Mode-by-mode fluid dynamics for relativistic heavy ion collisions

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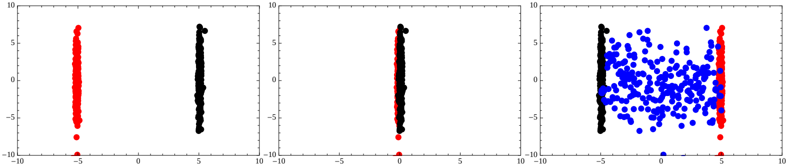
Ohio State University, April 10, 2014

based on work with Urs A. Wiedemann, Andrea Beraudo et al.

- Fluctuations around Bjorken Flow and the onset of turbulent phenomena, [JHEP 11, 100 (2011), with U. A. Wiedemann]
- Mode-by-mode fluid dynamics for relativistic heavy ion collisions [Phys. Lett. B, 728, 407 (2014), with U. A. Wiedemann]
- Characterization of initial fluctuations for the hydrodynamical description of heavy ion collisions, [Phys. Rev. C 88, 044906 (2013), with U. A. Wiedemann]
- Kinetic freeze-out, particle spectra and harmonic flow coefficients from mode-by-mode hydrodynamics, [arXiv:1311.7613, with U. A. Wiedemann]
- How (non-) linear is the hydrodynamics of heavy ion collisions? [arXiv:1312.5482, with U. A. Wiedemann, A. Beraudo, L. Del Zanna, G. Inghirami, V. Rolando]
- Mode-by-mode hydrodynamics: ideas and concepts [arXiv:1401.2339]

Introduction

Heavy Ion Collisions



- ions are strongly Lorentz-contracted
- *some* medium is produced after collision
- medium expands in longitudinal direction and gets diluted

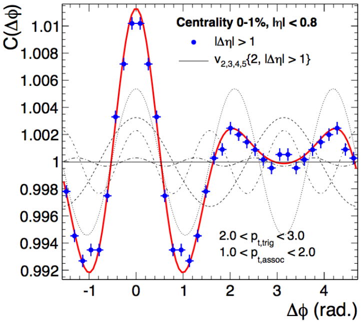
Evolution in time

- Non-equilibrium evolution at early times
  - initial state at from QCD? Color Glass Condensate? ...
  - thermalization via strong interactions, plasma instabilities, particle production, ...
- Local thermal and chemical equilibrium
  - strong interactions lead to short thermalization times
  - evolution from relativistic fluid dynamics
  - expansion, dilution, cool-down
- Chemical freeze-out
  - for small temperatures one has mesons and baryons
  - inelastic collision rates become small
  - particle species do not change any more
- Thermal freeze-out
  - elastic collision rates become small
  - particles stop interacting
  - particle momenta do not change any more

Fluid dynamic regime

- Assumes strong interaction effects leading to local equilibrium.
- Fluid dynamic variables
  - thermodynamic variables: e.g. $T(x)$, $\mu(x)$,
  - fluid velocity $u^\mu(x)$,
  - shear stress tensor $\pi^{\mu\nu}(x)$,
  - bulk viscous pressure $\pi_{\text{Bulk}}(x)$.
- Can be formulated as derivative expansion for $T^{\mu\nu}$.
- Hydrodynamics is universal: many details of microscopic theory not important.
- Some macroscopic properties are important:
  - ideal hydro: needs equation of state $p = p(T, \mu)$, $n = n(T, \mu)$ from thermodynamics
  - first order hydro: needs also transport coefficients like shear viscosity $\eta = \eta(T, \mu)$ and bulk viscosity $\zeta(T, \mu)$ from linear response theory
  - second order hydro: needs also relaxation times τ_{Shear}, τ_{Bulk} etc.

Experimental proof for fluctuations: v_3 *and* v_5



(ALICE 2011, similar pictures also from CMS, ATLAS, Phenix)

- One can expand two-point correlation function

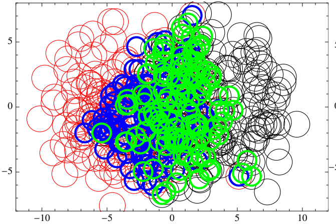
$$C(\Delta\phi) \sim 1 + \sum_{n=2}^{\infty} 2\, v_n\, \cos(n\,\Delta\phi)$$

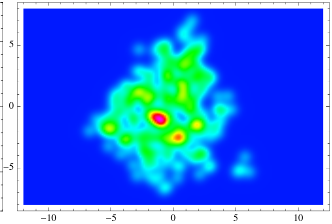
- Without fluctuations one would expect from mirror symmetry

$$v_3 = v_5 = \ldots = 0.$$

Event-by-event fluctuations

- Argument for $v_3 = v_5 = 0$ is based on smooth and symmetric energy density distribution.
- Deviations from this can come from event-by-event fluctuations.
- One example is Glauber model



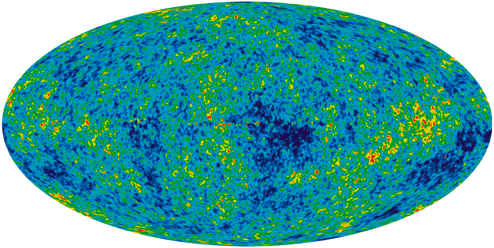


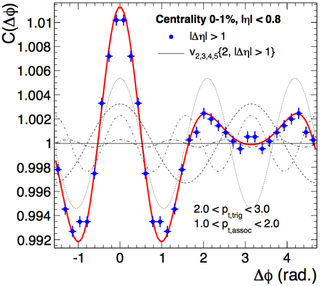
- The initial transverse density distribution fluctuates event-by-event and this leads to sizeable v_3 and v_5.
- More generally also other initial hydro fields may fluctuate: fluid velocity, shear stress, baryon number density etc.

What fluctuations are interesting and why?

- **Initial hydro fluctuations**: Event-by-event perturbations around the average of hydrodynamical fields at time τ_0:
  - energy density ϵ
  - fluid velocity u^μ
  - shear stress $\pi^{\mu\nu}$
  - more general also: baryon number density n_B, electric charge density, electromagnetic fields, ...
- measure for deviations from equilibrium
- contain interesting information from early times
- governed by universal evolution equations
- can be used to constrain thermodynamic and transport properties

Similarities to cosmic microwave background





- fluctuation spectrum contains info from early times
- many numbers can be measured and compared to theory
- can lead to detailed understanding of evolution and properties
- could trigger precision era in heavy ion physics

A program to understand fluctuations

1. Initial fluctuations at initialization time of hydro should be characterized and quantified completely.
2. Fluctuations have to be propagated through the hydrodynamical regime.
3. Contribution of different fluctuations to the particle spectra must be understood and quantified.
4. Fluctuations generated from non-hydro sources (such as jets) have to be taken into account.

Background-fluctuation splitting

- Background or average over many events is described by smooth fields

$$w_0 = \langle w \rangle$$
$$u_0^\mu = \langle u^\mu \rangle$$

- Fluctuations are added on top

$$w = w_0 + w_1$$
$$u^\mu = u_0^\mu + u_1^\mu$$

- For background one may assume Bjorken boost and azimuthal rotation invariance

$$w_0 = w_0(\tau, r)$$
$$u_0^\mu = (u_0^\tau(\tau, r), u_0^r(\tau, r), 0, 0)$$

Characterization of initial conditions

Characterization of transverse density 1

Fluctuations in initial transverse enthalpy density $w(r, \phi)$ can be characterized in terms of eccentricities $\epsilon_{n,m}$ and angles $\psi_{n,m}$ [Ollitrault, Teaney, Yan, Luzum, and others]

$$\epsilon_{n,m}\, e^{im\,\psi_{n,m}} = \frac{\int dr\, \int_0^{2\pi} d\varphi\, r^{n+1}\, e^{im\varphi}\, w(r,\varphi)}{\int dr\, \int_0^{2\pi} d\varphi\, r^{n+1}\, w(r,\varphi)}$$

- $w(r, \phi)$ completely determined by set of all $\epsilon_{n,m}$ and $\psi_{n,m}$
- closely related method is based on cumulants [Teaney, Yan]
- no positive transverse density can be associated to small set of cumulants (beyond Gaussian order) such that higher order cumulants vanish
- generalization to velocity and shear fluctuations not known

Characterization of transverse density 2

Characterization based on Bessel-Fourier expansion [Coleman-Smith, Petersen & Wolpert, 2012]

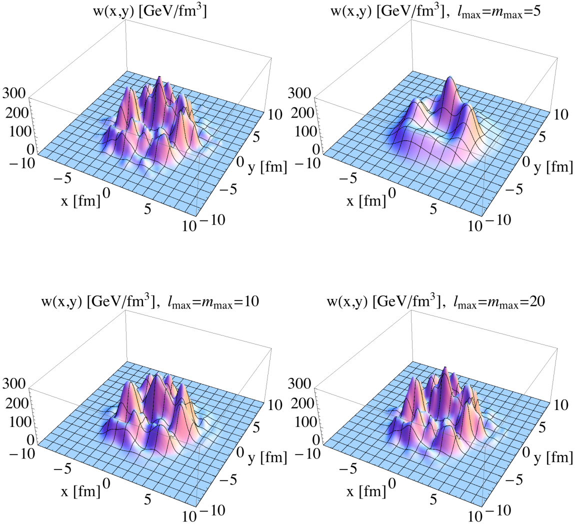
$$w(r,\phi) = \sum_{m,n} A_{m,n}\, e^{im\phi}\; J_m(\lambda_{m,n} \tfrac{r}{r_0})$$

Characterization based on Bessel-Fourier expansion and background density [Floerchinger & Wiedemann, 2013]

$$w(r,\phi) = w_{\text{BG}}(r) + w_{\text{BG}}(r) \sum_{m=-m_{\text{max}}}^{m_{\text{max}}} \sum_{l=1}^{l_{\text{max}}} \tilde{w}_l^{(m)}\, e^{im\phi}\; J_m(k_l^{(m)} r)$$

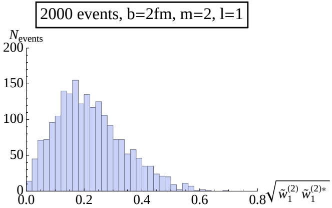
- $w(r,\phi)$ completely determined by set of all $\tilde{w}_l^{(m)}$
- higher l correspond to smaller spatial resolution
- single or few coefficients $\tilde{w}_l^{(m)}$ lead to *positive density*
- single modes can be propagated in hydro
- works similar for vectors (velocity) and tensors (shear stress)

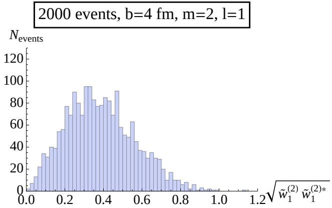
Transverse density from Glauber model

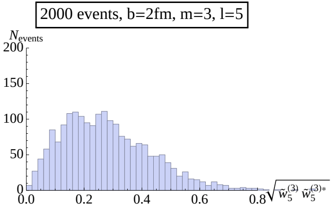


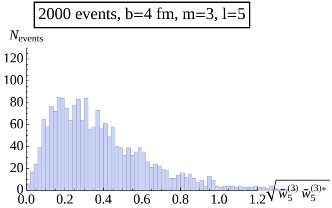
Distribution of weights

From Monte-Carlo Glauber model. Some examples:









Velocity fluctuation

- Initial velocity fluctuations at $\tau_0 \approx 0.5\,\mathrm{fm/c}$ are conceivable.
- Characterization similar as for density fluctuations. Two polarizations

$$u^r = u^r_{\mathsf{BG}} + \frac{1}{\sqrt{2}}(\tilde{u}^- + \tilde{u}^+)$$
$$u^\phi = \frac{i}{\sqrt{2}\,r}(\tilde{u}^- - \tilde{u}^+),$$

with

$$\tilde{u}^-(r,\phi) = \sum_{m,l} \tilde{u}_l^{-(m)}\, e^{im\phi}\, J_{m-1}\left(k_l^{(m)} r\right)$$
$$\tilde{u}^+(r,\phi) = \sum_{m,l} \tilde{u}_l^{+(m)}\, e^{im\phi}\, J_{m+1}\left(k_l^{(m)} r\right).$$

- Would be interesting to search for them in experimental data.

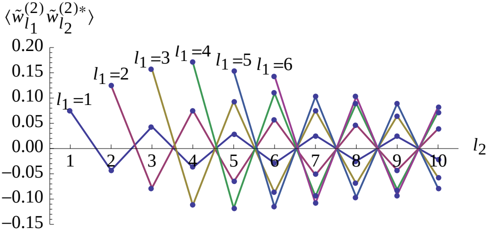
Event ensembles

- Event ensembles can be characterized in terms of functional probability distribution $p_{\tau_0}[w, u^\mu, \pi^{\mu\nu}, \ldots]$.
- Simplest case is Gaussian form

$$p_{\tau_0} \sim \exp\left[-\frac{1}{2} \sum_{m=-m_{\text{max}}}^{m_{\text{max}}} \sum_{l1,l2=1}^{l_{\text{max}}} T_{l_1 l_2}^{(m)} \, \tilde{w}_{l_1}^{(m)*} \tilde{w}_{l_2}^{(m)} \right]$$

- Fully determined by correlator

$$(T^{(m)})^{-1}_{l_1 l_2} = \langle \tilde{w}_{l_1}^{(m)} \tilde{w}_{l_2}^{(m)*} \rangle$$



Hydrodynamic evolution

Perturbative expansion

Write the hydrodynamic fields $h = (w, u^\mu, \pi^{\mu\nu}, \pi_{\text{Bulk}}, \ldots)$

- at initial time τ_0 as

$$h = h_0 + \epsilon\, h_1$$

  with h_0 the Background and $\epsilon\, h_1$ the fluctuation part

- at some later time $\tau > \tau_0$ as

$$h = h_0 + \epsilon\, h_1 + \epsilon^2 h_2 + \epsilon^3 h_3 + \ldots$$

Solve for time evolution in this scheme

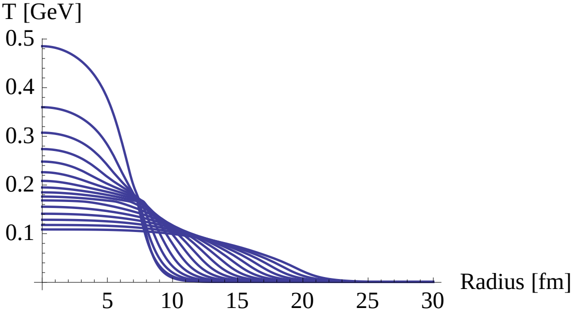
- h_0 is a solution of the full non-linear hydro equations but with higher symmetry: azimuthal rotation and Bjorken boost invariance
- h_1 is a solution of the linearized hydro equations around h_0, can be solved mode-by-mode
- h_2 can be obtained by from interactions between modes etc.

Background evolution

System of coupled $1+1$ dimensional non-linear partial differential equations for

- enthalpy density $w(\tau, r)$ or temperature $T(\tau, r)$
- fluid velocity $u^\tau(\tau, r), u^r(\tau, r)$
- two independent components of shear stress $\pi^{\mu\nu}(\tau, r)$

Can be easily solved numerically

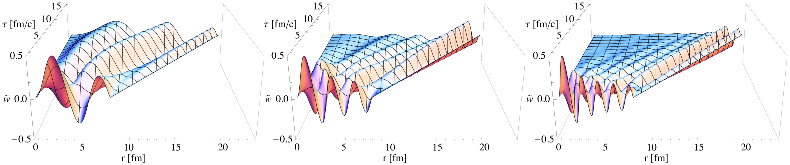


Evolving fluctuation modes

Linearized hydro equations are system of coupled $3+1$ dimensional, linear partial differential equations. Use Fourier expansion

$$h_j(\tau, r, \phi, \eta) = \sum_m \int \frac{dk_\eta}{2\pi}\, h_j^{(m)}(\tau, r, k_\eta)\, e^{i(m\phi + k_\eta \eta)}.$$

This gives $1+1$ dimensional linear partial differential equations that can be solved again numerically for initial conditions corresponding to each Bessel-Fourier mode.



Mode interactions

- Non-linear terms in the evolution equations for fluctuating fields lead to mode interaction terms of quadratic and higher order in the initial fluctuation fields.
- One can determine these terms from an iterative solution but that has not been fully worked out yet.
- The whole picture can be tested with complete numerical solution of the full hydro equations.

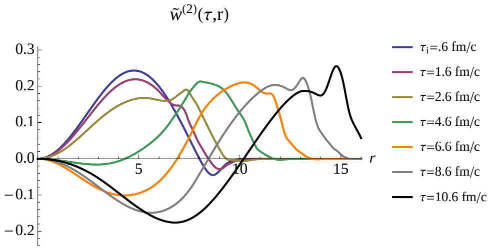
Scaling tests

- Start with single enthalpy density mode $(m = 2, l = 1)$ on top of background

$$w(\tau_0, r, \phi) = w_{\mathsf{BG}}(\tau_0, r)\left[1 + 2\,\tilde{w}_1^{(2)} J_2(k_1^{(2)} r)\,\cos(2\phi)\right].$$

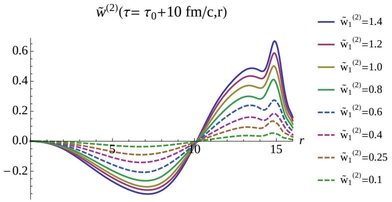
- Evolve this with hydro solver ECHO-QGP [Del Zanna *et al.*, EPJC 73, 2524 (2013)] and determine Fourier component

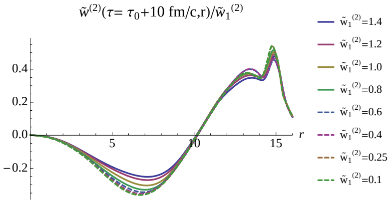
$$\tilde{w}^{(2)}(\tau, r) = w^{(2)}(\tau, r)/w_{\mathsf{BG}}(\tau, r).$$

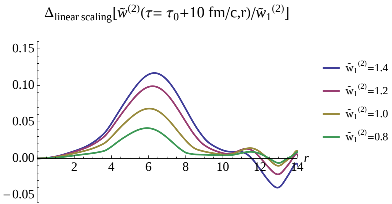


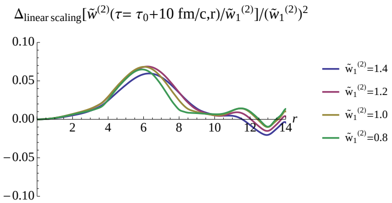
Scaling tests at first order

Compare enthalpy $\tilde{w}^{(2)}(\tau, r)$ at fixed τ for different initial weights





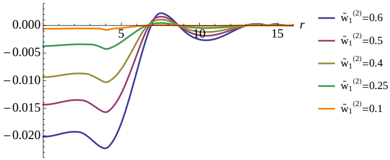




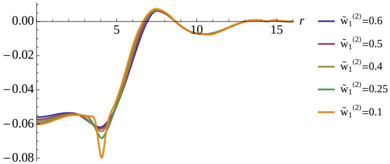
Scaling tests at second order

From symmetry considerations one expects that modes with $m = 0$ and $m = 4$ receive mainly quadratic contributions $\sim \left(\tilde{w}_1^{(2)}\right)^2$.

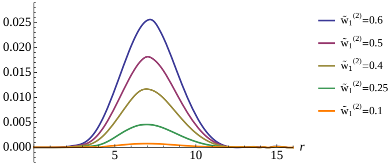
$w_{BG}(\tau,r)\tilde{w}^{(0)}(\tau,r)$ [GeV/fm^3], $\tau=\tau_0+10$ fm/c



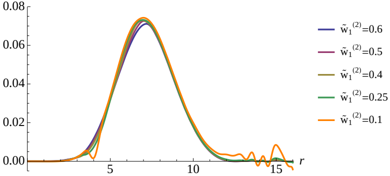
$w_{BG}(\tau,r)\tilde{w}^{(0)}(\tau,r)/(\tilde{w}_1^{(2)})^2$, $\tau=\tau_0+10$ fm/c



$w_{BG}(\tau,r)\tilde{w}^{(4)}(\tau,r)$ [GeV/fm^3], $\tau=\tau_0+10$ fm/c

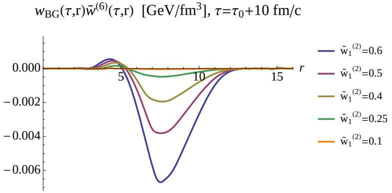


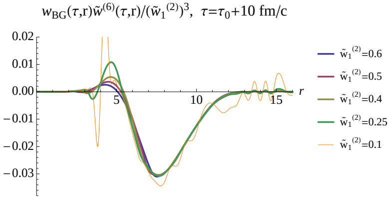
$w_{BG}(\tau,r)\tilde{w}^{(4)}(\tau,r)/(\tilde{w}_1^{(2)})^2$, $\tau=\tau_0+10$ fm/c



Scaling tests at third order

From symmetry considerations one expects that modes $m = 6$ receive mainly cubic contributions $\sim \left(\tilde{w}_1^{(2)}\right)^3$.



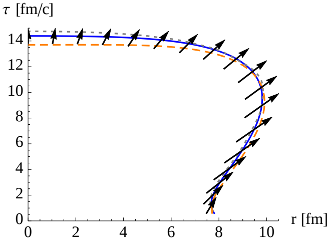


- Hydrodynamic response to initial enthalpy density perturbations perturbative.
- Non-linearities can be understood order-by-order and lead to characteristic "overtones".
- Results motivate a more thorough development of hydrodynamic perturbation theory.

Kinetic freeze-out

Freeze-out surface

- Background and fluctuations are propagated until $T_{\text{fo}} = 120\,\text{MeV}$ is reached.
- Distribution functions are determined and free streaming is assumed for later times [Cooper & Frye].
- Background-fluctuation splitting and expansion in powers of perturbations can be used also at freeze-out.
- Freeze-out surface is azimuthally symmetric as background [Floerchinger, Wiedemann 2013].
- Generalization to kinetic hadronic scattering and decay phase possible.



(solid: $\eta/s = 0.08$, dotted: $\eta/s = 0$, dashed: $\eta/s = 0.3$)

Contribution of modes to "single event spectrum"

Particle spectrum (or its logarithm) can be expanded in contribution from different modes. To linear order:

$$\ln\left(\frac{dN^{\text{single event}}}{p_T dp_T d\phi dy}\right) = \underbrace{\ln S_0(p_T)}_{\text{from background}} + \underbrace{\sum_{m,l} \tilde{w}_l^{(m)} e^{im\phi} \theta_l^{(m)}(p_T)}_{\text{from fluctuations}}$$

- Each mode has it's own angle $\tilde{w}_l^{(m)} = |\tilde{w}_l^{(m)}|\, e^{im\psi_l^{(m)}}$.
- p_T-dependence of different modes described by $\theta_l^{(m)}(p_T)$.
- To quadratic order this gets supplemented by

$$\sum_{m_1,m_2,l_1,l_2} \tilde{w}_{l_1}^{(m_1)} \tilde{w}_{l_2}^{(m_2)}\; e^{i(m_1+m_2)\phi}\; \kappa_{l_1,l_2}^{(m_1,m_2)}(p_T).$$

- The non-linearities encoded in $\kappa_{l_1,l_2}^{(m_1,m_2)}(p_T)$ arise both from hydro evolution and from kinetic freeze-out itself.

Harmonic flow coefficients

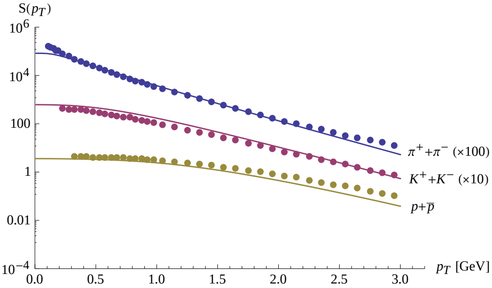
Double differential harmonic flow coefficient to lowest order

$$v_m^2\{2\}(p_T^a, p_T^b) = \sum_{l_1,l_2=1}^{l_{\text{max}}} \theta_{l_1}^{(m)}(p_T^a)\, \theta_{l_2}^{(m)}(p_T^b)\, \langle \tilde{w}_{l_1}^{(m)} \tilde{w}_{l_2}^{(m)*} \rangle$$

- intuitive matrix expression
- in general no factorization
- higher order corrections important for non-central collisions

One-particle spectrum

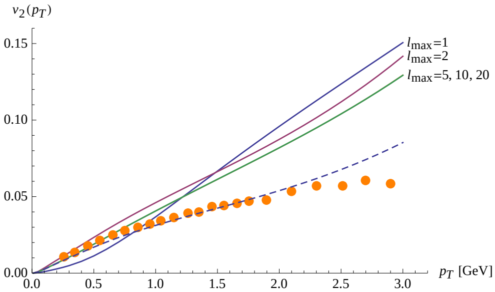
$$S(p_T) = dN/(2\pi p_T dp_T d\eta d\phi)$$



Points: 5% most central collisions, ALICE [PRL 109, 252301 (2012)]
Curves: Our calculation, no hadron rescattering and decays after freeze-out.

Harmonic flow coefficients for central collisions

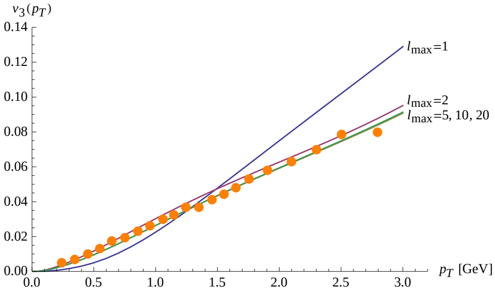
Elliptic flow for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Solid curves: Different maximal resolution $l_{\max}$
Dashed curve: Mode $(m = 2, l = 1)$ suppressed by factor 0.7

Harmonic flow coefficients for central collisions

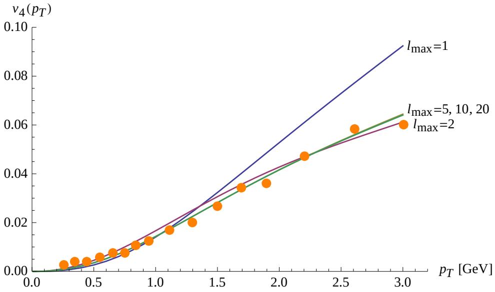
Triangular flow for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Curves: Different maximal resolution $l_{\max}$

Harmonic flow coefficients for central collisions

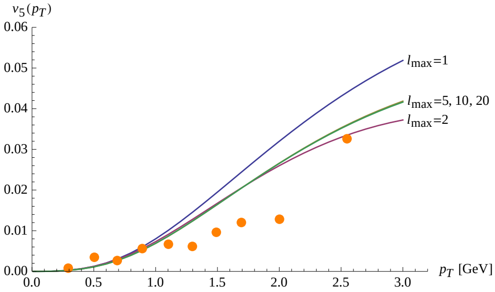
Flow coefficient v_4 for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Curves: Different maximal resolution l_{max}

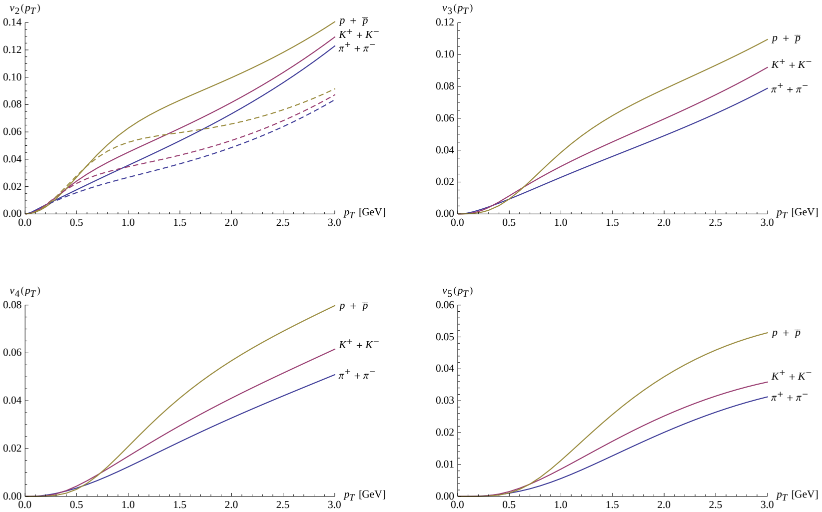
Harmonic flow coefficients for central collisions

Flow coefficient v_5 for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Curves: Different maximal resolution l_{max}

Harmonic flow coefficients, central, particle identified



Summary and Conclusions

Conclusions

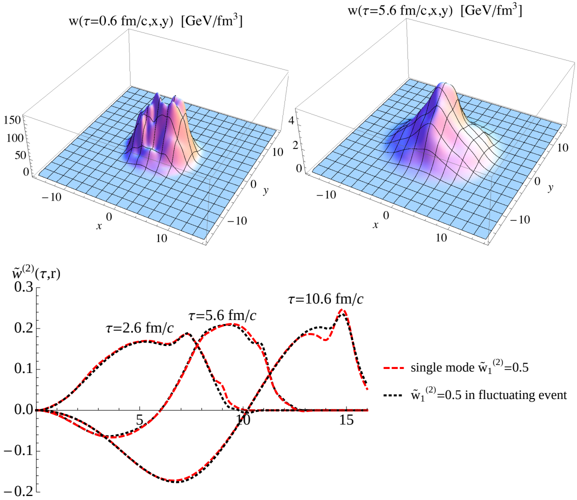
- Mode-by-mode perturbative hydrodynamic picture allows to determine response to initial density perturbations.
- Hydrodynamic evolution can be disentangled from initial state model.
- First study for enthalpy density fluctuations in Glauber model
  - yields good description of $v_m(p_T)$ for central collisions,
  - shows that fluctuations up to $l_{\text{max}} \approx 5$ can be resolved.
- Fluctuations to be studied:

	transverse plane	rapidity direction
enthalpy density / entropy	✓	-
fluid velocity	-	-
shear stress	-	-
baryon number density	-	-
electromagnetic fields	-	-
electric charge density	-	-
chiral order parameter	-	-

Backup

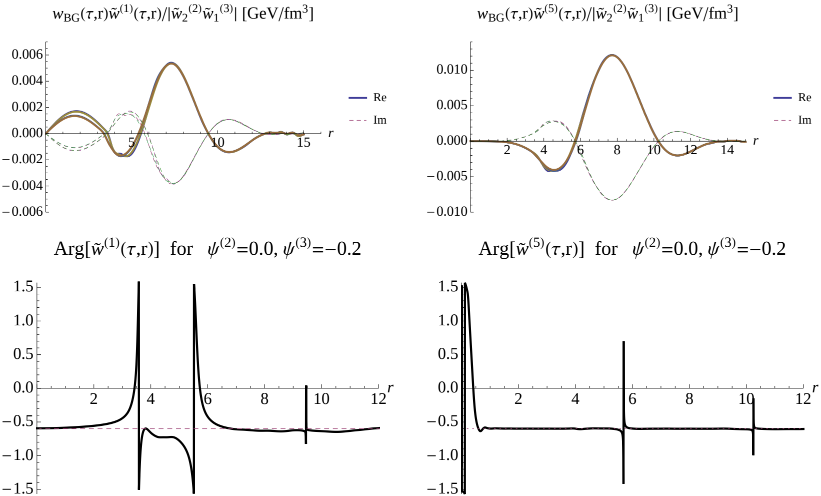
Scaling tests embedded in realistic event

Embed mode $(m = 2, l = 1)$ into realistic fluctuating event and compare to embedding into pure background.



Scaling tests with several initial modes

Start with linear combination of $(m = 2, l = 2)$ and $(m = 3, l = 1)$ modes and test scaling for $m = 1$ and $m = 5$ response.



Generalized Glauber model

- Fluctuations due to nucleon positions: used so far

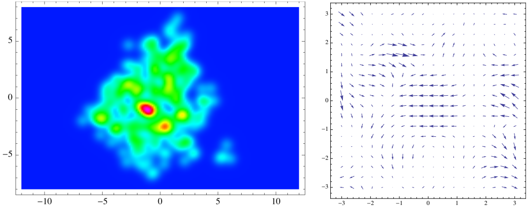
$$\epsilon(\tau, \mathbf{x}, y) = \sum_{i=1}^{N_{\text{part}}} \epsilon_w(\tau, \mathbf{x} - \mathbf{x}_i, y), \qquad u^\mu = (1, 0, 0, 0)$$

- can be generalized to include also velocity fluctuations

$$T^{\mu\nu}(\tau, \mathbf{x}, y) = \sum_{i=1}^{N_{\text{part}}} T_w^{\mu\nu}(\tau, \mathbf{x} - \mathbf{x}_i, y)$$

- More generally describe primordial fluid fields by
  - expectation values $\langle \epsilon(\tau_0, \mathbf{x}, y) \rangle, \langle u^\mu(\tau_0, \mathbf{x}, y) \rangle, \langle n_B(\tau_0, \mathbf{x}, y) \rangle$
  - correlation functions $\langle \epsilon(\tau_0, \mathbf{x}, y)\, \epsilon(\tau_0, \mathbf{x}', y') \rangle$, etc.
- Origin of this fluctuations is initial state physics and early-time, non-equilibrium dynamics.

Velocity fluctuations



- also the velocity field will fluctuate at the initialization time τ_0
- take here transverse velocity for every participant to be Gaussian distributed with width $0.1c$
- vorticity $|\partial_1 u^2 - \partial_2 u^1|$ and divergence $|\partial_1 u^1 + \partial_2 u^2|$