Fluid dynamic perturbations in heavy ion collisions

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mainly based on

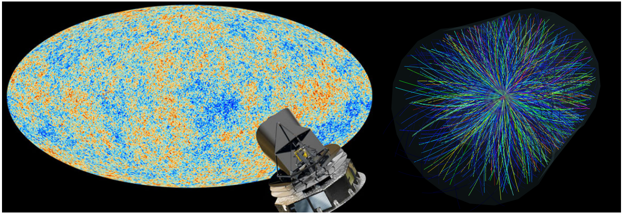
- Mode-by-mode fluid dynamics for relativistic heavy ion collisions
  [Phys. Lett. B, 728, 407 (2014), with U. A. Wiedemann]
- Statistics of initial density perturbations in heavy ion collisions and their fluid dynamic response [JHEP 1408 (2014) 005, with U. A. Wiedemann]
- Fluctuations of baryonic number around Bjorken background
  [work in progress, with M. Martinez]

Introduction

What perturbations are interesting and why?

- **Initial fluid perturbations**: Event-by-event fluctuations around a background or average of fluid fields at time τ_0:
  - energy density ϵ
  - fluid velocity u^μ
  - shear stress $\pi^{\mu\nu}$
  - more general also: baryon number density n, electric charge density, electromagnetic fields, ...
- governed by universal evolution equations
- can be used to constrain thermodynamic and transport properties
- contain interesting information from early times

Similarities to cosmological fluctuation analysis



- fluctuation spectrum contains info from early times
- many numbers can be measured and compared to theory
- can lead to detailed understanding of evolution
- to learn something about the evolution one needs to know some *universal* properties of initial state, for example $P(k) \sim k^{n_s - 1}$

A program to understand fluid perturbations

1. Characterize initial perturbations
2. Propagated them through fluid dynamic regime
3. Determine influence on particle spectra and harmonic flow coefficients
4. Take also perturbations from non-hydro sources (jets) into account
   [see work with K. Zapp, EPJC 74 (2014) 12, 3189]

Characterization of initial conditions

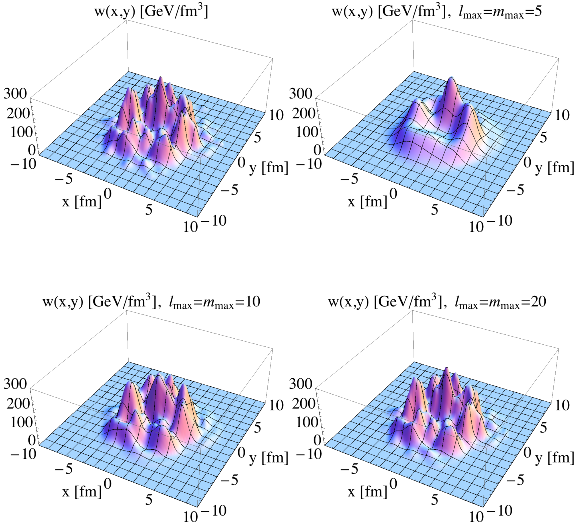
Transverse enthalpy density

Based on Bessel-Fourier expansion and background density
[Floerchinger & Wiedemann 2013, see also Coleman-Smith, Petersen & Wolpert 2012, Floerchinger & Wiedemann 2014]

$$w(r,\phi) = w_{\mathsf{BG}}(r) + w_{\mathsf{BG}}(r) \sum_{m,l} w_l^{(m)} \, e^{im\phi} \, J_m\left(z_l^{(m)} \rho(r)\right)$$

- azimuthal wavenumber m, radial wavenumber l
- $w_l^{(m)}$ dimensionless
- higher m and l correspond to finer spatial resolution
- coefficients $w_l^{(m)}$ can be related to eccentricienies
- works similar for vectors (velocity) and tensors (shear stress)

Transverse density from Glauber model



Event ensembles

- Initial conditions at beginning of fluid dynamic regime are governed by event-by-event probability distribution

$$p_{\tau_0}[w, u^\mu, \pi^{\mu\nu}, \ldots]$$

- Moments / correlation functions

$$\left\langle w_{l_1}^{(m_1)} w_{l_2}^{(m_2)} \ldots w_{l_n}^{(m_n)} \right\rangle$$

  - contain information from initial state physics / early dynamics
  - universal (model independent) properties would be nice to have
  - same information in cumulants (connected correlation functions)

Statistics of initial density perturbations

Independent point-sources model (IPSM)

$$w(\vec{x}) = \left[\frac{1}{\tau_0}\frac{dW_{\mathsf{BG}}}{d\eta}\right]\frac{1}{N}\sum_{j=1}^{N}\delta^{(2)}(\vec{x}-\vec{x}_j)$$

- random positions $\vec{x}_j$, independent and identically distributed
- probability distribution $p(\vec{x}_j)$ reflects collision geometry
- possible to determine correlation functions analytically for *central* and *non-central* collisions [Floerchinger & Wiedemann (2014)]
- Long-wavelength modes (small m and l) that don't resolve differences between point-like and extended sources have *universal statistics*.

Solution of IPSM and scaling with number of sources

- for IPSM one can exactly determine the correlation functions

$$\langle w_{l_1}^{(m_1)} \cdots w_{l_n}^{(m_n)} \rangle$$

[Floerchinger & Wiedemann, JHEP 1408 (2014) 005]

- connected correlation functions (cumulants) scale with N like
[see also Ollitrault & Yan (2014), Bzdak & Skokov (2014)]

$$\langle w_{l_1}^{(m_1)} \cdots w_{l_n}^{(m_n)} \rangle_c \sim \frac{1}{N^{n-1}}$$

(implies that distribution is non-Gaussian)

- scaling broken for non-central collisions
- impact parameter dependence of terms that break scaling is known

Fluid dynamic response

Response to density perturbations

For a single event

$$V_m^* = v_m e^{-i\, m\, \psi_m}$$
$$= \sum_l S_{(m)l}\, w_l^{(m)} + \sum_{\substack{m_1,m_2,\\ l_1,l_2}} S_{(m_1,m_2)l_1,l_2}\, w_{l_1}^{(m_1)}\, w_{l_2}^{(m_2)}\, \delta_{m,m_1+m_2} + \ldots$$

- $S_{(m)l}$ is linear dynamic response function
- $S_{(m_1,m_2)l_1,l_2}$ is quadratic dynamic response function etc.
- Symmetries imply conservation of azimuthal wavenumber
- Response functions depend on thermodynamic and transport properties, in particular viscosity.

Flow correlations from initial density correlations

Moments of flow coefficients

$$\left\langle V_{m_1}^* \cdots V_{m_n}^* \right\rangle = S_{(m_1)l_1} \cdots S_{(m_n)l_n} \left\langle w_{l_1}^{(m_1)} \cdots w_{l_n}^{(m_n)} \right\rangle$$

$$+ \text{ non-linear terms}$$

- combination of dynamical response coefficients and correlation functions of initial density perturbations
- linear, quadratic and higher-order terms
- For N independent sources and central collisions

$$v_m\{n\}^n \sim \frac{1}{N^{n-1}} \qquad \text{or} \qquad v_m\{n\} \sim \frac{1}{N^{1-\frac{1}{n}}}$$

  - explains why $v_m\{4\} \sim v_m\{6\} \sim v_m\{8\}$ are of almost equal size
  - holds also for extended sources
  - holds also for non-linear response contributions
  - can be extended to other correlation functions, e. g. $\langle V_2 V_3 V_5^* \rangle \sim \frac{1}{N^2}$
  - gets broken for non-central collisions
  - impact parameter dependence of corrections is known

Scaling with system size

- Large (PbPb) and small systems (pPb) may have different number of independent sources N and response functions $S_{(m)l}$
- For linear dynamics one has parametrically

$$v_m\{n\} \sim \frac{S_{(m)l}}{N^{1-\frac{1}{n}}}$$

- To have $v_m\{n\}|_{\text{PbPb}} = v_m\{n\}|_{\text{pPb}}$ one needs

$$\frac{S_{(m)l}|_{\text{pPb}}}{S_{(m)l}|_{\text{PbPb}}} = \left(\frac{N_{\text{pPb}}}{N_{\text{PbPb}}}\right)^{1-\frac{1}{n}}$$

  - For comparison at equal multiplicity one may have $N_{\text{pPb}} \approx N_{\text{PbPb}}$ so that response functions must be equal

$$S_{(m)l}|_{\text{pPb}} \approx S_{(m)l}|_{\text{PbPb}}$$

  - $S_{(m)l}$ depends on system size only via initial background $w_{\text{BG}}(r)$. Precise dependence can be investigated more closely.

Hydrodynamic evolution

Perturbative expansion

Write the hydrodynamic fields $h = (w, u^\mu, \pi^{\mu\nu}, \pi_{\text{Bulk}}, \ldots)$

- at initial time τ_0 as

$$h = h_0 + \epsilon\, h_1$$

  with background h_0, fluctuation part $\epsilon\, h_1$

- at later time $\tau > \tau_0$ as

$$h = h_0 + \epsilon\, h_1 + \epsilon^2 h_2 + \epsilon^3 h_3 + \ldots$$

Solve for time evolution in this scheme

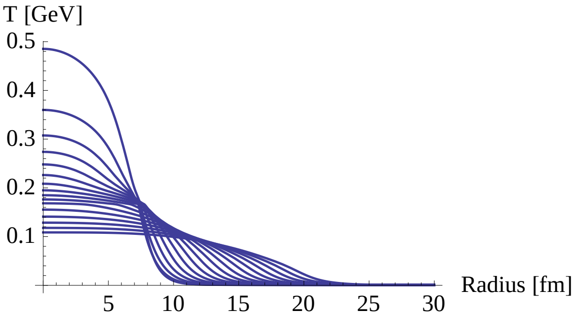
- h_0 is solution of full, non-linear hydro equations in symmetric situation: azimuthal rotation and Bjorken boost invariant
- h_1 is solution of linearized hydro equations around h_0, can be solved mode-by-mode
- h_2 can be obtained by from interactions between modes etc.

Background evolution

System of coupled $1+1$ dimensional non-linear partial differential equations for

- enthalpy density $w(\tau, r)$ (or temperature $T(\tau, r)$)
- fluid velocity $u^\tau(\tau, r), u^r(\tau, r)$
- two independent components of shear stress $\pi^{\mu\nu}(\tau, r)$

Can be easily solved numerically

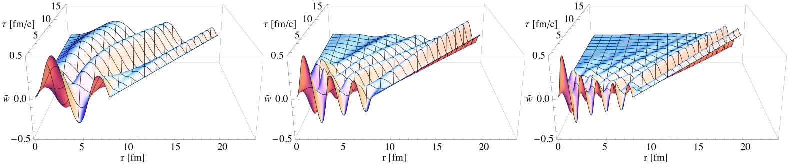


Evolving perturbation modes

- Linearized hydro equations: set of coupled $3+1$ dimensional, linear, partial differential equations.
- Use Fourier expansion

$$h_j(\tau, r, \phi, \eta) = \sum_m \int \frac{dk_\eta}{2\pi}\, h_j^{(m)}(\tau, r, k_\eta)\, e^{i(m\phi + k_\eta \eta)}.$$

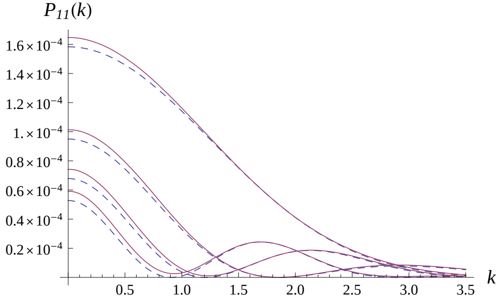
- Reduces to $1+1$ dimensions.
- Can be solved numerically for each initial Bessel-Fourier mode.

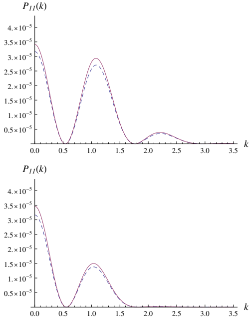


Evolution of spectrum of density perturbations

Density-density spectrum

$$P_{11}(\vec{k}) = \int d^2x\, e^{-i\vec{k}(\vec{x}-\vec{y})} \,\langle\, d(\vec{x}_1)\, d(\vec{x}_2)\,\rangle_c$$





dashed: linear evolution, solid: including first non-linear correction

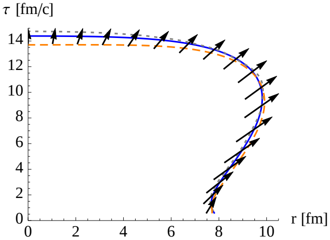
left: $\eta/s = 0.08$, $\tau = 1.5, 2.5, 3.5, 4.5$ fm/c, right: $\eta/s = 0.08$ and $\eta/s = 0.8$, $\tau = 7.5$ fm/c

[Brouzakis, Floerchinger, Tetradis & Wiedemann, arXiv:1411.2912]

Kinetic freeze-out

Freeze-out surface

- Perturbative expansion can be used also at freeze-out. [Floerchinger, Wiedemann 2013]
- Freeze-out surface is azimuthally symmetric as background.
- Generalization to kinetic hadronic scattering and decay phase possible.



(solid: $\eta/s = 0.08$, dotted: $\eta/s = 0$, dashed: $\eta/s = 0.3$)

Particle distribution

for single event

$$\ln\left(\frac{dN^{\text{single event}}}{p_T dp_T d\phi dy}\right) = \underbrace{\ln S_0(p_T)}_{\text{from background}} + \underbrace{\sum_{m,l} w_l^{(m)} e^{im\phi} \theta_l^{(m)}(p_T)}_{\text{from fluctuations}}$$

- each mode comes with an angle, $w_l^{(m)} = |w_l^{(m)}|\, e^{im\psi_l^{(m)}}$
- each mode has different p_T-dependence, $\theta_l^{(m)}(p_T)$
- quadratic order correction

$$\sum_{m_1,m_2,l_1,l_2} w_{l_1}^{(m_1)} w_{l_2}^{(m_2)}\; e^{i(m_1+m_2)\phi}\; \kappa_{l_1,l_2}^{(m_1,m_2)}(p_T)$$

- non-linearities from hydro evolution and freeze-out

Differential harmonic flow coefficients

Double differential harmonic flow coefficient (to lowest order)

$$v_m\{2\}^2(p_T^a, p_T^b) = \sum_{l_1, l_2} \theta_{l_1}^{(m)}(p_T^a)\, \theta_{l_2}^{(m)}(p_T^b)\, \langle w_{l_1}^{(m)} w_{l_2}^{(m)*} \rangle$$

- intuitive matrix expression
- in general no factorization
- can be generalized to higher order flow cumulants

Baryon number density fluctuations

Fluctuations around vanishing baryon number

- Evolution of baryon number density

$$u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

with diffusion current ν^α determined by heat conductivity κ

$$\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon + p} \right]^2 \Delta^{\alpha\beta} \partial_\beta \left(\frac{\mu}{T} \right)$$

- Consider situation with $\langle n(x) \rangle = \langle \mu(x) \rangle = 0$ but event-by-event fluctuation $\delta n \neq 0$
- Concentrate now on Bjorken flow profile for u^μ

$$\partial_\tau \delta n + \frac{1}{\tau} \delta n - \kappa \left[\frac{nT}{\epsilon + p} \right]^2 \left(\frac{\partial(\mu/T)}{\partial n} \right)_\epsilon \left(\partial_x^2 + \partial_y^2 + \frac{1}{\tau^2} \partial_\eta^2 \right) \delta n = 0$$

- Structures in transverse and rapidity directions are “flattened out” by heat conductive dissipation

Baryon number correlations experimentally

- Two-particle correlation function of baryons minus anti-baryons

$$C_{\text{Baryon}}(\phi_1 - \phi_2, \eta_1 - \eta_2) = \langle n(\phi_1, \eta_1) n(\phi_2, \eta_2) \rangle_c$$

- In Fourier representation

$$C_{\text{Baryon}}(\Delta\phi, \Delta\eta) = \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi} \, \tilde{C}_{\text{Baryon}}(m, q) \, e^{im\Delta\phi + iq\Delta\eta}$$

heat conductivity leads to exponential suppression

$$\tilde{C}_{\text{Baryon}}(m, q) = e^{-m^2 I_1 - q^2 I_2} \, \tilde{C}_{\text{Baryon}}(m, q)\Big|_{\kappa=0}$$

- I_1 and I_2 can be approximated as

$$I_1 \approx \int_{\tau_0}^{\tau_f} d\tau \, \frac{2}{R^2} \, \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_\epsilon$$

$$I_2 \approx \int_{\tau_0}^{\tau_f} d\tau \, \frac{2}{\tau^2} \, \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_\epsilon$$

- $I_2 \gg I_1$ would lead to long-range correlations in rapidity direction ("baryon number ridge")

Remarks on baryon number fluctuations

- Initial ("primordial") baryon number fluctuations are poorly understood so far but presumably non-vanishing.
- Heat conductivity of QCD also poorly understood theoretically so far
  - from perturbation theory [Danielewicz & Gyulassy, PRD 31, 53 (1985)]

$$\kappa \sim \frac{T^4}{\mu^2 \alpha_s^2 \ln \alpha_s} \qquad (\mu \ll T)$$

  - from AdS/CFT [Son & Starinets, JHEP 0603 (2006)]

$$\kappa = 8\pi^2 \frac{T}{\mu^2} \eta = 2\pi \frac{sT}{\mu^2} \qquad (\mu \ll T)$$

- More refined study needed to take transverse expansion properly into account.
- Seems to be interesting topic for further experimental and theoretical studies.

Summary and Conclusions

Conclusions

- Systematic expansion in initial fluid perturbations is possible (good convergence properties) and very useful.
- Formalism works in praxis (see backup slides for results of "proof of principle" study).
- Initial density perturbations have some universal properties that can help to better constrain thermodynamic and transport properties.
- Fluid dynamic response allows to access correlation functions of initial perturbations.
- Baryon number correlations could allow to constrain heat conductivity.

Backup

Characterization of transverse density via eccentricities

Fluctuations in initial transverse enthalpy density $w(r, \phi)$ can be characterized in terms of eccentricities $\epsilon_{n,m}$ and angles $\psi_{n,m}$ [Ollitrault, Teaney, Yan, Luzum, and others]

$$\epsilon_{n,m}\, e^{im\,\psi_{n,m}} = \frac{\int dr\, \int_0^{2\pi} d\varphi\, r^{n+1}\, e^{im\varphi}\, w(r,\varphi)}{\int dr\, \int_0^{2\pi} d\varphi\, r^{n+1}\, w(r,\varphi)}$$

- $w(r, \phi)$ completely determined by set of all $\epsilon_{n,m}$ and $\psi_{n,m}$
- closely related method is based on cumulants [Teaney, Yan]
- no positive transverse density can be associated to small set of cumulants (beyond Gaussian order) such that higher order cumulants vanish
- generalization to velocity and shear fluctuations not known

Scaling tests

- Start with single enthalpy density mode $(m = 2, l = 1)$ on top of background

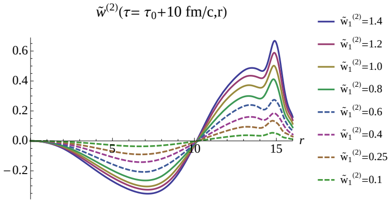
$$w(\tau_0, r, \phi) = w_{\mathsf{BG}}(\tau_0, r) \left[1 + 2\, \tilde{w}_1^{(2)} J_2(k_1^{(2)} r)\, \cos(2\phi)\right].$$

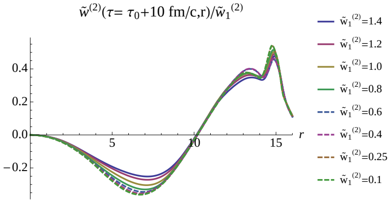
- Evolve this with hydro solver ECHO-QGP
  [Del Zanna *et al.*, EPJC 73, 2524 (2013), see also following talk]
- Determine Fourier components

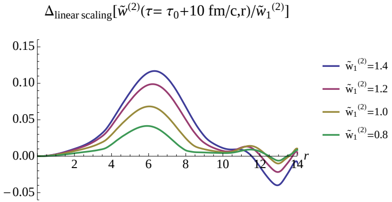
$$\tilde{w}^{(m)}(\tau, r) = \frac{1}{w_{\mathsf{BG}}(r)} \frac{1}{2\pi} \int d\phi\, e^{-im\phi}\, w(\tau, r, \phi)$$

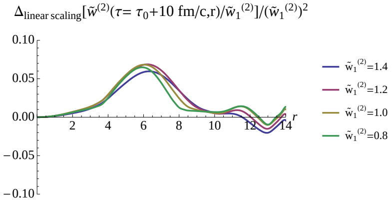
Scaling tests at first order

Compare enthalpy $\tilde{w}^{(2)}(\tau, r)$ at fixed τ for different initial weights



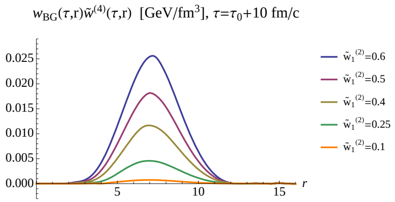


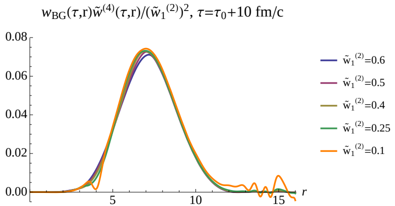




Scaling tests at second order

From symmetry considerations one expects that modes with $m = 0$ and $m = 4$ receive mainly quadratic contributions $\sim \left(\tilde{w}_1^{(2)}\right)^2$.

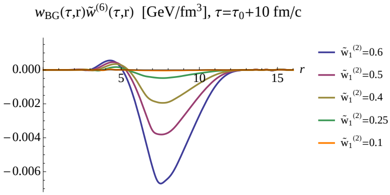


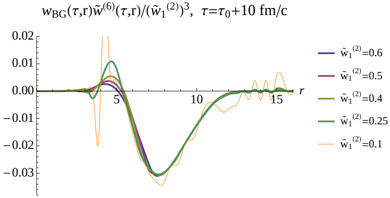


- Hydrodynamic response to initial enthalpy density fluctuations is perturbative.
- Non-linearities can be understood order-by-order and lead to characteristic "overtones".
- Results motivate more thorough development of fluid dynamic perturbation theory.

Scaling tests at third order

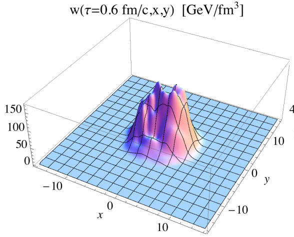
From symmetry considerations one expects that modes $m = 6$ receive mainly cubic contributions $\sim \left(\tilde{w}_1^{(2)}\right)^3$.

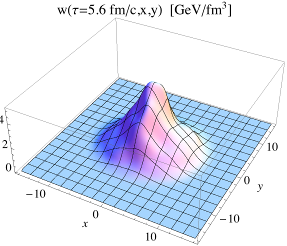


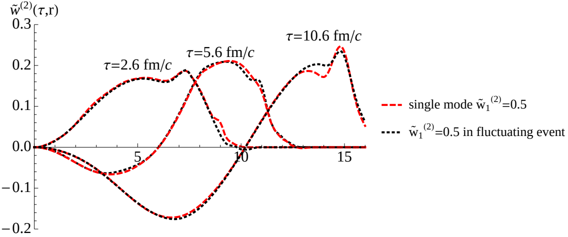


Scaling tests embedded in realistic event

Embed mode $(m = 2, l = 1)$ into realistic fluctuating event and compare to embedding into pure background.

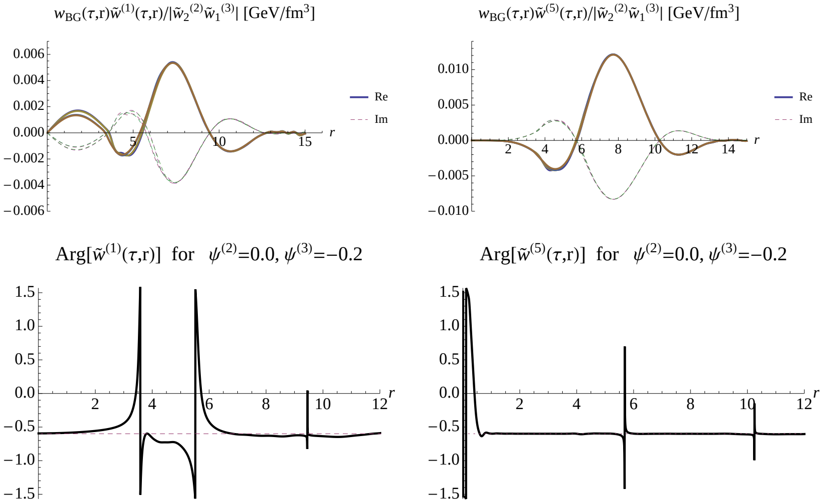






Scaling tests with several initial modes

Start with linear combination of $(m = 2, l = 2)$ and $(m = 3, l = 1)$ modes and test scaling for $m = 1$ and $m = 5$ response.



Generalized Glauber model

- Fluctuations due to nucleon positions: used so far

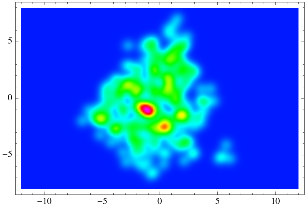
$$\epsilon(\tau, \mathbf{x}, y) = \sum_{i=1}^{N_{\text{part}}} \epsilon_w(\tau, \mathbf{x} - \mathbf{x}_i, y), \qquad u^\mu = (1, 0, 0, 0)$$

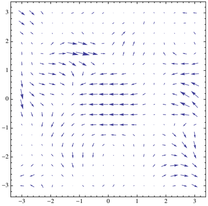
- can be generalized to include also velocity fluctuations

$$T^{\mu\nu}(\tau, \mathbf{x}, y) = \sum_{i=1}^{N_{\text{part}}} T_w^{\mu\nu}(\tau, \mathbf{x} - \mathbf{x}_i, y)$$

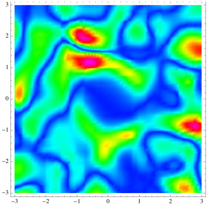
- More generally describe primordial fluid fields by
  - expectation values $\langle \epsilon(\tau_0, \mathbf{x}, y) \rangle, \langle u^\mu(\tau_0, \mathbf{x}, y) \rangle, \langle n_B(\tau_0, \mathbf{x}, y) \rangle$
  - correlation functions $\langle \epsilon(\tau_0, \mathbf{x}, y)\, \epsilon(\tau_0, \mathbf{x}', y') \rangle$, etc.
- Origin of this fluctuations is initial state physics and early-time, non-equilibrium dynamics.

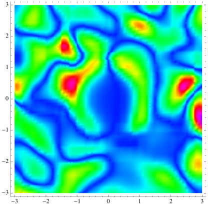
Velocity fluctuations





- also the velocity field will fluctuate at the initialization time τ_0
- take here transverse velocity for every participant to be Gaussian distributed with width $0.1c$
- vorticity $|\partial_1 u^2 - \partial_2 u^1|$ and divergence $|\partial_1 u^1 + \partial_2 u^2|$

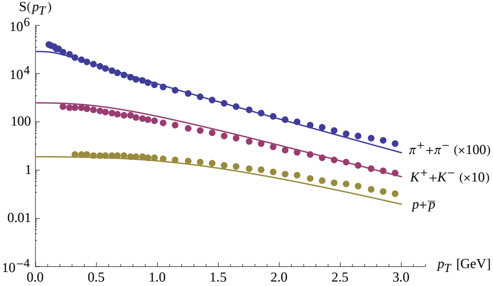




“Proof of principle” study: One-particle spectrum

Initial conditions from Glauber Monte Carlo Model

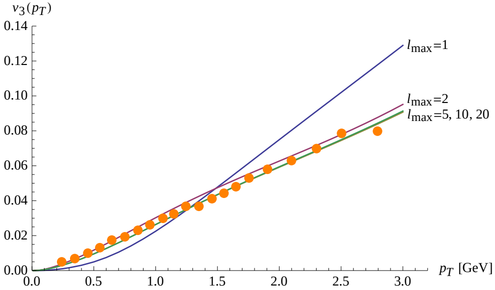
$$S(p_T) = dN/(2\pi p_T dp_T d\eta d\phi)$$



Points: 5% most central collisions, ALICE [PRL 109, 252301 (2012)]
Curves: Our calculation, no hadron rescattering and decays after freeze-out.

Harmonic flow coefficients for central collisions

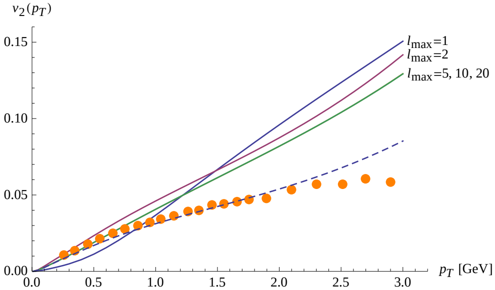
Triangular flow for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Curves: Different maximal resolution l_{max}

Harmonic flow coefficients for central collisions

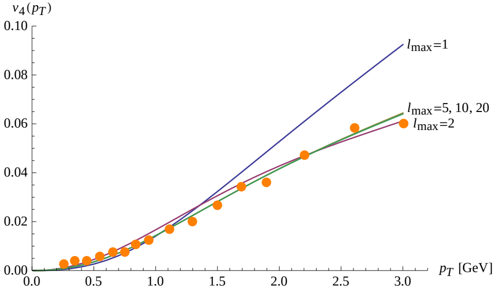
Elliptic flow for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Solid curves: Different maximal resolution l_{max}
Dashed curve: Mode $(m = 2, l = 1)$ suppressed by factor 0.7

Harmonic flow coefficients for central collisions

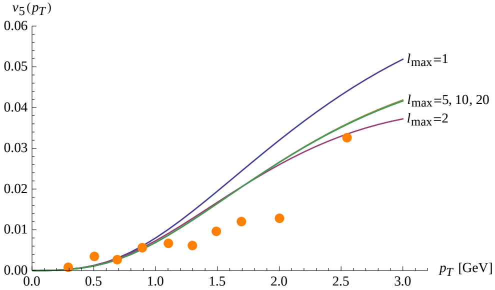
Flow coefficient v_4 for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Curves: Different maximal resolution $l_{\rm max}$

Harmonic flow coefficients for central collisions

Flow coefficient v_5 for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Curves: Different maximal resolution l_{max}

Harmonic flow coefficients, central, particle identified