

*Ultracold fermions with three components:  
Cooper-pairs, molecules and trions*

Stefan Flörchinger (CERN)

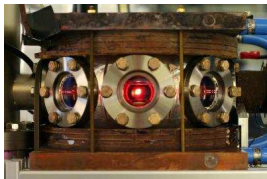
work done in Heidelberg in collaboration with

Richard Schmidt, Sergej Moroz and Christof Wetterich

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## *Cold atoms in a laser trap*



- typical density
  - particle number  $N = 10^6$
  - cloud volume  $V = 10^{-9} \text{ cm}^3$
  - interparticle distance  $d = 0.1 \text{ } \mu\text{m}$
- typical temperature
  - temperature  $T = 10^{-6} \text{ K}$
  - thermal de-Broglie length  $\lambda_T = 1 \text{ } \mu\text{m}$
- typical interaction parameters
  - interaction range  $\lambda_{\text{vdW}} = 10^{-4} \text{ } \mu\text{m}$
  - scattering length  $a = (0 \dots \infty) \text{ } \mu\text{m}$

## *Theoretical challenge*

- quantum effects are important
- many particles / nonzero density
- nonzero temperature
- large interaction strength
- possibly non-equilibrium dynamics
- similar problems as in QCD matter: Heavy ion collisions, Neutron stars, ...
- advantage for cold quantum gases: very well controlled, experiments on a table-top

## *Complexity problem with strong interaction*

- Strong interactions lead to strong effects. Qualitative features of a theory can change!
- Physical properties can become universal! Microscopic details become irrelevant.
- Strong interaction effects lead to fast Equilibration: Dynamics can be described by Close-to-Equilibrium methods.

## *Fermi gases with different physics*

- 1 component Fermi gas - no s-wave interaction
- 2 component Fermi gas - BCS-BEC crossover
- 3 component Fermi gas - ??

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(Efimov, Phys. Lett. **33B**, 563 (1970),

Review: Braaten and Hammer, Phys. Rep. **428**, 259 (2006))

# *Fermi gases with different physics*

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- 3 component Fermi gas - ??
  - Three-body problem: Efimov effect  
(Efimov, Phys. Lett. **33B**, 563 (1970),  
Review: Braaten and Hammer, Phys. Rep. **428**, 259 (2006))
  - On the lattice: Trion formation  
(Rapp, Zarand, Honerkamp, and Hofstetter, PRL **98**, 160405 (2007),  
Rapp, Hofstetter and Zarand, PRB **77**, 144520 (2008).)

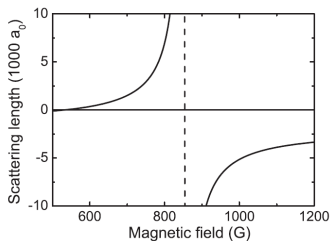
## *Single component Fermi gas*

- Most properties of dilute ultracold quantum gases are dominated by  $s$ -wave interactions.
- For identical fermions (only one spin component) wavefunction has to be antisymmetric in position space.
- $s$ -wave interaction suppressed by Pauli blocking.
- Behaves like ideal Fermi gas in many respects.

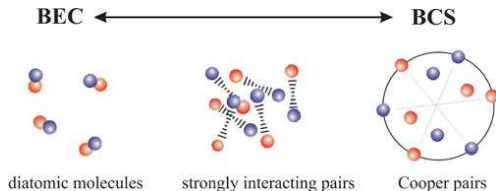


## Two component Fermi gas

- Two spin (or hyperfine-spin) components  $\psi_1$  and  $\psi_2$ .
- For equal mass  $M_{\psi_1} = M_{\psi_2}$ , density  $n_{\psi_1} = n_{\psi_2}$  etc. SU(2) spin symmetry
- $s$ -wave interaction measured by scattering length  $a$ .
- Repulsive microscopic interaction: Landau Fermi liquid.
- Attractive interaction leads to many interesting effects!
- Scattering length can be tuned experimentally with Feshbach resonances.



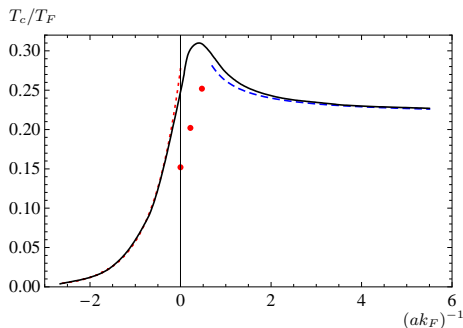
# BCS-BEC Crossover



- Small negative scattering length  $a \rightarrow 0_-$ 
  - Formation of Cooper pairs in momentum space
  - BCS-theory valid
  - superfluid at small temperatures
  - order parameter  $\varphi \sim \psi_1\psi_2$
- Small positive scattering length  $a \rightarrow 0_+$ 
  - Formation of dimers or molecules in position space
  - Bosonic mean field theory valid
  - superfluid at small temperatures
  - order parameter  $\varphi \sim \psi_1\psi_2$
- Between both limits: Continuous *BCS-BEC Crossover*
  - scattering length becomes large: strong interaction
  - superfluid, order parameter  $\varphi \sim \psi_1\psi_2$  at small  $T$

## Phase diagram BCS-BEC Crossover

- Crossover best parameterized by  $c^{-1} = (ak_F)^{-1}$ .
- Different methods give phase diagram
- Result of renormalization group study:



(Floerchinger, Scherer and Wetterich, PRA **81**, 063619 (2010).)

- More complicated phase diagram with population imbalance

## Three component Fermi gas

- For equal masses, densities etc. global SU(3) symmetry

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \rightarrow u \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \quad u \in \text{SU}(3).$$

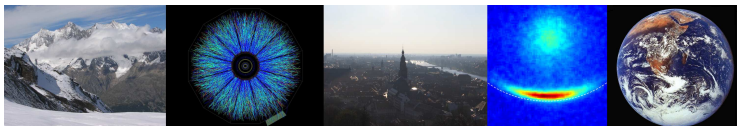
Similar to flavor symmetry in the Standard model!

- For small scattering length  $|a| \rightarrow 0$ 
  - BCS ( $a < 0$ ) or BEC ( $a > 0$ ) superfluidity at small T.
  - order parameter is conjugate triplet  $\bar{\mathbf{3}}$  under SU(3)

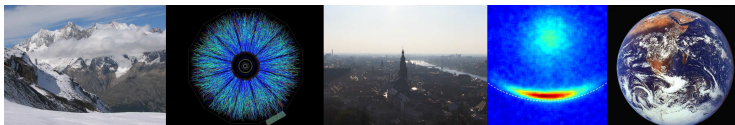
$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \sim \begin{pmatrix} \psi_2 \psi_3 \\ \psi_3 \psi_1 \\ \psi_1 \psi_2 \end{pmatrix}.$$

- SU(3) symmetry is broken spontaneously for  $\varphi \neq 0$ .
- What happens for large  $|a|$ ?

*How should we describe the world?*

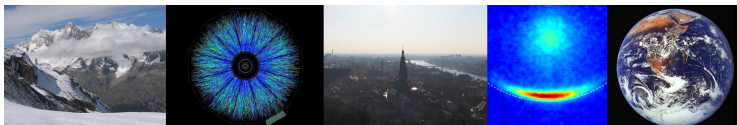


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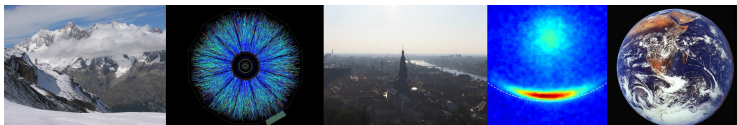
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- We look for a unified description.

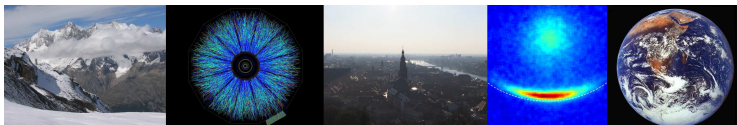
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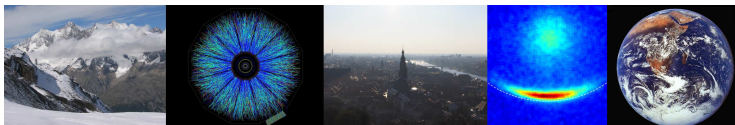


# *How should we describe the world?*



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- Einstein: *Everything should be made as simple as possible.*

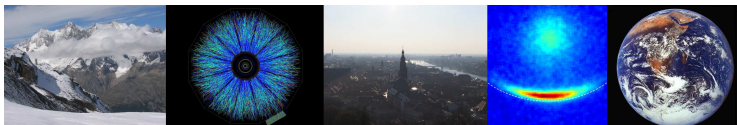
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FIELD THEORY.

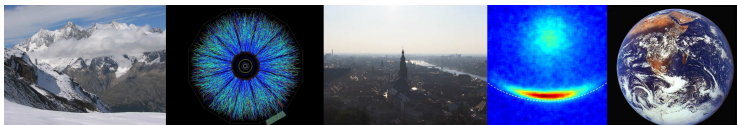
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QUANTUM FIELD THEORY.

## *Classical field theory*

- Describes electro-magnetic fields, waves, ... ( $\hbar \rightarrow 0$ ).
- Crucial object: classical action

$$S[\phi] = \int dt \int d^d x \mathcal{L}(\phi, \partial_t \phi, \vec{\nabla} \phi, \dots)$$

- Classical field equations from  $\frac{\delta S}{\delta \phi} = 0$ .
- Symmetries of  $S$  lead to conserved currents.
- All physical observables are easily obtained from  $S$ .

# Quantum field theory

- Describes electrons, atoms, quarks, gluons, protons,...  
...and cold quantum gases
- Crucial object: quantum effective action

$$\Gamma[\phi] = \int dt \int d^d x U(\phi) + \dots$$

- Quantum field equations from  $\frac{\delta \Gamma}{\delta \phi} = 0$
- Symmetries of  $\Gamma$  lead to conserved currents
- All physical observables are easily obtained from  $\Gamma$
- $\Gamma$  is generating functional of 1-PI Feynman diagrams and depends on external parameters like  $T, \mu$ , or  $\vec{B}$

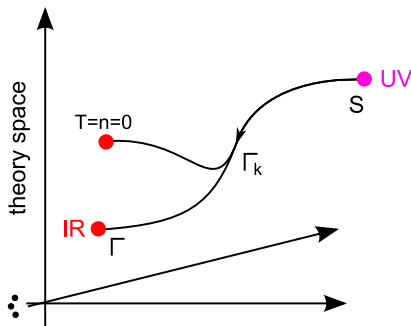
# *The renormalization group*

- very important in modern understanding of quantum field theory
- describes how (effective) theories evolve to other (effective) theories at smaller energy/momentum scales
- makes a simple, efficient and intuitive description of complex phenomena possible

# How do we obtain the quantum effective action $\Gamma[\phi]$ ?

Idea of functional renormalization:  $\Gamma[\phi] \rightarrow \Gamma_k[\phi]$

- $k$  is additional infrared cutoff parameter.
- $\Gamma_k[\phi] \rightarrow \Gamma[\phi]$  for  $k \rightarrow 0$ .
- $\Gamma_k[\phi] \rightarrow S[\phi]$  for  $k \rightarrow \infty$ .
- Dependence on  $T, \mu$  or  $\vec{B}$  trivial for  $k \rightarrow \infty$ .





# *How the flowing action flows*

Simple and exact flow equation (WETTERICH 1993)

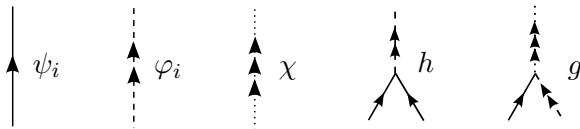
$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \text{STr} \left( \Gamma_k^{(2)}[\phi] + R_k \right)^{-1} \partial_k R_k.$$

- Differential equation for a functional.
- For most cases not solvable exactly.
- Approximate solutions can be found from Truncations.
  - Ansatz for  $\Gamma_k$  with a finite number of parameters.
  - Derive ordinary differential equations for this parameters or couplings from the flow equation for  $\Gamma_k$ .
  - Solve these equations numerically.

# Simple truncation for fermions with three components

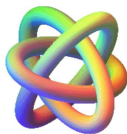
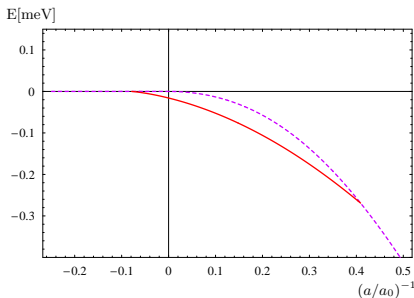
$$\begin{aligned}\Gamma_k = & \int_x \psi^\dagger (\partial_\tau - \vec{\nabla}^2 - \mu) \psi + \varphi^\dagger (\partial_\tau - \frac{1}{2} \vec{\nabla}^2 + m_\varphi^2) \varphi \\ & + \chi^* (\partial_\tau - \frac{1}{3} \vec{\nabla}^2 + m_\chi^2) \chi \\ & + h \epsilon_{ijk} (\varphi_i^* \psi_j \psi_k + h.c.) + g (\varphi_i \psi_i^* \chi + h.c.).\end{aligned}$$

- Units are such that  $\hbar = k_B = 2M = 1$
- Wavefunction renormalization for  $\psi$ ,  $\varphi$  and  $\chi$  is implicit.
- $\Gamma_k$  contains terms for
  - fermion field  $\psi = (\psi_1, \psi_2, \psi_3)$
  - bosonic field  $\varphi = (\varphi_1, \varphi_2, \varphi_3) \sim (\psi_2 \psi_3, \psi_3 \psi_1, \psi_1 \psi_2)$
  - trion field  $\chi \sim \psi_1 \psi_2 \psi_3$



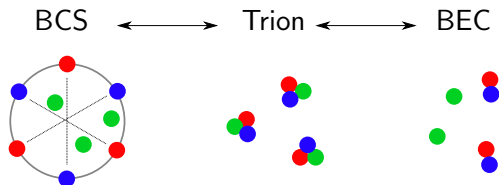
# Binding energies

- Vacuum limit  $T \rightarrow 0$ ,  $n \rightarrow 0$ .



- Binding energy per atom for
  - molecule or dimer  $\varphi$  (dashed line)
  - trion or trimer  $\chi$  (solid line)
- For large scattering length  $a$  trion is energetically favorable!
- Three-body bound state even for  $a < 0$ .

# Quantum phase diagram

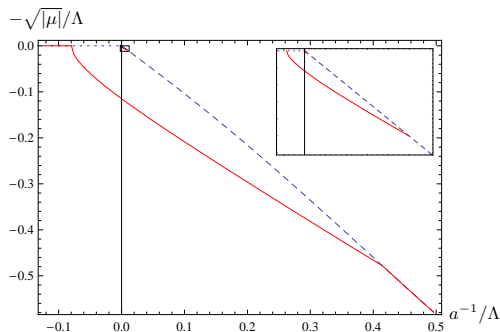


- BCS-Trion-BEC transition

(Floerchinger, Schmidt, Moroz and Wetterich, PRA **79**, 013603 (2009)).

- $a \rightarrow 0_-$ : Cooper pairs,  $SU(3) \times U(1) \rightarrow SU(2) \times U(1)$ .
  - $a \rightarrow 0_+$ : BEC of molecules,  $SU(3) \times U(1) \rightarrow SU(2) \times U(1)$ .
  - $a \rightarrow \pm\infty$ : Trion phase,  $SU(3)$  unbroken.
- Quantum phase transitions
  - from BCS to Trion phase
  - from Trion to BEC phase.

# Efimov effect



- Self-similarity in energy spectrum.
- Efimov trimers become more and more shallow. At  $a = \infty$

$$E_{n+1} = e^{-2\pi/s_0} E_n.$$

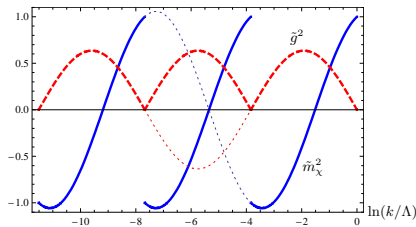
- Simple truncation:  $s_0 \approx 0.82$ .
- Advanced truncation:  $s_0 \approx 1.006$  (exact result)  
(Moroz, Floerchinger, Schmidt and Wetterich, PRA **79**, 042705 (2009).)

## Renormalization group limit cycle

- For  $\mu = 0$  and  $a^{-1} = 0$  flow equations for rescaled couplings

$$k \frac{\partial}{\partial k} \begin{pmatrix} \tilde{g}^2 \\ \tilde{m}_\chi^2 \end{pmatrix} = \begin{pmatrix} 7/25 & -13/25 \\ 36/25 & 7/25 \end{pmatrix} \begin{pmatrix} \tilde{g}^2 \\ \tilde{m}_\chi^2 \end{pmatrix}.$$

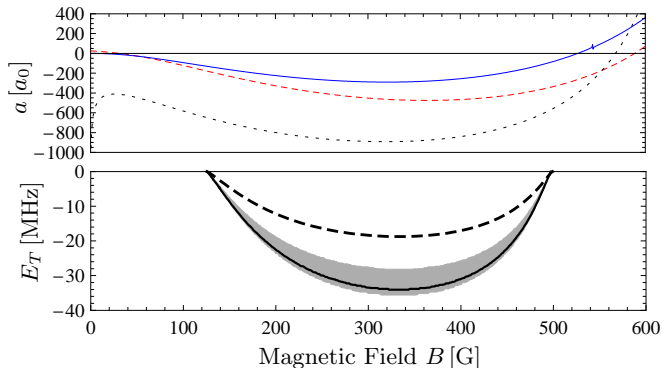
- Solution is log-periodic in scale.



- Every zero-crossing of  $\tilde{m}_\chi^2$  corresponds to a new bound state.
- For  $\mu \neq 0$  or  $a^{-1} \neq 0$  limit cycle scaling stops at some scale  $k$ . Only finite number of Efimov trimers.

## Contact to experiments

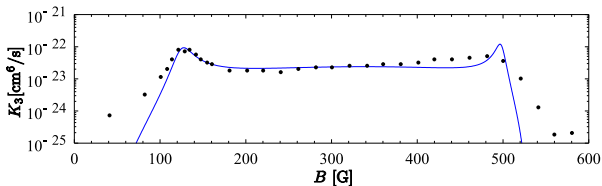
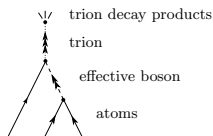
- Model can be generalized to case without SU(3) symmetry (Floerchinger, Schmidt and Wetterich, PRA A **79**, 053633 (2009)).
- Hyperfine states of  $^6\text{Li}$  have large scattering lengths.



- Binding energies might be measured using RF-spectroscopy.
- Lifetime is quite short  $\sim 10\text{ns}$ .

## Three-body loss rate

- Three-body loss rate measured experimentally (Ottenstein et al., PRL **101**, 203202 (2008); Huckans et al., PRL **102**, 165302 (2009))



- Trion may decay into deeper bound molecule states
- Calculate  $B$ -field dependence of loss process above.
- Left resonance (position and width) fixes model parameters.
- Form of curve for large  $B$  is prediction.
- Similar results obtained by other methods  
(Braaten, Hammer, Kang and Platter, PRL **103**, 073202 (2009);  
Naidon and Ueda, PRL **103**, 073203 (2009).)

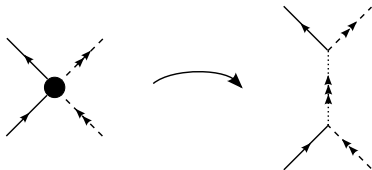


# Conclusions

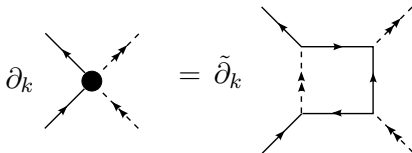
- Physics of ultracold fermions with three components quite interesting.
- Few-body physics (Efimov effect) well described by functional renormalization.
- Many-body physics shows parallels to QCD
  - BCS – “Color” – superfluidity for small negative  $a$ .
  - Trion – “Hadron” – phase for large  $|a|$ .
  - BEC – “Color” – superfluidity for small positive  $a$ .
- Very nice playground for renormalization group methods.
- Experimental tests seem possible.

## *“Refermionization”*

- Trion field is introduced via a generalized Hubbard-Stratonovich transformation



- Fermion-boson coupling is regenerated by the flow



- Express this again by trion exchange  
(Gies and Wetterich, PRD **65**, 065001 (2002),  
Floerchinger and Wetterich, PLB **680**, 371 (2009).)

# Truncations

For many purposes *derivative expansions* are suitable approximations. For example we use for the BCS-BEC Crossover

$$\Gamma_k = \int_{\tau, \vec{x}} \left\{ \psi^\dagger (\partial_\tau - \vec{\nabla}^2 - \mu) \psi + \varphi^* (Z_\varphi \partial_\tau - A_\varphi \frac{1}{2} \vec{\nabla}^2) \varphi \right. \\ \left. - h(\varphi^* \psi_1 \psi_2 + h.c.) + \frac{1}{2} \lambda_\psi (\psi^\dagger \psi)^2 + U_k(\varphi^* \varphi, \mu) \right\}$$

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- The coefficients  $Z_\varphi$ ,  $A_\varphi$ ,  $\lambda_\psi$ ,  $h$  and the effective potential  $U_k$  are scale-dependent.

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- The effective potential  $U_k$  contains no derivatives - describes homogeneous fields.
- Wave-function renormalization and self-energy corrections for fermions can be included as well.

## *The effective potential*

- We use a Taylor expansion around the minimum  $\rho_0$

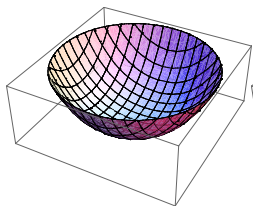
$$U_k(\varphi^* \varphi) = -p + m^2 (\varphi^* \varphi - \rho_0) + \frac{1}{2} \lambda (\varphi^* \varphi - \rho_0)^2.$$

## The effective potential

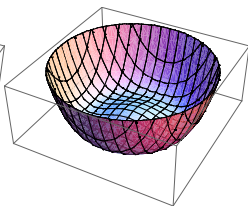
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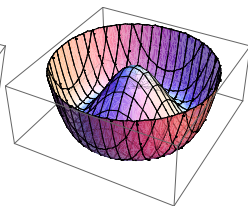
- Symmetry breaking:



$$T > T_c$$



$$T < T_c$$



$$T \ll T_c$$



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- Typical flow:

