Dissipative properties of the cosmological fluid

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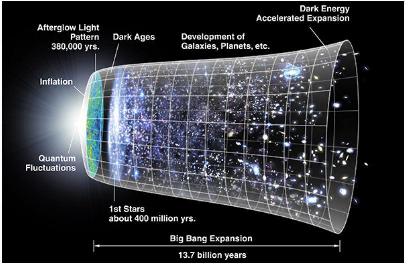
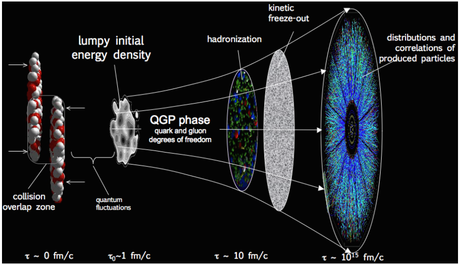
Content

- Why are dissipative properties of dark matter interesting ?
- How to understand large scale structure formation ?
- Could dissipation affect the cosmological expansion ?
- How do dissipation and thermalization work in general relativity ?

Why are dissipative properties of dark matter interesting ?

Big bang – little bang analogy





Cosmology

- cosmol. scale: MPc$= 3.1 \times 10^{22}$ m
- Gravity + QED + Dark sector
- one big event

Heavy ion collisions

- nuclear scale: fm$= 10^{-15}$ m
- QCD
- very many events

- initial conditions not directly accessible
- all information must be reconstructed from final state
- **dynamical description as a (viscous) fluid**

The dark matter fluid

- Heavy ion collisions

$$\mathscr{L}_{\text{QCD}} \quad \to \quad \text{fluid properties}$$

- Late time cosmology

$$\text{fluid properties} \quad \to \quad \mathscr{L}_{\text{dark matter}}$$

- Until direct detection of dark matter, it can only be observed via

$$T^{\mu\nu}_{\text{dark matter}}$$

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

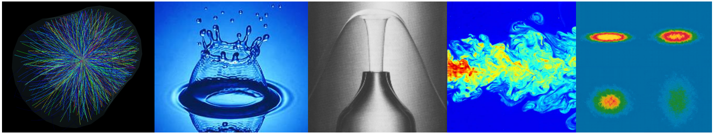
$$T^{\mu\nu} = (\epsilon + p + \pi_{\text{bulk}})u^\mu u^\nu + (p + \pi_{\text{bulk}})g^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition w. r. t. fluid velocity u^μ
- pressure $p = p(\epsilon, n)$
- constitutive relations for viscous terms in derivative expansion
  - bulk viscous pressure $\pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu + \ldots$
  - shear stress $\pi^{\mu\nu} = -\eta \left[\Delta^{\mu\alpha}\nabla_\alpha u^\nu + \Delta^{\nu\alpha}\nabla_\alpha u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\right] + \ldots$
  - diffusion current $\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon+p}\right]^2 \Delta^{\alpha\beta}\partial_\beta \left(\frac{\mu}{T}\right) + \ldots$

Fluid dynamic equations from covariant **conservation laws**

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \nabla_\mu N^\mu = 0.$$

Fluid dynamics



- Long distances, long times or strong enough interactions
- Needs **macroscopic** fluid properties
  - equation of state $p(\epsilon, n)$
  - shear viscosity $\eta(\epsilon, n)$
  - bulk viscosity $\zeta(\epsilon, n)$
  - heat conductivity $\kappa(\epsilon, n)$
  - relaxation times, ...
- For QCD no full *ab initio* calculation of transport properties possible yet but in principle fixed by **microscopic** properties encoded in $\mathscr{L}_{\text{QCD}}$
- Ongoing experimental and theoretical effort to understand this in detail

Ideal fluid versus collision-less gas

- Many codes used in cosmology describe dark matter as **ideal, cold and pressure-less fluid**

$$T^{\mu\nu} = \epsilon \, u^\mu u^\nu$$

- Equation of state $p = 0$
- No shear stress and bulk viscous pressure $\pi^{\mu\nu} = \pi_{\mathsf{bulk}} = 0$
- Dark matter is also modeled as **collision-less gas** of massive particles, interacting via gravity only
- Two pictures are in general **not consistent**

Dissipative properties

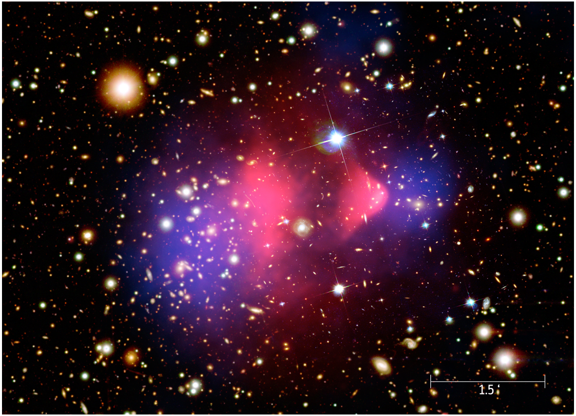
Viscosities

- Diffusive transport of momentum [Maxwell (1860)]
- Depend strongly on interaction properties
- Example: non-relativistic gas of particles with mass m, mean peculiar velocity $\bar{v}$, elastic $2 \to 2$ cross-section σ_{el}

$$\eta = \frac{m\,\bar{v}}{3\,\sigma_{\text{el}}} \qquad\qquad \zeta = 0$$

- Interesting additional **information about dark matter**

Self-interaction of dark matter

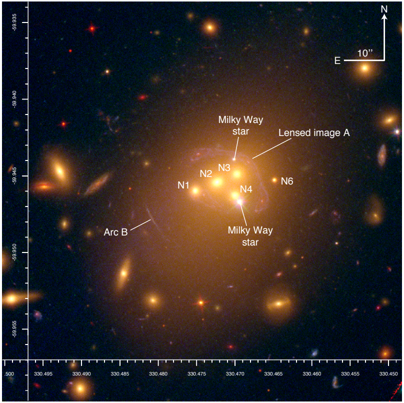


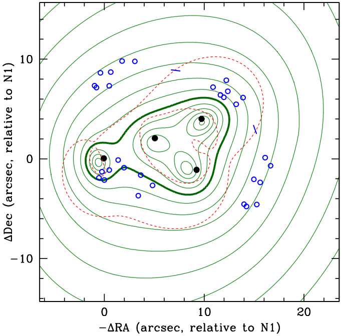
Gravitational lensing and x-ray image of "bullet cluster" 1E0657-56

- so far: dark matter is non-interacting → can collide without stopping

$$\frac{\sigma_{\mathrm{el}}}{m} \lesssim 1.2\,\frac{\mathrm{cm}^2}{\mathrm{g}}$$

Is dark matter self-interacting?





Galaxy cluster Abell 3827

[Massey *et al.*, MNRAS 449, 3393 (2015)]

- Offset between stars and dark matter falling into cluster
- Is this a first indication for a dark matter self-interaction?

$$\frac{\sigma_{\text{el}}}{m} \approx 3\,\frac{\text{cm}^2}{\text{g}} \approx 0.5\,\frac{\text{b}}{\text{GeV}} \qquad \text{(under debate)}$$

[Kahlhoefer, Schmidt-Hoberg, Kummer & Sarkar, MNRAS 452, 1 (2015)]

Precision cosmology can measure shear stress

- Scalar excitations in gravity

$$ds^2 = a^2 \left[-(1+2\psi)d\eta^2 + (1-2\phi)dx_i dx_i\right]$$

  with two Newtonian potentials ψ and ϕ.
- Einsteins equations imply

$$\left(\partial_i\partial_j - \tfrac{1}{3}\delta_{ij}\partial_k^2\right)(\phi - \psi) = 8\pi G_{\mathsf{N}} a^2 \; \pi_{ij}\big|_{\text{scalar}}$$

  with scalar part of shear stress

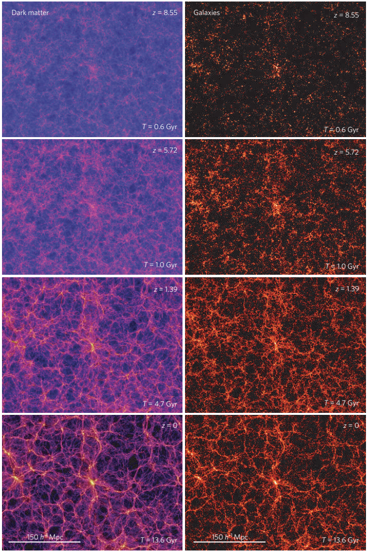
$$\pi_{ij}\big|_{\text{scalar}} = \left(\partial_i\partial_j - \tfrac{1}{3}\delta_{ij}\partial_k^2\right)\tilde{\pi}$$

- Detailed data at small redshift e.g. from Euclid satellite (esa, 2020) [Amendola *et al.* (2012)]
  - ψ can be measured via acceleration of matter
  - $\psi + \phi$ can be meaured by weak lensing and Sachs-Wolfe effect
  - fluid velocity can be accessed by redshift space distortions
- New quantitative precise insights into fluid properties of dark matter

Cosmological structure formation

- How do **viscosities influence structure formation**?
- Does viscous fluid dynamics help to **understand large scale structure (semi) analytically**?

Dark matter   Visible galaxies



[Springel, Frenk & White, Nature 440, 1137 (2006)]

How is structure formation modified?

Linear dynamics

- energy conservation ($\theta = \vec{\nabla} \cdot \vec{v}$)

$$\dot{\delta\epsilon} + 3\tfrac{\dot{a}}{a}\delta\epsilon + \bar{\epsilon}\,\theta = 0$$

- Navier-Stokes equation

$$\bar{\epsilon}\left[\dot{\theta} + \tfrac{\dot{a}}{a}\theta - k^2\psi\right] + \tfrac{1}{a}\left(\zeta + \tfrac{4}{3}\eta\right)k^2\theta = 0$$

- Poisson equation

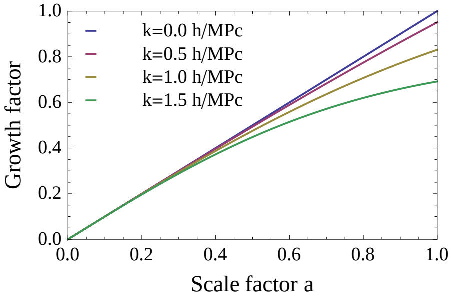
$$-k^2\psi = 4\pi G_{\mathsf{N}} a^2 \delta\epsilon$$

Scalar perturbations ($\delta = \frac{\delta\epsilon}{\bar{\epsilon}}$)

$$\ddot{\delta} + \left[\frac{\dot{a}}{a} + \frac{\zeta + \frac{4}{3}\eta}{a\bar{\epsilon}}k^2\right]\dot{\delta} - 4\pi G_{\mathsf{N}}\bar{\epsilon}\,\delta = 0$$

Viscosites slow down gravitational collapse but do not wash out structure

Structure formation with viscosities



[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP 1511, 049 (2015)]

- k-dependent growth factor for scalar modes
- Could be tested by observation of large scale structure
- Depends on $\zeta + \frac{4}{3}\eta$ as function of time (or density)

How to understand large scale structure formation ?

Formation of large scale structure

- Formation of large scale structure is interesting
  - tests physics of dark matter
  - tests physics of dark energy
  - gets tested by missions like Euclid, ...
- Cosmological perturbation theory breaks down when density contrast

$$\delta(\mathbf{k}) = \frac{\delta\rho(\mathbf{k})}{\bar{\rho}} \gg 1$$

  grows large at late times and for small scales.
- Numerical simulations (N-body) are expensive and time-consuming
- One would like to have better analytical understanding

Renormalization group apprach

[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP 1511, 049 (2015)]

[Floerchinger, Garny, Tetradis & Wiedemann, in preparation]

- Start from ideal fluid approximation
- Large scale structure formation can be formulated as classical field theory with stochastic initial conditions
- Leads to classical statistical field theory
- Initial state fluctuations can be treated by functional renormalization group, similar to thermal or quantum fluctuations in other contexts [Matarrese & Pietroni (2007)]
- Modify theory by cutting off the initial spectrum in the IR

$$P_k^0(\mathbf{q}) = P^0(\mathbf{q})\,\Theta(|\mathbf{q}| - k)$$

- Use flow equation for 1PI effective action [Wetterich (1993)]

$$\partial_k \Gamma_k[\phi, \chi] = \frac{1}{2}\,\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\phi, \chi] - i\left(P_k^0 - P^0\right)\right)^{-1} \partial_k P_k^0\right\}$$

Renormalization of effective viscosity and pressure

- Effective theory at scale k has additional terms in equations of motion
- Order them by derivative expansion.
- Lowest order: ideal fluid
- Next-to-lowest order: effective sound velocity parameter

$$\gamma_s = \frac{c_s^2}{\mathcal{H}^2} = \frac{dp/d\rho}{\mathcal{H}^2}.$$

and effective viscosity parameter

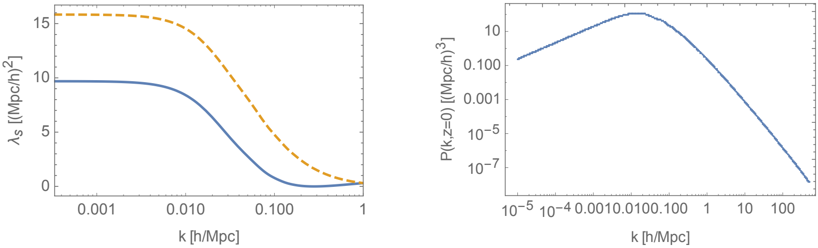
$$\gamma_\nu = \frac{4\eta/3 + \zeta}{(\rho + p)\mathcal{H}a}.$$

- Both depend on cosmological time or scale factor a

$$\gamma_s = \lambda_s\, a^\kappa, \qquad \gamma_\nu = \lambda_\nu\, a^\kappa$$

with exponent $\kappa \approx 2$.

RG flow of effective sound velocity parameter

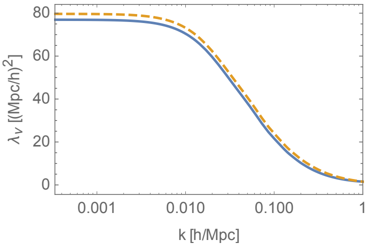


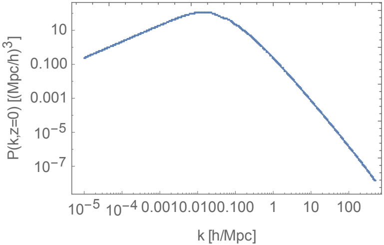
- left: RG flow of effective sound velocity today $\lambda_s = \frac{c_s^2}{H_0^2} = \frac{dp/d\rho}{H_0^2}$
  - dashed line: one-loop approximation

$$\partial_k \lambda_s = -\frac{4\pi}{3}\frac{31}{70}P^0(k)$$

  - solid line: functional RG
- right: linear density power spectrum

RG flow of effective viscosity parameter



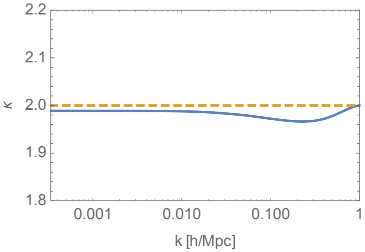


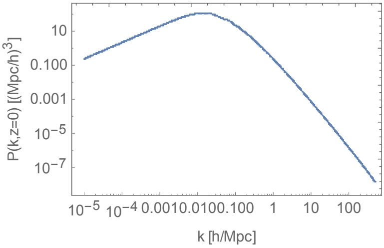
- left: RG flow of effective viscosity today $\lambda_\nu = \frac{4\eta/3+\zeta}{(\rho+p)_0 H_0}$
  - dashed line: one-loop approximation

$$\partial_k \lambda_\nu = -\frac{4\pi}{3}\frac{78}{35}P^0(k)$$

  - solid line: functional RG
- right: linear density power spectrum

RG flow of exponent κ



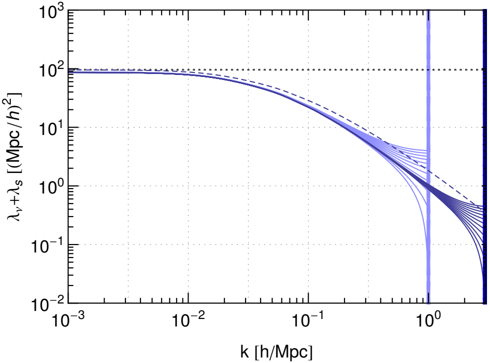


- left: RG flow of exponent κ
  - dashed line: one-loop approximation

$$\partial_k \kappa = \frac{4\pi}{3} P^0(k) \frac{78(\kappa - 2)}{35\tilde{\lambda}_\nu}$$

  - solid line: functional RG
- right: linear density power spectrum

Fixed point behavior



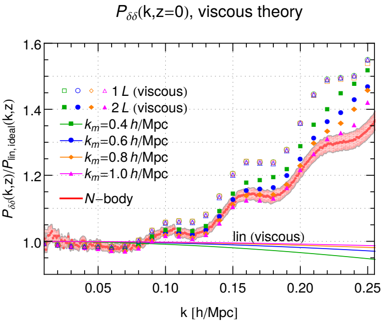
- growing mode is sensitive to $\lambda_s + \lambda_\nu$
- functional RG has IR fixed points

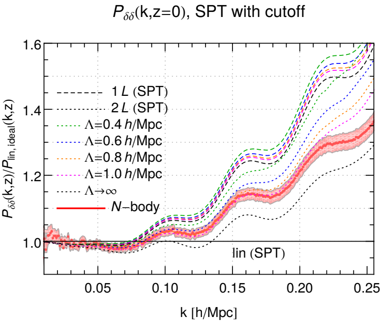
Functional RG + perturbation theory

[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP 1511, 049 (2015)]

[Floerchinger, Garny, Tetradis & Wiedemann, in preparation]

- RG evolution to determine effective viscosity and sound velocity at intermediate scale k_m
- Perturbation theory for power spectrum for scales $0 < |\mathbf{q}| < k_m$
- Theory with effective parameters





Could dissipation affect the cosmological expansion ?

Bulk viscosity

- Bulk viscous pressure is negative for expanding universe

$$\pi_{\text{bulk}} = -\zeta \, \nabla_\mu u^\mu = -\zeta \, 3H < 0$$

- Negative effective pressure

$$p_{\text{eff}} = p + \pi_{\text{bulk}} < 0$$

would act similar to dark energy in Friedmann's equations
[Murphy (1973), Padmanabhan & Chitre (1987), Fabris, Goncalves & de Sa Ribeiro (2006), Li & Barrow (2009), Velten & Schwarz (2011), Gagnon & Lesgourgues (2011), ...]

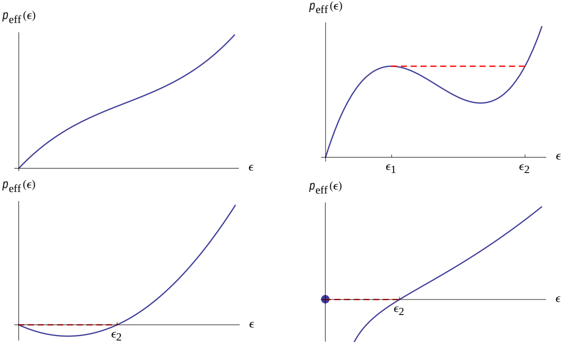
- **Is negative effective pressure physical?**
- In context of heavy ion physics: instability for $p_{\text{eff}} < 0$ ("cavitation")
[Torrieri & Mishustin (2008), Rajagopal & Tripuraneni (2010), Buchel, Camanho & Edelstein (2014), Habich & Romatschke (2015), Denicol, Gale & Jeon (2015)]
- What precisely happens at the instability?

Is negative effective pressure physical?

- Kinetic theory

$$p_{\text{eff}}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{3E_{\vec{p}}} f(x, \vec{p}) \geq 0$$

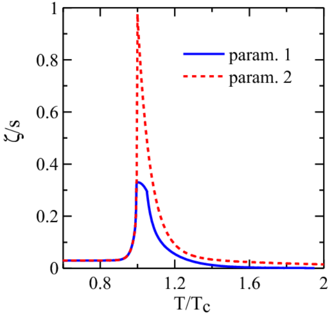
- Stability argument



If there is a vacuum with $\epsilon = p_{\text{eff}} = 0$, phases with $p_{\text{eff}} < 0$ cannot be mechanically stable. (But could be metastable.)

Bulk viscosity in heavy ion physics

- In heavy ion physics people start now to consider bulk viscosity.
- Becomes relevant close to chiral crossover



[Denicol, Gale & Jeon (2015)]

- Is there a first-order phase transition triggered by the expansion?
- What is the relation to chemical and kinetic freeze-out?
- More detailed understanding needed, both for heavy ion physics and cosmology

Backreaction: General idea

- for $0+1$ dimensional, non-linear dynamics

$$\dot{\varphi} = f(\varphi) = f_0 + f_1\,\varphi + \tfrac{1}{2} f_2\,\varphi^2 + \dots$$

- one has for expectation values $\bar{\varphi} = \langle \varphi \rangle$

$$\dot{\bar{\varphi}} = f_0 + f_1\,\bar{\varphi} + \tfrac{1}{2} f_2\,\bar{\varphi}^2 + \tfrac{1}{2} f_2\,\langle (\varphi - \bar{\varphi})^2 \rangle + \dots$$

- evolution equation for expectation value $\bar{\varphi}$ depends on two-point correlation function or spectrum $P_2 = \langle (\varphi - \bar{\varphi})^2 \rangle$
- evolution equation for spectrum depends on bispectrum and so on
- more complicated for higher dimensional theories
- more complicated for gauge theories such as gravity

Backreaction in gravity

- Einstein's equations are non-linear.
- Important question [G. F. R. Ellis (1984)]: If Einstein's field equations describe small scales, including inhomogeneities, do they also hold on large scales?
- Is there a sizable backreaction from inhomogeneities to the cosmological expansion?
- Difficult question, has been studied by many people [Ellis & Stoeger (1987); Mukhanov, Abramo & Brandenberger (1997); Unruh (1998); Buchert (2000); Geshnzjani & Brandenberger (2002); Schwarz (2002); Wetterich (2003); Räsänen (2004); Kolb, Matarrese & Riotto (2006); Brown, Behrend, Malik (2009); Gasperini, Marozzi & Veneziano (2009); Clarkson & Umeh (2011); Green & Wald (2011); ...]
- Recent reviews: [Buchert & Räsänen, Ann. Rev. Nucl. Part. Sci. 62, 57 (2012); Green & Wald, Class. Quant. Grav. 31, 234003 (2014)]
- No general consensus but most people believe now that **gravitational backreaction is rather small**.
- In the following we look at a new **backreaction on the matter side** of Einstein's equations.

Fluid equation for energy density

First order viscous fluid dynamics

$$u^\mu \partial_\mu \epsilon + (\epsilon + p)\nabla_\mu u^\mu - \zeta\Theta^2 - 2\eta\sigma^{\mu\nu}\sigma_{\mu\nu} = 0$$

For $\vec{v}^2 \ll c^2$ and Newtonian potentials $\Phi, \Psi \ll 1$

$$\dot{\epsilon} + \vec{v}\cdot\vec{\nabla}\epsilon + (\epsilon + p)\left(3\frac{\dot{a}}{a} + \vec{\nabla}\cdot\vec{v}\right)$$
$$= \frac{\zeta}{a}\left[3\frac{\dot{a}}{a} + \vec{\nabla}\cdot\vec{v}\right]^2 + \frac{\eta}{a}\left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}(\vec{\nabla}\cdot\vec{v})^2\right]$$

Fluid dynamic backreaction in Cosmology

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

Expectation value of energy density $\bar{\epsilon} = \langle \epsilon \rangle$

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p} - 3\bar{\zeta}H\right) = D$$

with dissipative backreaction term

$$D = \frac{1}{a^2}\langle \eta \left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}\partial_i v_i \partial_j v_j\right]\rangle + \frac{1}{a^2}\langle \zeta [\vec{\nabla} \cdot \vec{v}]^2 \rangle + \frac{1}{a}\langle \vec{v} \cdot \vec{\nabla}\,(p - 6\zeta H)\rangle$$

- D vanishes for unperturbed homogeneous and isotropic universe
- D has contribution from shear & bulk viscous dissipation and thermodynamic work done by contraction against pressure gradients
- dissipative terms in D are positive semi-definite
- for spatially constant viscosities and scalar perturbations only

$$D = \frac{\bar{\zeta} + \frac{4}{3}\bar{\eta}}{a^2} \int d^3q\; P_{\theta\theta}(q)$$

Dissipation of perturbations

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

- Dissipative backreaction does not need negative effective pressure

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p}_{\text{eff}}\right) = D$$

- D is an integral over perturbations, could become large at late times.
- Can it potentially accelerate the universe?
- Need additional equation for scale parameter a
- Use trace of Einstein's equations $R = 8\pi G_{\text{N}} T^{\mu}{}_{\mu}$

$$\frac{1}{a}\dot{H} + 2H^2 = \frac{4\pi G_{\text{N}}}{3}\left(\bar{\epsilon} - 3\bar{p}_{\text{eff}}\right)$$

  does not depend on unknown quantities like $\langle(\epsilon + p_{\text{eff}})u^{\mu}u^{\nu}\rangle$
- To close the equations one needs equation of state $\bar{p}_{\text{eff}} = \bar{p}_{\text{eff}}(\bar{\epsilon})$ and dissipation parameter D

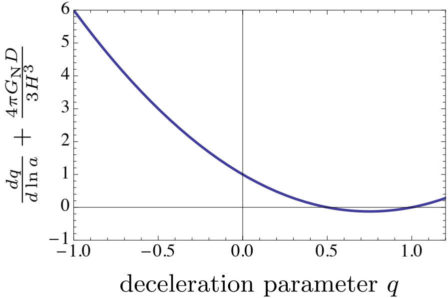
Deceleration parameter

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

- assume now vanishing effective pressure $\bar{p}_{\text{eff}} = 0$
- obtain for deceleration parameter $q = -1 - \frac{\dot{H}}{aH^2}$

$$-\frac{dq}{d\ln a} + 2(q-1)\left(q - \frac{1}{2}\right) = \frac{4\pi G_{\text{N}} D}{3H^3}$$

- for $D = 0$ attractive fixed point at $q_* = \frac{1}{2}$ (deceleration)
- for $D > 0$ fixed point shifted towards $q_* < 0$ (acceleration)



Estimating viscous backreaction D

- For $\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx 4$ one could explain the current accelerated expansion ($q \approx -0.6$) by dissipative backreaction.
- Is this possible?
- In principle one can determine D for given equation of state and viscous properties from dynamics of structure formation.
- So far only rough estimates. If shear viscosity dominates:

$$D = \frac{1}{a^2}\langle \eta \left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \tfrac{2}{3}\partial_i v_i \partial_j v_j\right]\rangle \approx c_D \bar{\eta} H^2$$

  with $c_D = \mathcal{O}(1)$. Corresponds to $\Delta v \approx 100\,\mathsf{km/s}$ for $\Delta x \approx 1\,\mathsf{MPc}$

- Leads to

$$\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx \frac{c_D \bar{\eta} H}{2\rho_c}$$

  with $\rho_c = \frac{3H^2}{8\pi G_{\mathsf{N}}}$

Viscosities

- Relativistic particles / radiation contribute to shear viscosity

$$\eta = c_\eta \, \epsilon_R \, \tau_R$$

  - prefactor $c_\eta = \mathcal{O}(1)$
  - energy density of radiation ϵ_R
  - mean free time τ_R

- Bulk viscosity vanishes in situations with conformal symmetry but can be large when conformal symmetry is broken.
- For massive scalar particles with $\lambda\varphi^4$ interaction [Jeon & Yaffe (1996)]

$$\zeta \sim \frac{m^6}{\lambda^4 T^3} e^{2m/T}, \qquad \eta \sim \frac{m^{5/2} T^{1/2}}{\lambda^2} \qquad \text{for} \qquad \frac{T}{m} \ll 1$$

Estimating viscous backreaction D

Consider shear viscosity from radiation

$$\eta = c_\eta \, \epsilon_R \, \tau_R$$

Backreaction term

$$\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx \frac{c_D c_\eta}{2} \frac{\epsilon_R}{\rho_c} \tau_R H$$

- fluid approximation needs $\tau_R H < 1$
- for sizeable effect one would need $\epsilon_R / \rho_c = \mathcal{O}(1)$
- unlikely that D becomes large enough in this scenario

Needed refinements:

- full dynamics of perturbations
- second order fluid dynamics
- complete model(s)

How do dissipation and thermalization work in general relativity ?

Gravity and thermalization

Consider ensemble of massive particles interacting via gravity only. Start with some velocity distribution. Is there **equilibration/thermalization**...

- ... in Newtonian gravity?
- ... in classical General relativity?
- ... in quantized gravity?

Analogy to other gauge theories suggests that **quantum properties are important for thermalization**

Dissipation by gravity

- Gravitational waves in viscous fluid have life time [Hawking (1966)]

$$\tau_G = \frac{1}{16\pi G_\mathsf{N} \eta}$$

- Diffusive momentum transport by graviton radiation induces viscosity

$$\eta \approx \epsilon_G \, \tau_G$$

  with energy density of gravitational field ϵ_G
- Can be solved for η and τ_G [Weinberg (1972)]

$$\eta = \sqrt{\frac{\epsilon_G}{16\pi G_\mathsf{N}}}, \qquad \tau_G = \sqrt{\frac{1}{16\pi G_\mathsf{N} \epsilon_G}}$$

- Can this really be independent of dark matter mass and density?
- Thermalization time $\sim m_\mathsf{P}/T^2$ is very large
- What determines dissipation on shorter time scales, when classical fields dominate?

Conclusions

Dissipative properties of the cosmological fluid...

- ... are quite interesting
- ... could provide more detailed understanding of dark matter
- ... can be tested by precision cosmology
- ... can help to better understand large scale structure
- ... could even affect the cosmological expansion

Backup slides

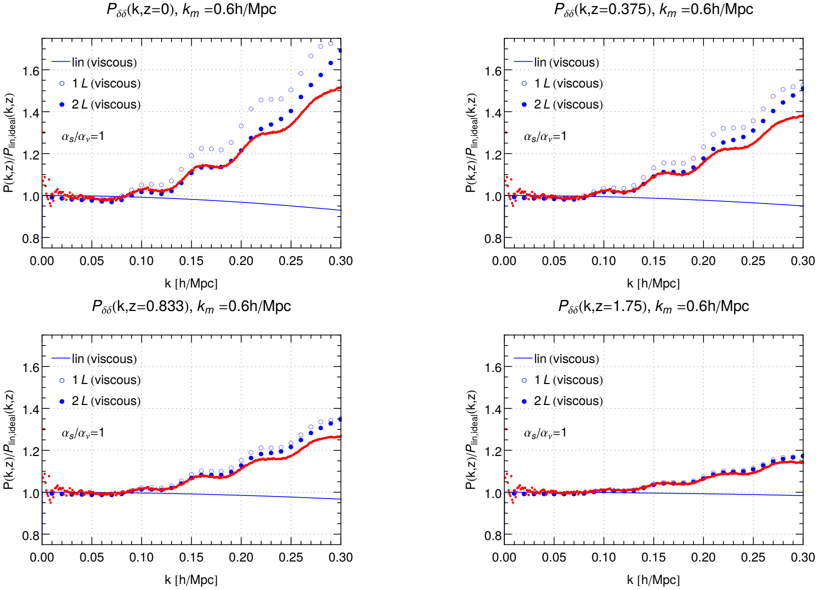
"Fundamental" and "effective" viscosity

Two types of viscosities for cosmological fluid

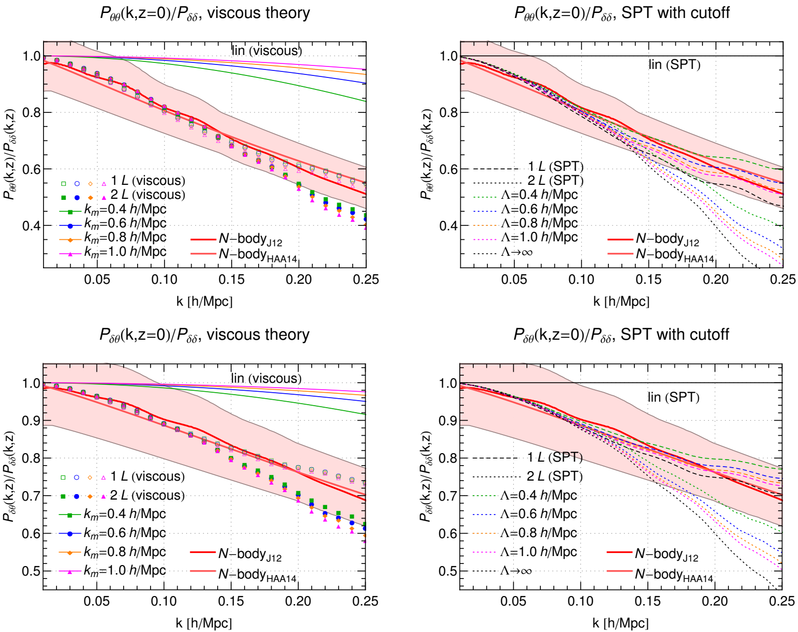
1. Momentum transport by particles or radiation
   - governed by interactions
   - from linear response theory [Green (1954), Kubo (1957)]
   - close to equilibrium

2. Momentum transport in the inhomogeneous, coarse-grained fluid
   - governed by non-linear fluid mode couplings
   - determined perturbatively [Blas, Floerchinger, Garny, Tetradis & Wiedemann]
   - non-equilibrium
   - heavy ions: anomalous plasma viscosity [Asakawa, Bass & Müller (2006)]
     eddy viscosity [Romatschke (2008)]

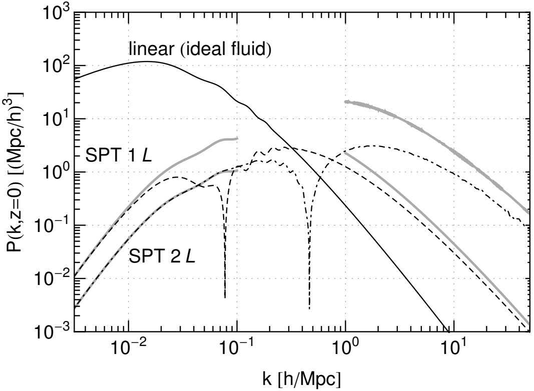
Power spectrum at different redshifts



Velocity spectra



Power spectrum, standard perturbation theory



[D. Blas, M. Garny and T. Konstandin, JCAP 1309 (2013) 024]

Could viscous backreaction lead to ΛCDM*-type expansion?*

[Floerchinger, Tetradis & Wiedemann, 1506.00407]

- Backreaction term $D(z)$ will be *some* function of redshift.
- For given dissipative properties $D(z)$ can be determined, but calculation is involved.
- One may ask simpler question: For what form of $D(z)$ would the expansion be as in the ΛCDM model?
- The *ad hoc* ansatz $D(z) = \text{const} \cdot H(z)$ leads to modified Friedmann equations

$$\bar{\epsilon} - \frac{D}{4H} = \frac{3}{8\pi G_\text{N}} H^2, \qquad \bar{p}_\text{eff} - \frac{D}{12H} = -\frac{1}{8\pi G_\text{N}} \left(2 \frac{1}{a} \dot{H} + 3H^2 \right)$$

- In terms of $\hat{\epsilon} = \bar{\epsilon} - \frac{D}{3H}$ one can write

$$\frac{1}{a} \dot{\hat{\epsilon}} + 3H(\hat{\epsilon} + \bar{p}_\text{eff}) = 0, \qquad R + \frac{8\pi G_\text{N} D}{3H} = -8\pi G_\text{N} (\hat{\epsilon} - 3\bar{p}_\text{eff})$$

- For $\bar{p}_\text{eff} = 0$ these are standard equations for ΛCDM model with

$$\Lambda = \frac{2\pi G_\text{N} D}{3H}$$

Modification of Friedmann's equations by backreaction 1

- For universe with fluid velocity inhomogeneities one cannot easily take direct average of Einstein's equations.
- However, fluid equation for energy density and trace of Einstein's equations can be used.
- By integration one finds modified Friedmann equation

$$H(\tau)^2 = \frac{8\pi G_{\mathsf{N}}}{3} \left[\bar{\epsilon}(\tau) - \int_{\tau_{\mathsf{I}}}^{\tau} d\tau' \left(\frac{a(\tau')}{a(\tau)} \right)^4 a(\tau') D(\tau') \right]$$

- Additive deviation from Friedmann's law for $D(\tau') > 0$
- Part of the total energy density is due to dissipative production

$$\bar{\epsilon} = \bar{\epsilon}_{\mathsf{nd}} + \bar{\epsilon}_{\mathsf{d}}$$

- Assume for dissipatively produced part

$$\dot{\bar{\epsilon}}_{\mathsf{d}} + 3\frac{\dot{a}}{a}(1 + \hat{w}_{\mathsf{d}})\bar{\epsilon}_{\mathsf{d}} = aD$$

Modification of Friedmann's equations by backreaction 2

Leads to another variant of Friedmann's equation

$$H(\tau)^2 = \frac{8\pi G_{\mathsf{N}}}{3}\left[\bar{\epsilon}_{\mathsf{nd}}(\tau) + \int_{\tau_{\mathsf{I}}}^{\tau} d\tau' \left[\left(\frac{a(\tau')}{a(\tau)}\right)^{3+3\hat{w}_{\mathsf{d}}} - \left(\frac{a(\tau')}{a(\tau)}\right)^{4}\right] a(\tau')D(\tau')\right]$$

- If the dissipative backreaction D produces pure radiation, $\hat{w}_{\mathsf{d}} = 1/3$, it does not show up in effective Friedmann equation at all!
- For $\hat{w}_{\mathsf{d}} < 1/3$ there is a new component with positive contribution on the right hand side of the effective Friedmann equation.
- To understand expansion, parametrize for late times

$$D(\tau) = H(\tau)\left(\frac{a(\tau)}{a(\tau_0)}\right)^{-\kappa}\tilde{D}$$

  with constants $\tilde{D}$ and κ.
- Hubble parameter as function of $(a_0/a) = 1 + z$

$$H(a) = H_0\sqrt{\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_D\left(\frac{a_0}{a}\right)^\kappa}$$

- For $\kappa \approx 0$ the role of Ω_Λ and Ω_D would be similar.

Inhomogeneities in heavy ion collisions

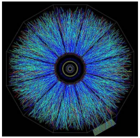
Inhomogeneities are main source of information in cosmology.

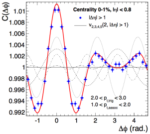
Similarly, in heavy ion collisions:

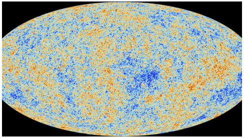
- **Initial fluid perturbations**: Event-by-event fluctuations around averaged fluid fields at time τ_0 and their evolution:
  - energy density ϵ
  - fluid velocity u^μ
  - shear stress $\pi^{\mu\nu}$
  - more general also: baryon number density n, electric charge density, electromagnetic fields, ...
- governed by universal evolution equations
- determine particle distributions after freeze-out, e.g. $v_n(p_T)$
- usefull to constrain **thermodynamic and transport properties of QCD**
- contain interesting information from early times

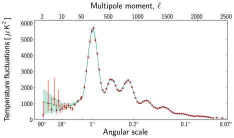
Fluid dynamic perturbation theory for heavy ions

[Floerchinger & Wiedemann, PLB 728, 407 (2014); JHEP 08 (2014) 005]









- goal: understand dynamics of heavy ion collisions better, constrain fluid properties of QCD from experimental results
- before: fully numerical fluid simulations e.g. [Heinz & Snellings (2013)]
- new: solve fluid equations for smooth and symmetric background and order-by-order in perturbations
- similar to cosmological perturbation theory
- good convergence properties [Floerchinger *et al.*, PLB 735, 305 (2014), Brouzakis *et al.* PRD 91, 065007 (2015)]
- some insights from heavy ion physics might be useful for cosmology

Fluid dynamics in cosmology and heavy ion collisions

Cosmology

- Large part of cosmological evolution is governed by equilibrium thermodynamics or ideal fluid dynamics.
- Free-streaming of photons and neutrinos at late times.
- Matter in equilibrium at early times, drops out of equilibrium later.
- Gravitational interaction is long range and treated explicitely.

Heavy ion collisions

- Expansion governed by viscous fluid dynamics.
- Free streaming of hadrons at late times.
- Strong interactions are confined at low T and screened at high T, treated implicitly.

First steps towards fluid dynamic perturbation or response theory

- Linear perturbations around Bjorken flow [Floerchinger & Wiedemann (2011)]
- Linear perturbations around Gubser solution for conformal fluids
  [Gubser & Yarom (2010), Staig & Shuryak (2011), Springer & Stephanov (2013)]
- More detailed investigation of linear perturbations and first steps towards non-linear perturbations around Gubser solution
  [Hatta, Noronha, Torrieri, Xiao (2014)]
- Linear perturbations around general azimuthally symmetric initial state, realistic equation of state
  [Floerchinger & Wiedemann (2013)]
- Characterization of initial conditions by Bessel-Fourier expansion
  [Coleman-Smith, Petersen & Wolpert (2012), Floerchinger & Wiedemann (2013)]
- Comparison to full numerical solution shows good convergence properties of perturbative expansion
  [Floerchinger, Wiedemann, Beraudo, Del Zanna, Inghirami, Rolando (2013)]
- Related response formalism for expansion in eccentricities
  [Teaney & Yan (2012), Yan & Ollitrault (2015]