Hadronization at high μ_B in the chiral model

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ECT* Trento, 08.10.2014

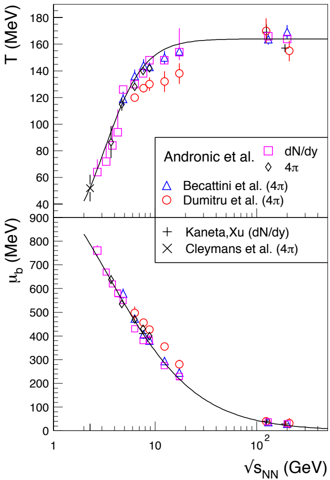
based on

- S. Floerchinger and C. Wetterich *Chiral freeze-out in heavy ion collisions at large baryon densities*, [Nucl. Phys. A, 890-891, 11 (2012)]

see also a related follow-up work

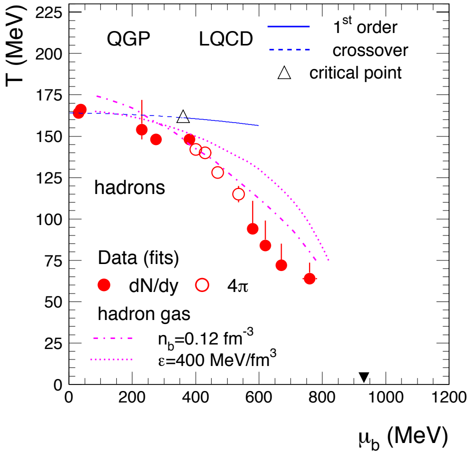
- M. Drews, T. Hell, B. Klein and W. Weise, *Thermodynamic phases and mesonic fluctuations in a chiral nucleon-meson model*, [Phys. Rev. D, 88, 069011 (2013)]

Chemical freeze-out for different collision energies



[Andronic, Braun-Munzinger, Stachel (2009)]

A phase diagram from chemical freeze-out?



[Andronic, Braun-Munzinger, Stachel (2009), LQCD from Fodor, Katz (2004)]

Is the chemical freeze-out universal?

- Is the chemical freeze-out dominated by the universal thermodynamic properties of QCD?
- Or do the chemical freeze-out points rather reflect the dynamics of a heavy-ion collision?

Thought experiment 1:

- Consider a universe filled by a quark-gluon plasma with a very slow expansion
- Temperature decreases only very slowly
- Chemical equilibrium would be maintained down to very small temperatures

Thought experiment 2:

- Consider a universe filled by a quark-gluon plasma with a very fast expansion.
- No time for equilibration
- Quarks and gluons would hadronize like jets

Is chemical freeze-out universal? (2)

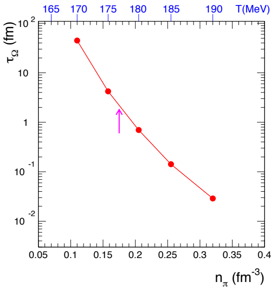
- Thought experiment 1 would lead to thermal particle yields but at very low temperature
- Thought experiment 2 might lead to non-thermal particle yields but hard to quantity since hadronization is purely understood
- Conclusion: Freeze-out temperature inevitably depends on dynamics of the expansion
- Dependence *might* be weak for realistic expansions
- Freeze-out *might* become effectively universal

Chemical freeze-out and chiral crossover

It has been argued that chemical-freeze out at RHIC energies is essentially on the chiral crossover:

[Braun-Munzinger, Stachel, Wetterich, Phys. Lett. B, 596, 61 (2004)]

- Expansion rate is approximately $r_T = |\dot{T}/T| \approx 0.13/\text{fm}$
- Rates for $2 \to 2$-particle scattering with strangeness exchange too small to maintain chemical equilibrium, typically $r_{2\to 2} \approx 0.018/\text{fm}$
- Multi-hadron strangeness exchange reactions with N_{in} incoming particles have rates that depend strongly on density, $\sim n^{N_{\text{in}}}$
- Many such processes become important close to T_c

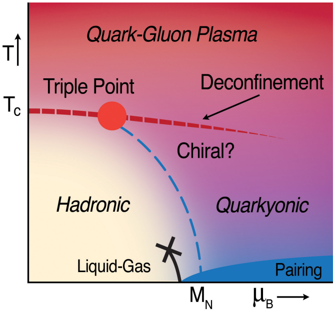


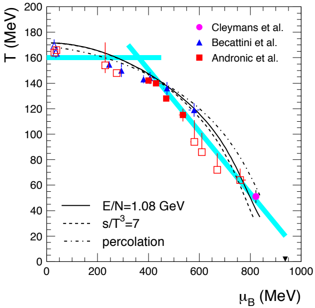
Time to bring Ω baryons into equilibrium

Is chemical freeze-out close to a phase transition everywhere?

It was also proposed that the freeze-out line corresponds to

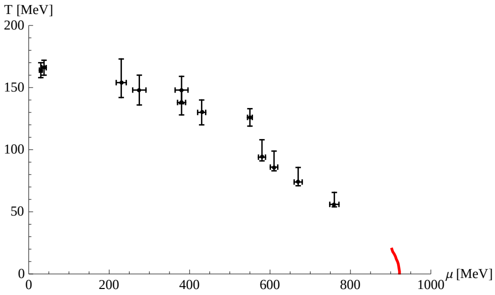
- chiral & deconfinement transition at small μ
- transition to quarkyonic matter at small T





[Andronic, Blaschke, Braun-Munzinger, Cleymans, Fukushima, McLerran, Oeschler, Pisarski, Redlich, Sasaki, Satz, Stachel, Nucl. Phys. A 837 (2010) 65.]

Normal nuclear matter and the droplet model



- Normal nuclear matter is sitting on a first order phase transition at $T = 0$, $\mu = \mu_c$.
- For densities $n < n_{\text{nucl}} = 0.153/\text{fm}^3$ one has phase separation: vacuum $(n = 0)$ or nuclear matter $(n = n_{\text{nucl}})$
- Nuclei can be seen as droplets of normal nuclear matter

The chiral nucleon-meson model

- Low-energy degrees of freedom of QCD
  - nucleons (protons, neutrons)

$$\psi_a = \begin{pmatrix} p \\ n \end{pmatrix}$$

  - scalar mesons σ, pseudscalar mesons (pions) π^0, $\pi^\pm$ combined into

$$\phi_{ab} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\sigma + i\pi^0) & i\pi^- \\ i\pi^+ & \frac{1}{\sqrt{2}}(\sigma - i\pi^0) \end{pmatrix}$$

  - isospin singlet vector mesons ω_μ

- Chiral transformations are linear

$$\psi \to \left(1 + \frac{i}{2}\boldsymbol{\alpha}_V \boldsymbol{\tau} + \frac{i}{2}\boldsymbol{\alpha}_A \boldsymbol{\tau}\gamma_5 + \frac{i}{2}\beta_V + \frac{i}{2}\beta_A\gamma_5\right)\psi,$$

$$\phi \to \phi - \frac{i}{2}\boldsymbol{\alpha}_V[\boldsymbol{\tau}, \phi] - \frac{i}{2}\boldsymbol{\alpha}_A\{\boldsymbol{\tau}, \phi\} + i\beta_A\phi,$$

$$\omega_\mu \to \omega_\mu.$$

The chiral nucleon-meson model (2)

$$\begin{aligned}\mathscr{L} =& \bar{\psi}_a \, i\gamma^\nu(\partial_\nu - i\, g\, \omega_\nu - i\, \mu\, \delta_{0\nu})\, \psi_a \\ &+ h\sqrt{2}\left[\bar{\psi}_a \left(\tfrac{1+\gamma_5}{2}\right)\phi_{ab}\psi_b + \bar{\psi}_a \left(\tfrac{1-\gamma_5}{2}\right)(\phi^\dagger)_{ab}\psi_b\right] \\ &+ \tfrac{1}{2}\phi^*_{ab}(-\partial_\mu\partial^\mu)\phi_{ab} + U_{\text{mic}}(\rho,\sigma) \\ &+ \frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu) + \frac{1}{2}m_\omega^2\,\omega_\mu\omega^\mu.\end{aligned}$$

- kinetic terms for nucleons ψ_a, scalars ϕ_{ab} and vectors ω_ν
- Yukawa coupling between nucleons and vectors g
- Yukawa coupling between nucleons and scalars h
- effective potential for scalars

$$U_{\text{mic}}(\rho,\sigma) = \bar{U}(\rho) - m_\pi^2 f_\pi \sigma$$

  with the chiral invariant combination

$$\rho = \frac{1}{2}\phi^*_{ab}\phi_{ab} = \frac{1}{2}(\sigma^2 + \boldsymbol{\pi}^2).$$

- extended and chiral version of Waleckas model
  [Walecka, Ann. Phys. 83, 491 (1974)]

The chiral nucleon-meson model (3)

- No mass terms for nucleons ψ_a, effective masses come from chiral symmetry breaking only.
- $\mathscr{L}$ is invariant under chiral symmetry except for an explicit symmetry breaking term from quark masses $\sim m_\pi^2 f_\pi \sigma$
- Parameters of the model are
  - pion mass $m_\pi = 135\,\text{MeV}$
  - pion decay constant $f_\pi = 93\,\text{MeV}$
  - vector boson mass $m_\omega = 783\,\text{MeV}$

  as well as
  - vector Yukawa couplings g
  - scalar Yukawa coupling h
  - the form of the effective potential $\bar{U}(\rho)$.

Bosonic field expectation values

From symmetries at non-zero density

- time component of vector field $\omega_0 \neq 0$
  leads to shift in nucleon chemical potential $\mu_{\text{eff}} = \mu + g\,\omega_0$
- spatial components of vector field $\omega_j = 0$
- scalar field $\sigma \neq 0$
  leads to effective nucleon mass $m_{\text{nucl}} = h\,\sigma$
- pseudoscalar field $\pi^0 = \pi^+ = \pi^- = 0$

Thermodynamic and chiral properties from *quantum* effective potential

$$U(\sigma, \omega_0)$$

Effective potential

Calculation of $U(\sigma, \omega_0)$ from microscopic model is in general difficult
Simplifications from two points:

- explicit breaking of chiral symmetry comes only from linear term

  $$U(\sigma, \omega_0) = U(\rho, \omega_0) - m_\pi^2 f_\pi \sigma, \qquad \rho = \sigma^2/2$$

  with chiral invariant potential $U(\rho, \omega_0)$
- at $T = 0$ and $\mu = \mu_c$ many properties of U are known from nuclear droplet model. Write therefore

  $$U(\sigma, \omega_0; T, \mu) = U(\sigma, \omega_0; 0, \mu_c) + \Delta$$

  where the difference Δ is much easier to compute than the full potential U

Effective potential (2)

Most important medium modification of U is from nucleon fluctuations

$$U(\sigma,\omega_0;T,\mu) = U_{\mathsf{vac}}(\sigma,\omega_0) - 4\,p_{\mathsf{FG}}(T,\mu+g\omega_0,h\sigma) + \text{boson fluctuations},$$

For the vacuum part use Taylor expansion around minimum

$$\begin{aligned} U_{\mathsf{vac}}(\sigma,\omega_0) =& \frac{m_\pi^2}{2}\left(\sigma^2-f_\pi^2\right)+\frac{\lambda}{8}\left(\sigma^2-f_\pi^2\right)^2 \\ &+\frac{\gamma_3}{3f_\pi^2}\left(\sigma^2-f_\pi^2\right)^3+\frac{\gamma_4}{4f_\pi^4}\left(\sigma^2-f_\pi^2\right)^4 \\ &-m_\pi^2 f_\pi\left(\sigma-f_\pi\right)-\frac{1}{2}m_\omega^2\omega_0^2. \end{aligned}$$

Fermionic fluctuation part has factor 4 from spin + isospin degeneracy and

$$p_{\mathsf{FG}}(T,\mu,m)=\int\frac{d^3p}{(2\pi)^3}\frac{\vec{p}^2}{3\sqrt{\vec{p}^2+m^2}}\left[\frac{1}{e^{\frac{1}{T}(\sqrt{\vec{p}^2+m^2}-\mu)}+1}+\frac{1}{e^{\frac{1}{T}(\sqrt{\vec{p}^2+m^2}+\mu)}+1}\right].$$

Bosonic fluctuations can be included by solving functional renormalization group equations [done by Drews, Hell, Klein & Weise (2013)]

Fixing parameters

In vacuum $T = 0, \mu = 0$ the field equations

$$\frac{\partial}{\partial \sigma} U(\sigma, \omega_0) = \frac{\partial}{\partial \omega_0} U(\sigma, \omega_0) = 0$$

have the solution

$$\sigma = f_\pi, \qquad \omega_0 = 0.$$

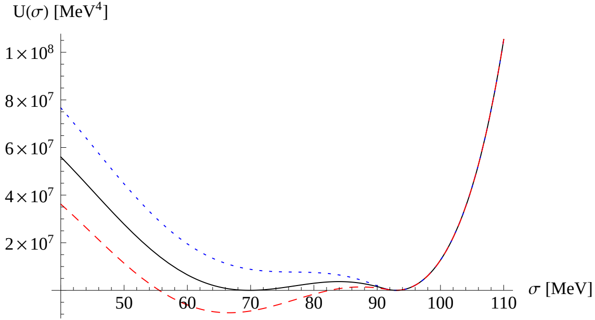
- Linear coefficient in $\boldsymbol{\pi}^2$ determines the pion mass m_π
- Nucleon mass is given by $m_N = h f_\pi$ which gives $h = 10$
- Quadratic coefficient in σ gives $m_\sigma^2 = m_\pi^2 + \lambda f_\pi^2$

First order quantum phase transition

- At vanishing temperature $T = 0$ one has a first-order phase transition at some value μ_c
- Solve field equation for ω_0:

$$\frac{\partial}{\partial \omega_0} U(\sigma, \omega_0) = 0 \quad \rightarrow \quad U(\sigma) = U(\sigma, \omega_{0,\mathsf{min}})$$

- Effective potential $U(\sigma)$ for $T = 0$:



$\mu = 915$ MeV (dotted), $\mu = 922.7$ MeV (solid), $\mu = 930$ MeV (dashed).

Fixing parameters (2)

- Directly at $\mu = \mu_c$ one has two minima:
  - usual vacuum minimum at $\sigma = f_\pi = 93\,\text{MeV}$
  - nuclear matter minimum at $\sigma = \sigma_{\text{nucl}}$
- The baryon density and vector field condensate have values
  - $n = 0$, $\omega_0 = 0$ at $\sigma = f_\pi$
  - $n = n_{\text{nucl}}$, $\omega_0 = \omega_{0,\text{nucl}}$ at $\sigma = \sigma_{\text{nucl}}$
- From experimental values for
  - nuclear binding energy $\epsilon_{\text{bind}} = -16.3\,\text{MeV}$,
  - nuclear saturation density $n_{\text{nucl}} = 0.153/\text{fm}^3$
  - Landau mass $m_L = \mu_c + g\omega_{0,\text{nucl}} = 0.80\, m_N$

  one can determine
  - the vector Yukawa coupling $g = 9.5$,
  - the critical chemical potential $\mu_c = m_N + \epsilon_{\text{bind}} = 922.7\,\text{MeV}$
  - and the condensates $\sigma_{\text{nucl}} = 69.8\,\text{MeV}$, $\omega_{0,\text{nucl}} = -18\,\text{MeV}$.

Fixing parameters (3)

- Two of the remaining parameters λ, γ_3, γ_4 get fixed by the constraints for a first order phase transition at μ_c

$$U(\sigma_{\mathsf{nucl}}, \omega_{0,\mathsf{nucl}}) = U(f_\pi, 0)$$

and

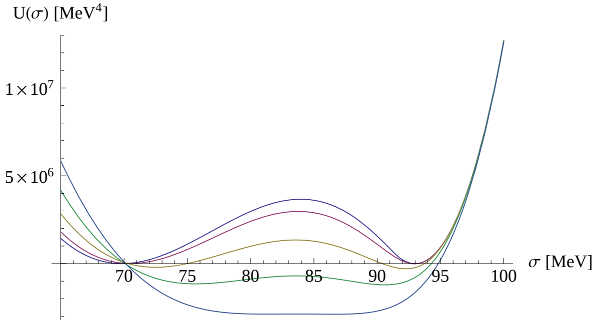
$$\frac{\partial}{\partial\sigma} U(\sigma_{\mathsf{nucl}}, \omega_{0,\mathsf{nucl}}) = 0.$$

- The last parameter can be fixed from other properties of normal nuclear matter.
- The choice $\lambda = 50$, $\gamma_3 = 3$, $\gamma_4 = 50$ gives
  - vacuum mass of σ-meson $m_\sigma = 670\,\mathsf{MeV}$
  - compressibility module $K = \frac{9n}{\partial n/\partial\mu} = 300\,\mathsf{MeV}$
  - surface tension of nuclear droplet $\Sigma = \int_{\sigma_{\mathsf{nucl}}}^{f_\pi} \sqrt{2U(\sigma)}d\sigma = 42000\ \mathsf{MeV}^3$.

  in reasonable agreement with the experimentally established values.

The liquid-gas phase transition

Phase transition can be followed also at $T > 0$. For $\mu = \mu_c(T)$



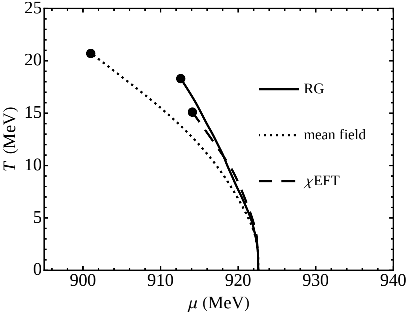
$T = \{0, 5, 10, 15, 20\}$ MeV.

For increasing temperature

- $U(\sigma)$ at minima becomes more negative: pressure p increases
- potential barrier gets smaller: droplet surface tension Σ decreases

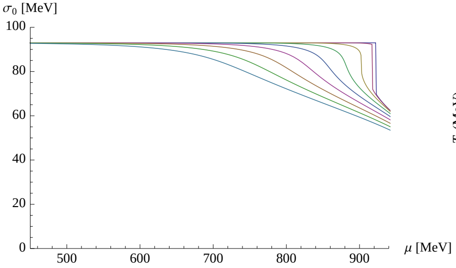
First order line and critical end point

- The first order phase transition line ends in a critical end point at some temperature T_* and chemical potential μ_*
- The form of the transition line is somewhat changed by the effect of bosonic fluctuations

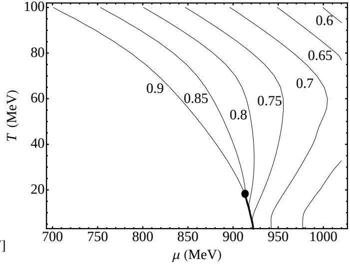


[Drews, Hell, Klein, Weise, PRD 88, 096011 (2013)]

Chiral order parameter



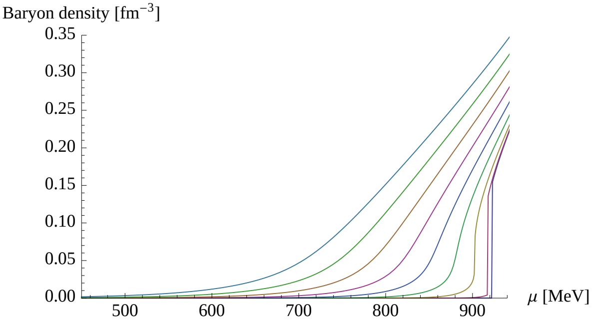
Left: $T = \{0, 10, 20, 30, 40, 50, 60, 70, 80\}$ MeV.



Right: Contour plot for σ_0/f_π.

- σ_0 decreases for larger chemical potential and temperature
- Effective nucleon mass $m_{N,\text{eff}} = h\sigma_0$ gets smaller
- First order phase transition becomes quickly rather smooth crossover

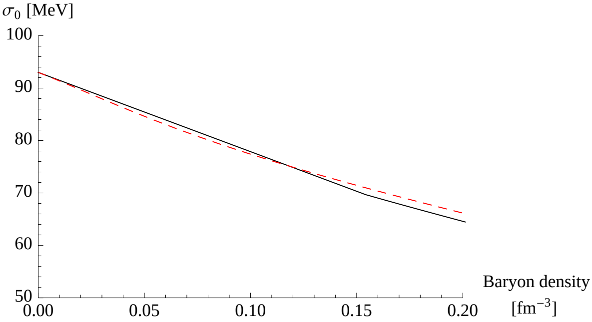
Baryon number density



$T = \{0, 10, 20, 30, 40, 50, 60, 70, 80\}$ MeV.

- n_B increases for larger chemical potential and temperature
- First order phase transition becomes quickly rather smooth crossover

Chiral order parameter as a function of density

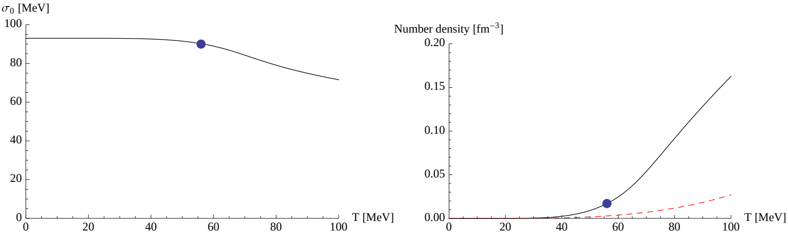


$T = 0$ (solid), $T = 50\,\text{MeV}$ (dashed).

- Chiral order parameter σ_0 decreases with baryon density
- Temperature dependence rather small

Chemical freeze out

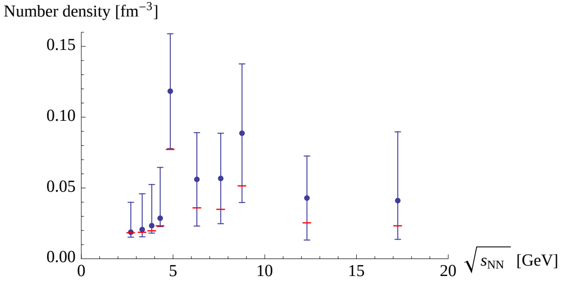
- Is there any sign of a phase transition in the region of chemical freeze-out at large μ?
- Consider for example the region around highest baryon density $\mu_{\text{ch}} = 760 \pm 23\,\text{MeV}$, $T_{\text{ch}} = 56^{+9.6}_{-2.0}\,\text{MeV}$



Left: Chiral condensate σ_0 for $\mu = 750\,\text{MeV}$. The dot marks chemical freeze-out.
Right: Baryon number density (solid) and pion number density (dashed).

Chemical freeze-out (2)

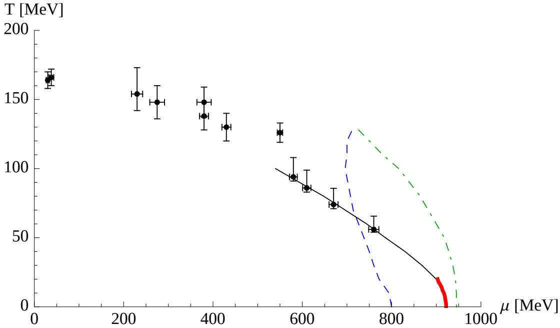
- Freeze-out at large μ does not seem to be related to phase transition or rapid crossover.
- Baryon number density does seem to change quickly there, however.
- Chemical freeze-out at constant number density does make physical sense when rates for strangeness exchange processes depend strongly on density.



Baryon density due to protons, neutrons and Delta baryons in statistical model.

Chemical freeze-out (3)

At small collision energy or high baryon chemical potential chemical freeze-out takes place at constant baryon number density!



Red line: First order liquid-gas phase transition.

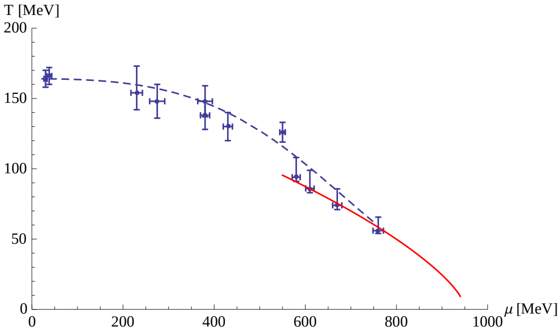
Black line: Constant baryon number density $n_{\mathrm{Baryons}} = 0.15\, n_{\mathrm{nucl}}$.

Conclusions

- "Chemical freeze-out = Phase transition" is too simple
- No sign of any phase transition or rapid crossover in the region of chemical freeze-out for low-energy experiments
- Chemical freeze-out seems still related to thermodynamic properties of QCD: $n_{\text{Baryons}} = 0.15\, n_{\text{nucl}}$
- Chiral nucleon-meson model gives rather detailed and realistic description of nuclear matter as well as thermodynamic properties of QCD at intermediate μ and low temperatures
- Would also be interesting to determine transport properties i.e. viscosities and conductivities
- Bottom-up approach to QCD phase diagram might be explored further, could also be extended to smaller chemical potential

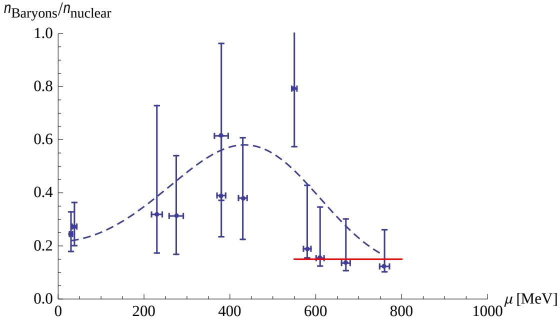
BACKUP

Parametrization in T-μ-plane is dangerous



Parametrization of freeze-out curve seems reasonable with respect to temperature and chemical potential...

Parametrization in T-μ-plane is dangerous (2)



...but is quite far from data points for baryon density.