not need negative effective pressure

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p}_{\text{eff}}\right) = D$$

- D is an integral over perturbations, could become large at late times.
- Can it potentially accelerate the universe?
- Need additional equation for scale parameter a
- Use trace of Einstein's equations $R = 8\pi G_{\text{N}} T^{\mu}{}_{\mu}$

$$\frac{1}{a}\dot{H} + 2H^2 = \frac{4\pi G_{\text{N}}}{3}\left(\bar{\epsilon} - 3\bar{p}_{\text{eff}}\right)$$

  does not depend on unknown quantities like $\langle(\epsilon + p_{\text{eff}})u^{\mu}u^{\nu}\rangle$
- To close the equations one needs equation of state $\bar{p}_{\text{eff}} = \bar{p}_{\text{eff}}(\bar{\epsilon})$ and dissipation parameter D

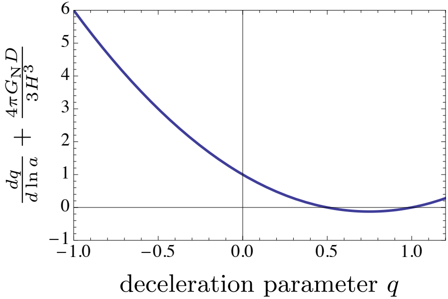
Deceleration parameter

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

- assume now vanishing effective pressure $\bar{p}_{\text{eff}} = 0$
- obtain for deceleration parameter $q = -1 - \frac{\dot{H}}{aH^2}$

$$-\frac{dq}{d\ln a} + 2(q-1)\left(q - \frac{1}{2}\right) = \frac{4\pi G_{\text{N}} D}{3H^3}$$

- for $D = 0$ attractive fixed point at $q_* = \frac{1}{2}$ (deceleration)
- for $D > 0$ fixed point shifted towards $q_* < 0$ (acceleration)



Estimating viscous backreaction D

- For $\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx 4$ one could explain the current accelerated expansion ($q \approx -0.6$) by dissipative backreaction.
- Is this possible?
- In principle one can determine D for given equation of state and viscous properties from dynamics of structure formation.
- So far only rough estimates. If shear viscosity dominates:

$$D = \frac{1}{a^2} \langle \eta \left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3} \partial_i v_i \partial_j v_j \right] \rangle \approx c_D \bar{\eta} H^2$$

  with $c_D = \mathcal{O}(1)$. Corresponds to $\Delta v \approx 100\,\mathsf{km/s}$ for $\Delta x \approx 1\,\mathsf{MPc}$
- Leads to

$$\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx \frac{c_D \bar{\eta} H}{2\rho_c}$$

  with $\rho_c = \frac{3H^2}{8\pi G_{\mathsf{N}}}$

Viscosities

- Relativistic particles / radiation contribute to shear viscosity

$$\eta = c_\eta \, \epsilon_R \, \tau_R$$

  - prefactor $c_\eta = \mathcal{O}(1)$
  - energy density of radiation ϵ_R
  - mean free time τ_R
- Bulk viscosity vanishes in situations with conformal symmetry but can be large when conformal symmetry is broken.
- For massive scalar particles with $\lambda\varphi^4$ interaction [Jeon & Yaffe (1996)]

$$\zeta \sim \frac{m^6}{\lambda^4 T^3} e^{2m/T}, \qquad \eta \sim \frac{m^{5/2} T^{1/2}}{\lambda^2} \qquad \text{for} \qquad \frac{T}{m} \ll 1$$

Estimating viscous backreaction D

Consider shear viscosity from radiation

$$\eta = c_\eta \, \epsilon_R \, \tau_R$$

Backreaction term

$$\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx \frac{c_D c_\eta}{2} \frac{\epsilon_R}{\rho_c} \tau_R H$$

- fluid approximation needs $\tau_R H < 1$
- for sizeable effect one would need $\epsilon_R/\rho_c = \mathcal{O}(1)$
- unlikely that D becomes large enough in this scenario

Needed refinements:

- full dynamics of perturbations
- second order fluid dynamics
- complete model(s)

Could viscous backreaction lead to ΛCDM-type expansion?

[Floerchinger, Tetradis & Wiedemann, 1506.00407]

- Backreaction term $D(z)$ will be *some* function of redshift.
- For given dissipative properties $D(z)$ can be determined, but calculation is involved.
- One may ask simpler question: For what form of $D(z)$ would the expansion be as in the ΛCDM model?
- The *ad hoc* ansatz $D(z) = \text{const} \cdot H(z)$ leads to modified Friedmann equations

$$\bar{\epsilon} - \frac{D}{4H} = \frac{3}{8\pi G_\mathsf{N}} H^2, \qquad \bar{p}_\mathsf{eff} - \frac{D}{12H} = -\frac{1}{8\pi G_\mathsf{N}} \left(2\frac{1}{a}\dot{H} + 3H^2 \right)$$

- In terms of $\hat{\epsilon} = \bar{\epsilon} - \frac{D}{3H}$ one can write

$$\frac{1}{a}\dot{\hat{\epsilon}} + 3H(\hat{\epsilon} + \bar{p}_\mathsf{eff}) = 0, \qquad R + \frac{8\pi G_\mathsf{N} D}{3H} = -8\pi G_\mathsf{N}(\hat{\epsilon} - 3\bar{p}_\mathsf{eff})$$

- For $\bar{p}_\mathsf{eff} = 0$ these are standard equations for ΛCDM model with

$$\Lambda = \frac{2\pi G_\mathsf{N} D}{3H}$$

Modification of Friedmann's equations by backreaction 1

- For universe with fluid velocity inhomogeneities one cannot easily take direct average of Einstein's equations.
- However, fluid equation for energy density and trace of Einstein's equations can be used.
- By integration one finds modified Friedmann equation

$$H(\tau)^2 = \frac{8\pi G_{\mathsf{N}}}{3}\left[\bar{\epsilon}(\tau) - \int_{\tau_{\mathsf{I}}}^{\tau} d\tau' \left(\frac{a(\tau')}{a(\tau)}\right)^4 a(\tau') D(\tau')\right]$$

- Additive deviation from Friedmann's law for $D(\tau') > 0$
- Part of the total energy density is due to dissipative production

$$\bar{\epsilon} = \bar{\epsilon}_{\mathsf{nd}} + \bar{\epsilon}_{\mathsf{d}}$$

- Assume for dissipatively produced part

$$\dot{\bar{\epsilon}}_{\mathsf{d}} + 3\frac{\dot{a}}{a}(1 + \hat{w}_{\mathsf{d}})\bar{\epsilon}_{\mathsf{d}} = aD$$

Modification of Friedmann's equations by backreaction 2

Leads to another variant of Friedmann's equation

$$H(\tau)^2 = \frac{8\pi G_\mathsf{N}}{3}\left[\bar{\epsilon}_\mathsf{nd}(\tau) + \int_{\tau_\mathsf{I}}^{\tau} d\tau' \left[\left(\frac{a(\tau')}{a(\tau)}\right)^{3+3\hat{w}_\mathsf{d}} - \left(\frac{a(\tau')}{a(\tau)}\right)^4\right] a(\tau')D(\tau')\right]$$

- If the dissipative backreaction D produces pure radiation, $\hat{w}_\mathsf{d} = 1/3$, it does not show up in effective Friedmann equation at all!
- For $\hat{w}_\mathsf{d} < 1/3$ there is a new component with positive contribution on the right hand side of the effective Friedmann equation.
- To understand expansion, parametrize for late times

$$D(\tau) = H(\tau)\left(\frac{a(\tau)}{a(\tau_0)}\right)^{-\kappa}\tilde{D}$$

  with constants $\tilde{D}$ and κ.
- Hubble parameter as function of $(a_0/a) = 1 + z$

$$H(a) = H_0\sqrt{\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_D\left(\frac{a_0}{a}\right)^\kappa}$$

- For $\kappa \approx 0$ the role of Ω_Λ and Ω_D would be similar.