Mode-by-mode hydrodynamics for heavy ion collisions

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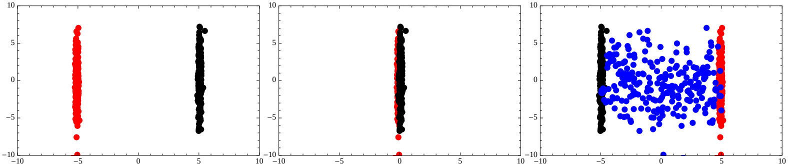
based on work with Urs A. Wiedemann

- Mode-by-mode fluid dynamics for relativistic heavy ion collisions, arXiv:1307.3453.
- Characterization of initial fluctuations for the hydrodynamical description of heavy ion collisions, arXiv:1307.7611.
- Fluctuations around Bjorken Flow and the onset of turbulent phenomena, JHEP 11, 100 (2011).

Heavy Ion Collisions



- ions are strongly Lorentz-contracted
- *some* medium is produced after collision
- medium expands in longitudinal direction and gets diluted

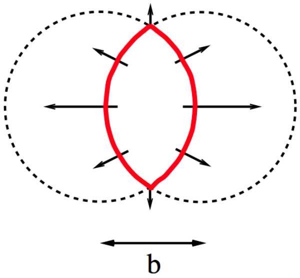
Evolution in time

- Non-equilibrium evolution at early times
  - initial state at from QCD? Color Glass Condensate? ...
  - thermalization via strong interactions, plasma instabilities, particle production, ...
- Local thermal and chemical equilibrium
  - strong interactions lead to short thermalization times
  - evolution from relativistic fluid dynamics
  - expansion, dilution, cool-down
- Chemical freeze-out
  - for small temperatures one has mesons and baryons
  - inelastic collision rates become small
  - particle species do not change any more
- Thermal freeze-out
  - elastic collision rates become small
  - particles stop interacting
  - particle momenta do not change any more

Fluid dynamic regime

- assumes strong interaction effects leading to local equilibrium
- fluid dynamic variables
  - thermodynamic variables: e.g. $T(x)$, $\mu(x)$
  - fluid velocity $u^\mu(x)$
- can be formulated as derivative expansion for $T^{\mu\nu}$
- hydrodynamics is universal: many details of microscopic theory not important.
- some macroscopic properties are important:
  - ideal hydro: needs equation of state $p = p(T, \mu)$ from thermodynamics
  - first order hydro: needs also transport coefficients like viscosity $\eta = \eta(T, \mu)$ from linear response theory
  - second order hydro: needs also relaxation times

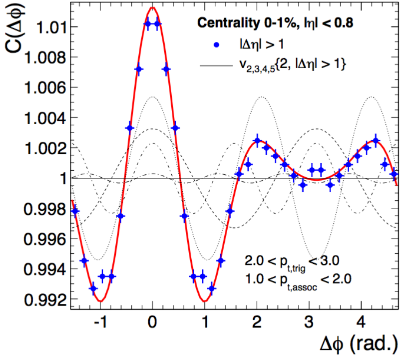
Elliptic flow v_2



- non-central collisions lead to deviations from rotation symmetry
- pressure gradients larger in one direction
- larger fluid velocity in this direction
- more particles will fly in this direction
- can be quantified in terms of elliptic flow v_2

$$C(\Delta\phi) \sim 1 + 2\, v_2\; \cos(2\,\Delta\phi)$$

A puzzle: v_3 and v_5



(ALICE 2011, similar pictures also from CMS, ATLAS, Phenix)

- quite generally, one can expand

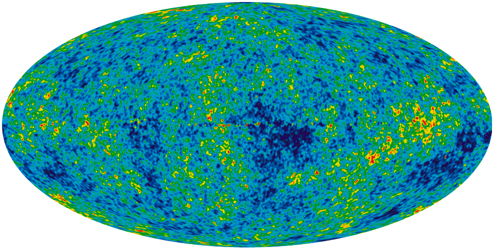
$$C(\Delta\phi) \sim 1 + \sum_{n=2}^{\infty} 2\, v_n\, \cos(n\,\Delta\phi)$$

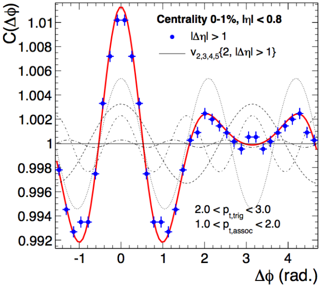
- from symmetry reasons one expects naively $v_3 = v_5 = \ldots = 0$

Why are fluctuations interesting?

- **Hydrodynamic fluctuations**: Local and event-by-event perturbations around the average of hydrodynamical fields:
  - energy density ϵ
  - fluid velocity u^μ
  - shear stress $\pi^{\mu\nu}$
  - more general also: baryon number density n_B, ...
- measure for deviations from equilibrium
- contain interesting information from early times
- an be used to constrain thermodynamic and transport properties

Similarities to cosmic microwave background





- fluctuation spectrum contains info from early times
- many numbers can be measured and compared to theory
- detailed understanding of evolution needed
- could trigger precision era in heavy ion physics

A complete story about fluctuations

1. Initial fluctuations at initialization time of hydro should be characterized and quantified completely.
2. Fluctuations have to be propagated through the hydrodynamical regime.
3. Contribution of fluctuations to the particle spectra at freeze-out must be understood and quantified.
4. Fluctuations generated from non-hydro sources (such as jets) have to be taken into account.

Characterization of single events

Fluctuations in initial transverse enthalpy density $w(\tau, r)$

- Traditional characterization based on eccentricities

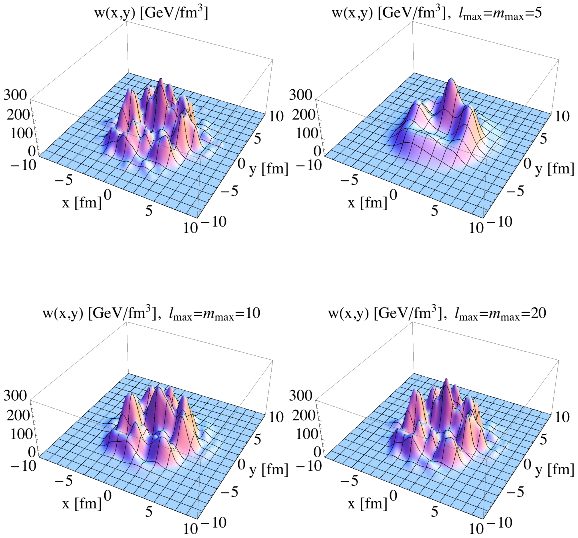
$$\epsilon_{n,m} = \frac{\int dr\ \int_0^{2\pi} d\varphi\, r^{n+1}\, e^{im\varphi}\, w(r,\varphi)}{\int dr\ \int_0^{2\pi} d\varphi\, r^{n+1}\, w(r,\varphi)}$$

  - resolves radial dependence only poorly
  - "inverse transform" ill defined
- More differential way based on Bessel functions (S.F. and U. A. Wiedemann, 2013)

$$w(r,\varphi) = w_{\mathsf{BG}}(r) + w_{\mathsf{BG}}(r) \sum_{m=-m_{\max}}^{m_{\max}} \sum_{l=1}^{l_{\max}} \tilde{w}_l^{(m)}\, e^{im\varphi}\ J_m(k_l^{(m)} r)$$

  - higher l correspond to smaller spatial resolution
  - can be inverted
  - single modes can be propagated
  - generalizable to vectors (velocity) and tensors (shear stress)

Transverse density from Glauber model



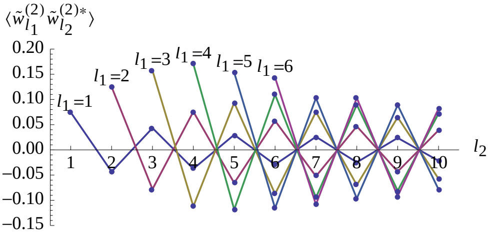
Event ensembles

- Event ensembles can be characterized in terms of probability distribution $p_{\tau_0}[w, u^\mu, \pi^{\mu\nu}, \ldots]$.
- Simplest case is Gaussian form

$$p_{\tau_0} = \frac{1}{\mathcal{N}} \exp\left[-\frac{1}{2} \sum_{m=-m_{\text{max}}}^{m_{\text{max}}} \sum_{l1,l2=1}^{l_{\text{max}}} T^{(m)}_{l_1 l_2}\ \tilde{w}^{(m)*}_{l_1} \tilde{w}^{(m)}_{l_2} \right]$$

- Fully determined by correlator

$$(T^{(m)})^{-1}_{l_1 l_2} = \langle \tilde{w}^{(m)}_{l_1} \tilde{w}^{(m)*}_{l_2} \rangle$$



Background-fluctuation splitting

- Background or average over many events is described by smooth fields

$$w_{\mathsf{BG}} = \langle w \rangle$$
$$u^\mu_{\mathsf{BG}} = \langle u^\mu \rangle$$

- Fluctuations are added on top

$$w = w_{\mathsf{BG}} + \delta w$$
$$u^\mu = u^\mu_{\mathsf{BG}} + \delta u^\mu$$

- For background one can assume Bjorken boost and azimuthal rotation invariance

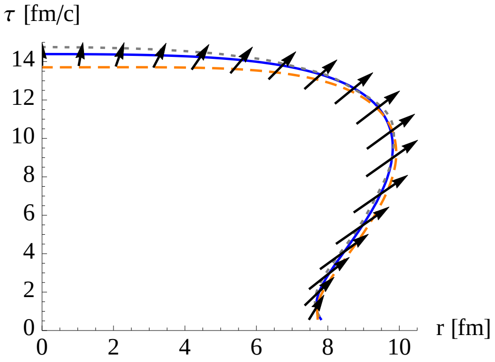
$$w_{\mathsf{BG}} = w_{\mathsf{BG}}(\tau, r)$$
$$u^\mu_{\mathsf{BG}} = (u^\tau_{\mathsf{BG}}, u^r_{\mathsf{BG}}, 0, 0)$$

Evolving fluctuations

...

Freeze-out surface

Background and fluctuations are propagated until $T_{\mathrm{fo}} = 120\,\mathrm{MeV}$ is reached.

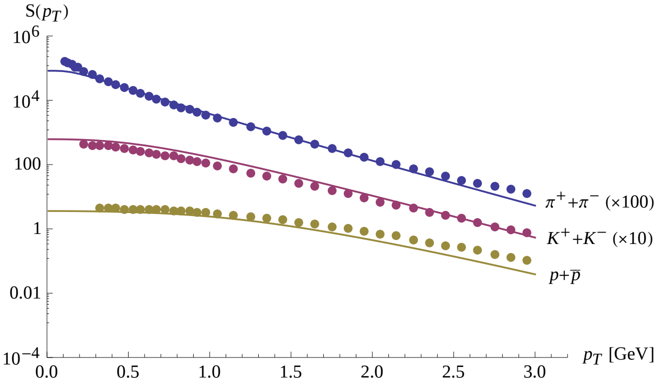


(solid: $\eta/s = 0.08$, dotted: $\eta/s = 0$, dashed: $\eta/s = 0.3$)

Distribution functions are determined and free streaming is assumed for later times (Cooper-Frye freeze out).

One-particle spectrum

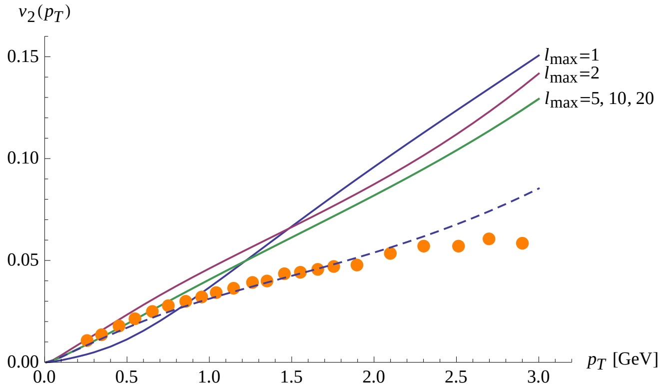
$$S(p_T) = dN/(2\pi p_T dp_T d\eta d\phi)$$



Points: 5% most central collisions, ALICE, PRL **109**, 252301 (2012), see also Talk by Ortiz Velasquez on 04/09.
Curves: Our calculation, no hadron rescattering and decays after freeze-out.

Harmonic flow coefficients for central collisions

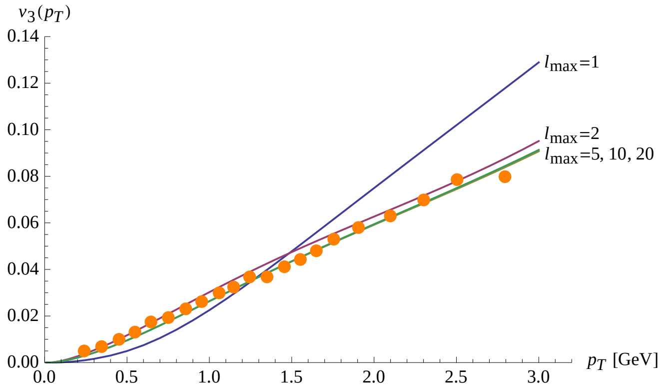
Elliptic flow for charged particles



Points: 2% most central collisions, ALICE, PRL **107**, 032301 (2011),
Solid curves: Different maximal resolution l_{max}
Dashed curve: Mode $(m = 2, l = 1)$ suppressed by factor 0.7

Harmonic flow coefficients for central collisions

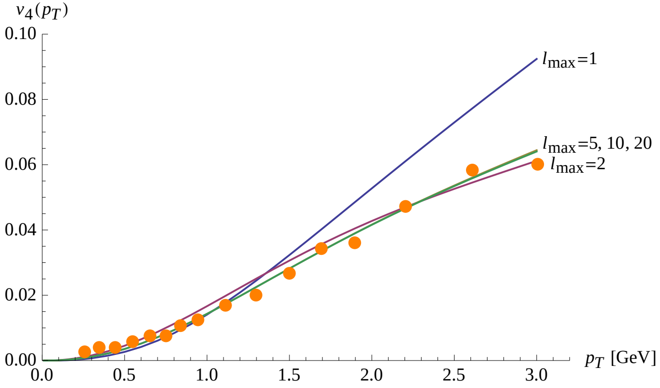
Triangular flow for charged particles



Points: 2% most central collisions, ALICE, PRL **107**, 032301 (2011),
Curves: Different maximal resolution l_{max}

Harmonic flow coefficients for central collisions

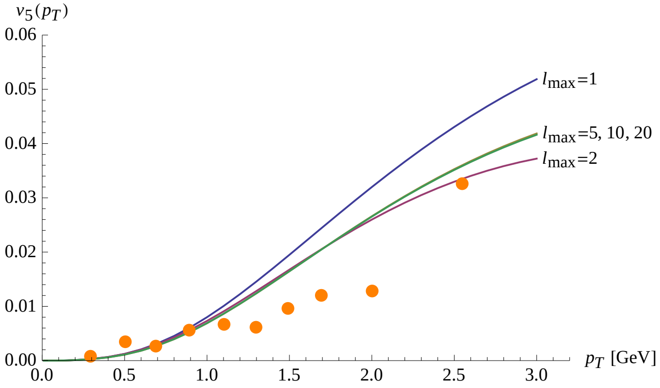
Flow coefficient v_4 for charged particles



Points: 2% most central collisions, ALICE, PRL **107**, 032301 (2011),
Curves: Different maximal resolution l_{max}

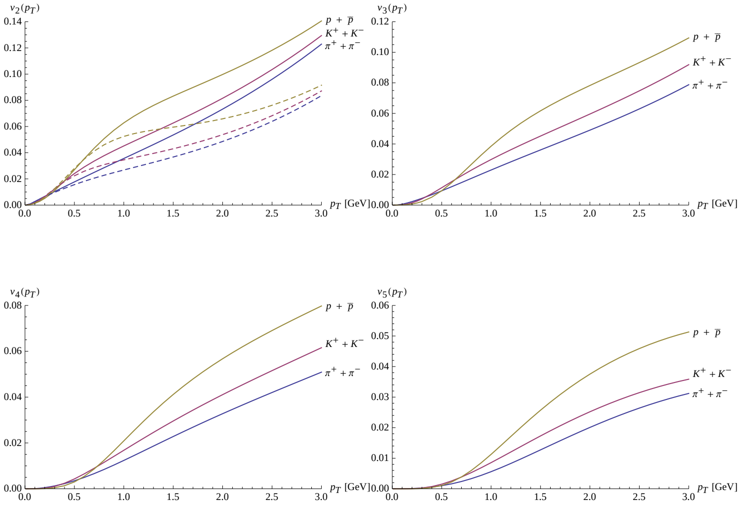
Harmonic flow coefficients for central collisions

Flow coefficient v_5 for charged particles



Points: 2% most central collisions, ALICE, PRL **107**, 032301 (2011),
Curves: Different maximal resolution l_{max}

Harmonic flow coefficients, central, particle identified



Conclusions

- Method to characterize and propagate initial fluctuations in hydrodynamical fields has been developed
- First study for enthalpy density fluctuations in Glauber model
  - yields good description of $v_m(p_T)$ for central collisions
  - shows that fluctuations up to $l_{\text{max}} \approx 5$ can be resolved
- Fluctuations to be studied:

	transverse plane	rapidity direction
enthalpy density / entropy	✓	-
fluid velocity	-	-
shear stress	-	-
baryon number density	-	-
electromagnetic fields	-	-
electric charge density	-	-
chiral order parameter	-	-