Ultracold quantum gases and functional renormalization

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ERG 2010, Corfu, 18/09/2010

Collaborations

- Based on work done in Heidelberg in collaboration with: S. Diehl, H. Gies, S. Moroz, J. M. Pawlowski, M. M. Scherer, R. Schmidt, C. Wetterich
- Related work was done in
  - Frankfurt: L. Bartosch, A. Ferraz, S. Ledowski, N. Hasselmann and P. Kopietz
  - Manchester: M. C. Birse, B. Krippa, J. A. McGovern and N. R. Walet
  - Paris: N. Dupuis and K. Sengupta

Outline

Introduction

Why are Ultracold quantum gases interesting?

- Ultracold gases in the bulk are simple systems!
  - for example: Fermi surface is usually a sphere.
- Both fermions and bosons can be studied.
- Interactions can be tuned to arbitrary values.
- Lower dimensional systems can be realized.

Very nice model system to test methods of quantum and statistical *field theory*!

Lagrangians

We use a local field theory to describe the microscopic model.
Examples:

1. Bose gas with pointlike interaction

$$\mathcal{L} = \varphi^* \left(\partial_\tau - \tfrac{1}{2M}\vec{\nabla}^2 - \mu\right)\varphi + \frac{1}{2}\lambda_\varphi \ (\varphi^*\varphi)^2 \,.$$

2. Fermions in the BCS-BEC-Crossover

$$\mathcal{L} = \sum_{i=1}^{2} \psi_i^\dagger(\partial_\tau - \tfrac{1}{2M}\vec{\nabla}^2 - \mu)\psi_i + \lambda_\psi \ \psi_1^\dagger\psi_1\psi_2^\dagger\psi_2$$

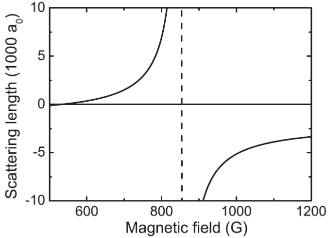
These are effective theories on the length scale of the Bohr radius or van-der-Waals length.

Single component Bose gas

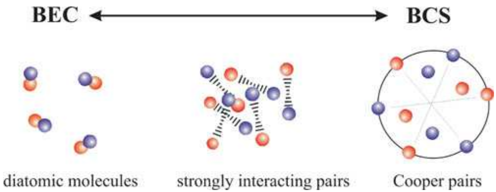
- For repulsive interaction $\lambda_\varphi > 0$ stable, shows Bose-Einstein Condensation at small temperatures.
- For attractive interaction $\lambda_\varphi < 0$ unstable.

Two component Fermi gas

- Two spin (or hyperfine-spin) components ψ_1 and ψ_2.
- For equal mass $M_{\psi_1} = M_{\psi_2}$, density $n_{\psi_1} = n_{\psi_2}$ etc. SU(2) spin symmetry
- s-wave interaction measured by scattering length a.
- Repulsive microscopic interaction: Landau Fermi liquid.
- Attractive interaction leads to many interesting effects!
- Scattering length can be tuned experimentally with Feshbach resonances.



BCS-BEC Crossover



- Small negative scattering length $a \to 0_-$
  - Formation of Cooper pairs in momentum space
  - BCS-theory valid
  - superfluid at small temperatures
  - order parameter $\varphi \sim \psi_1 \psi_2$
- Small positive scattering length $a \to 0_+$
  - Formation of dimers or molecules in position space
  - Bosonic mean field theory valid
  - superfluid at small temperatures
  - order parameter $\varphi \sim \psi_1 \psi_2$
- Between both limits: Continuous *BCS-BEC Crossover*
  - scattering length becomes large: strong interaction
  - superfluid, order parameter $\varphi \sim \psi_1 \psi_2$ at small T

How to construct a truncation

General ideas

- Make an ansatz for flowing action in terms of some expansion

$$\Gamma_k[\phi] = \sum_{i=1}^{\infty} c_i(k)\, \mathcal{O}_i[\phi]$$

- Include all terms that are allowed by symmetries
- Truncate expansion at finite order

$$\Gamma_k[\phi] = \sum_{i=1}^{N} c_i(k)\, \mathcal{O}_i[\phi]$$

- Improve truncation by including more terms

Symmetries of nonrelativistic field theories

- Global U(1) (particle number conservation)

$$\varphi \to e^{i\alpha}\varphi, \quad \varphi^* \to e^{-i\alpha}\varphi^*$$

- Translations and Rotations
- Galilean boost transformations

$$\varphi(t,\vec{x}) \to e^{i\left(\frac{M\vec{v}^2}{2}t - M\vec{v}\vec{x}\right)}\varphi(t,\vec{x}-\vec{v}t)$$

- Energy shift symmetry

$$\varphi(t,\vec{x}) \to e^{i\Delta E t}\varphi(t,\vec{x}), \quad \mu \to \mu + \Delta E$$

- Possibly conformal symmetries

Derivative expansion

For many purposes *derivative expansion* is a suitable approximation. For example for a Bose gas

$$\Gamma_k = \int_0^{1/T} d\tau \int d^d x \Big\{ U(\varphi^* \varphi) + Z_1\, \varphi^* (\partial_\tau) \varphi + Z_2\, \varphi^* (-\vec{\nabla}^2) \varphi + V_1\, \varphi^* (-\partial_\tau^2) \varphi + V_2\, \varphi^* (\partial_\tau \vec{\nabla}^2) \varphi + V_3\, \varphi^* (-\vec{\nabla}^4) \varphi + \dots \Big\}.$$

- the effective potential U contains no derivatives - describes homogeneous fields
- the coefficients Z_i, V_i and the effective potential U are k-dependent
- at higher order the Z_i and V_i are functions of $\rho = \varphi^* \varphi$

Hubbard-Stratonovich transformation 1

- Sometimes it is useful to employ auxiliary fields to rewrite the effective action.
- Add bosonic auxiliary field with Gaussian action to the BCS-BEC crossover model

$$
\begin{aligned}
S = \int_{\tau,\vec{x}} \sum_{i=1}^{2} \psi_i^\dagger (\partial_\tau - \tfrac{1}{2M}\vec{\nabla}^2 - \mu)\psi_i + \lambda_\psi \; \psi_1^\dagger \psi_1 \psi_2^\dagger \psi_2 \\
+ \left(\varphi^* - \psi_2^\dagger \psi_1^\dagger \frac{h}{\partial_\tau - \frac{1}{4M}\vec{\nabla}^2 + m^2} \right) \\
\times \left(\partial_\tau - \tfrac{1}{4M}\vec{\nabla}^2 + m^2 \right) \left(\varphi - \frac{h}{\partial_\tau - \frac{1}{4M}\vec{\nabla}^2 + m^2} \psi_1 \psi_2 \right)
\end{aligned}
$$

- For $h \to \infty$, $m^2 \to \infty$ with $\frac{h^2}{m^2} + \lambda_\psi = 0$ the terms $\sim \psi_1^\dagger \psi_1 \psi_2^\dagger \psi_2$ cancel.

Hubbard-Stratonovich transformation 2

- One is left with a Yukawa-type theory

$$S = \int_{\tau,\vec{x}} \psi_i^\dagger (\partial_\tau - \tfrac{1}{2M}\vec{\nabla}^2 - \mu)\psi_i - h(\varphi^* \psi_1 \psi_2 + \psi_2^\dagger \psi_1^\dagger \varphi) + \varphi^* \left(\partial_\tau - \tfrac{1}{4M}\vec{\nabla}^2 + m^2\right)\varphi$$

- One can now use a truncation in terms of a derivative expansion

$$\Gamma_k = \int_{\tau,\vec{x}} \psi_i^\dagger Z_\psi (\partial_\tau - \tfrac{\vec{\nabla}^2}{2M} - \mu + \Delta m_\psi)\psi_i - h(\varphi^* \psi_1 \psi_2 + \psi_2^\dagger \psi_1^\dagger \varphi) + \varphi^* (Z_\varphi \partial_\tau - A_\varphi \tfrac{\vec{\nabla}^2}{4M})\varphi + U(\varphi^* \varphi)$$

- The coefficients Z_ψ, Δm_ψ, Z_φ, A_φ, h, and the effective potential U are now k-dependent.

Choice of cutoff 1

The cutoff term

$$\Delta S_k = \int_p \varphi^*(p) R_k(p) \varphi(p)$$

should

- be an infrared cutoff $\lim_{p\to 0} R_k(p) \approx k^2$
- fall off for large momenta $\lim_{p\to\infty} R_k(p) \to 0$
- respect symmetries
- allow to perform loop integrations
- be simple
- be “optimal” (cf. Litim, Pawlowski)

Choice of cutoff 2

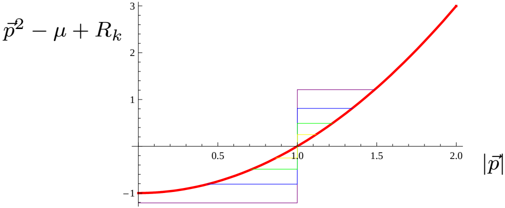
- Example for nonrelativistic bosons (Litim)

$$R_k(p) = Z_\varphi(k^2 - \vec{p}^2)\theta(k^2 - \vec{p}^2)$$

- Example for nonrelativistic fermions

$$R_k(p) = Z_\psi(\mathsf{sign}(x)k^2 - x)\theta(k^2 - |x|)$$

with $x = \vec{p}^2 - p_F^2$



How to derive flow equations

General ideas

- start from exact flow equation (Wetterich 1993)

$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathsf{STr}\left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1} \partial_k R_k .$$

- plug in the truncation

$$\Gamma_k[\phi] = \sum_{i=1}^{N} c_i(k)\, \mathcal{O}_i[\phi]$$

- project both sides onto the operators $\mathcal{O}_i[\phi]$
- obtain ordinary (sometimes partial) differential equations for "running couplings"

$$\partial_k\, c_i = \beta_i(c_1, \ldots, c_N; k)$$

Loop expressions

- useful to rewrite flow equation

$$\partial_k \Gamma_k[\phi] = \tilde{\partial}_k \frac{1}{2} \mathsf{STr} \, \ln \left(\Gamma_k^{(2)}[\phi] + R_k \right)$$

- projecting the right hand side by taking functional derivatives gives one-loop expressions
- performing the cutoff derivative $\tilde{\partial}_k$ leads to IR and UV finite expressions

The effective potential

- for constant fields $\varphi(x) = \sqrt{\rho}, \psi = 0$

$$\Gamma_k = \int_{\tau,\vec{x}} U_k(\rho)$$

- flow equation is partial differential equation, for BCS-BEC Crossover

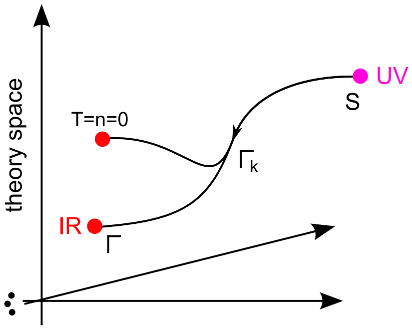
$$\frac{\partial}{\partial \ln k} U_k(\rho) = \eta_\varphi \, \rho \, U_k'(\rho) + k^{d+2} \, s_B\left(\frac{U_k'}{k^2}, \frac{\rho U_k''}{k^2}, \frac{T}{k^2}, \eta_\varphi, \frac{Z_\varphi}{A_\varphi}\right) - k^{d+2} \, s_F\left(\frac{h^2 \rho}{k^4}, \frac{\mu + \Delta m_\psi}{k^2}, \frac{T}{k^2}, \eta_\psi\right)$$

- *threshold functions* s_B, s_F depend on cutoff R_k
- $\eta_\varphi = -\frac{\partial \ln A_\varphi}{\partial \ln k}$ and $\eta_\psi = -\frac{\partial \ln Z_\psi}{\partial \ln k}$ are anomalous dimensions

How to solve flow equations

General ideas

- ordinary differential equations can be solved numerically
- UV values at $k = \Lambda$ determined from few-body physics
- consider first vacuum limit $T \to 0, \mu \to 0$
- "couplings" at $k = 0$ can be related to few-body observables
- flow deviates from vacuum solution for $k^2 \lesssim T, \mu$



Few-body physics

- for $\mu = T = 0$ one can easily perform analytic continuation to real time
- formalism should be equivalent to quantum mechanics
- flow equations simplify substantially
- often analytic solutions can be found
- interesting physics: universal bound states, Efimov effect, ...
- more details: talks by M. Birse, S. Moroz, B. Krippa
- one finds a useful hierarchy of flow equations

Few-body hierarchy 1

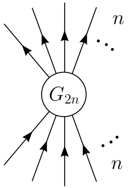
- use vertex expansion (e.g. for Bose gas, $\Gamma_k^{(i,j)}[\varphi] \sim \varphi^{*i}\varphi^j$)

$$\begin{aligned}\Gamma_k[\varphi] = \Gamma_k^{(0,0)}[\varphi] + \Gamma_k^{(0,1)}[\varphi] + \Gamma_k^{(0,2)}[\varphi] + \dots \\ + \Gamma_k^{(1,0)}[\varphi] + \Gamma_k^{(1,1)}[\varphi] + \Gamma_k^{(1,2)}[\varphi] + \dots \\ + \dots\end{aligned}$$

- U(1) symmetry implies

$$\Gamma_k[\varphi] = \Gamma_k^{(0,0)}[\varphi] + \Gamma_k^{(1,1)}[\varphi] + \Gamma_k^{(2,2)}[\varphi] + \dots$$

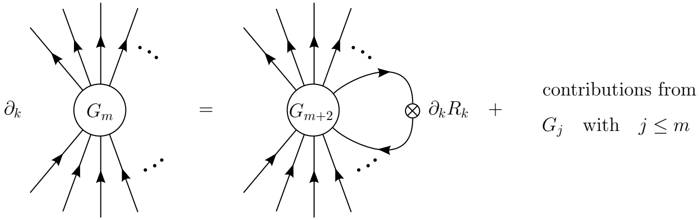
- $\Gamma_k^{(n,n)}$ contains information about $2n$-point function G_{2n}



Few-body hierarchy 2

Theorem: The flow equation for G_m depends only on the functions G_j with $j \leq m$ and the propagator G_2 is not renormalized.

- the n-particle problem can be solved without solving the $n+1$-particle problem
- the flow equations can be integrated
  - solve two-body problem
  - then three-body problem, ...
- sketch of proof:

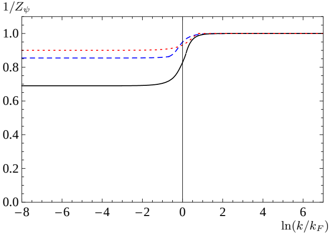


The loop vanishes since all poles are in one half-plane.

Fermion self energy corrections

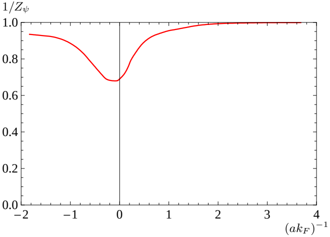
(Floerchinger, Scherer and Wetterich, PRA **81**, 063619 (2010))

- Flow of fermion wavefunction renormalization Z_ψ



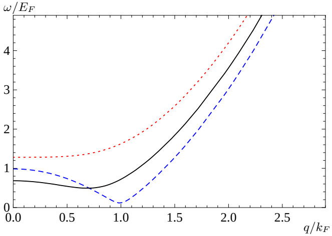
solid: $(ak_F)^{-1} = 0$,
long-dashed: $(ak_F)^{-1} = -1$,
short-dashed: $(ak_F)^{-1} = 1$

- At the macroscopic scale $k = 0$



Dispersion relation and gap at zero temperature

- Dispersion relation $\omega = \pm\sqrt{\Delta^2 + (\vec{q}^2 - p_F^2)^2}$



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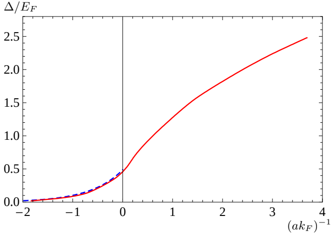
- Single particle gap



Unitarity $(ak_F = \infty)$	μ/E_F	Δ/E_F
Carlson *et al.*	0.43	0.54
Perali *et al.*	0.46	0.53
Haussmann *et al.*	0.36	0.46
Diehl *et al.*	0.55	0.60
Bartosch *et al.*	0.32	0.61
present work	0.51	0.46

The effective potential

- Taylor expansion around the minimum ρ_0

$$U_k(\varphi^*\varphi) = -\,p + m^2\,\varphi^*\varphi + \frac{1}{2}\lambda\,(\varphi^*\varphi)^2 \qquad \text{for} \quad \rho_0 = 0,$$

$$U_k(\varphi^*\varphi) = -\,p + \frac{1}{2}\lambda\,(\varphi^*\varphi - \rho_0)^2 \qquad \text{for} \quad \rho_0 > 0.$$

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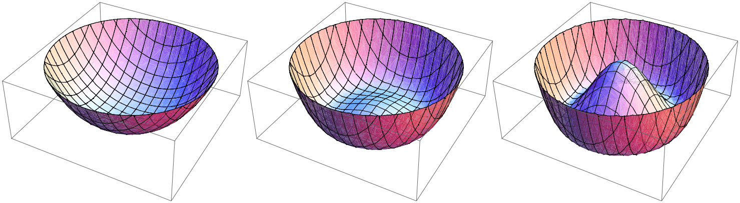
- Symmetry breaking:



$T > T_c$ $\qquad$ $T < T_c$ $\qquad$ $T \ll T_c$

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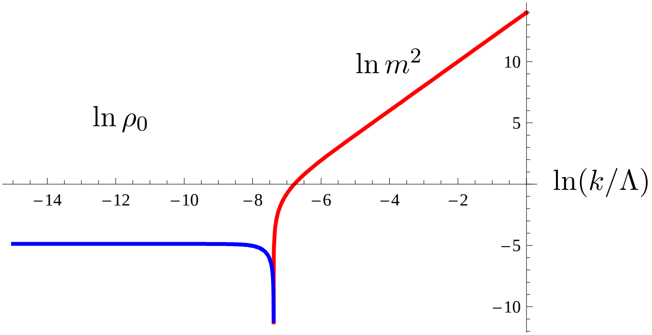
$$U_k(\varphi^*\varphi) = -\,p + \frac{1}{2}\lambda\,(\varphi^*\varphi - \rho_0)^2 \qquad \text{for} \quad \rho_0 > 0.$$

- Typical flow:



Phase diagram

- information on phase diagram is contained in form of the effective potential $U_k(\rho, \mu, T)$ at $k = 0$
- solution for different $T, \mu, \ldots$ gives phase diagram
- very nice generalization of Landau's theory!

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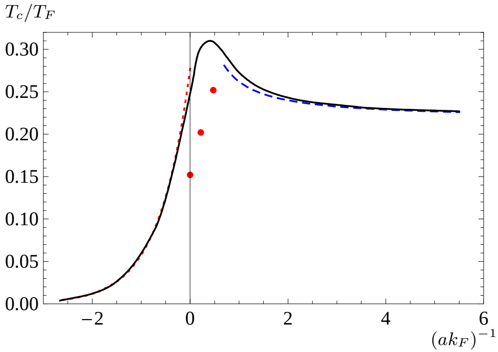
- example: BCS-BEC Crossover

  (Floerchinger, Scherer and Wetterich, PRA **81**, 063619 (2010))



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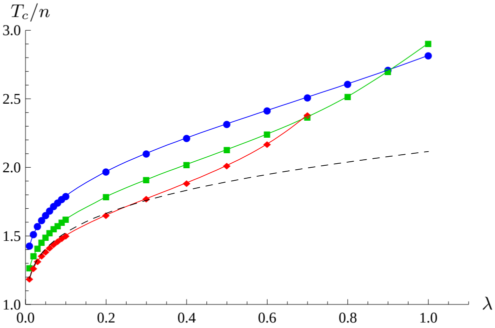
- example: Superfluid Bose gas in $d = 2$

(Floerchinger and Wetterich, PRA 79, 013601 (2009))



Thermodynamic observables

From grand canonical potential

$$dU = -dp = -s\,dT - n\,d\mu$$

take derivatives e. g. for Bose gas in $d = 3$

(Floerchinger and Wetterich, PRA 79, 063602 (2009))

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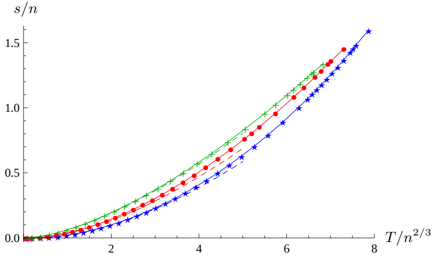

- entropy density $s = -\frac{\partial U}{\partial T}$,

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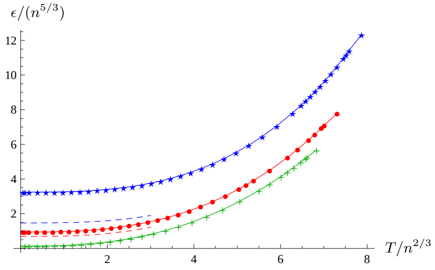

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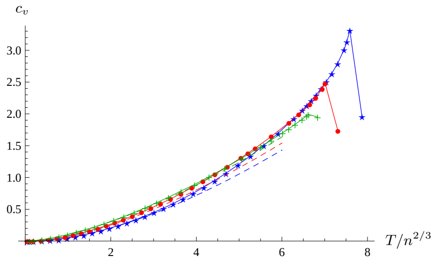

- entropy density $s = -\frac{\partial U}{\partial T}$,
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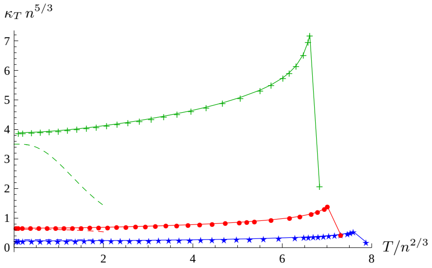

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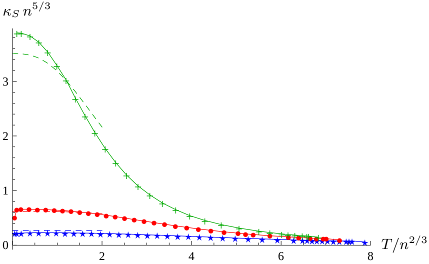

- entropy density $s = -\frac{\partial U}{\partial T}$,
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- adiab. compressibility κ_S,

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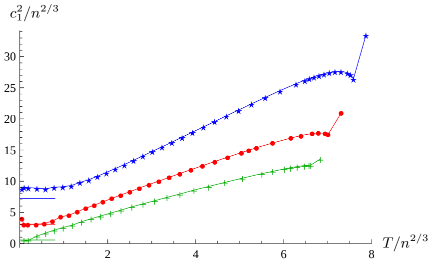

- entropy density $s = -\frac{\partial U}{\partial T}$,
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- velocity of sound I,

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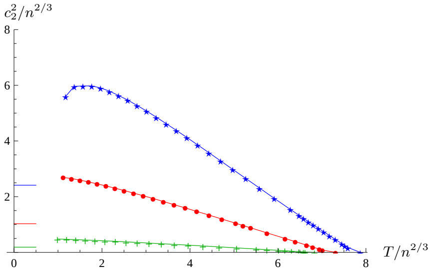

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Thermodynamic observables

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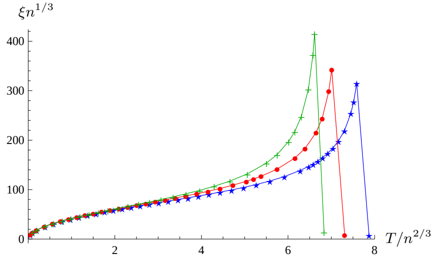


- entropy density $s = -\frac{\partial U}{\partial T}$,
- energy density $\epsilon = -p + Ts + \mu n$,
- specific heat c_v,
- isoth. compressibility κ_T,
- adiab. compressibility κ_S,
- velocity of sound I,
- velocity of sound II,
- correlation length.

Occupation numbers

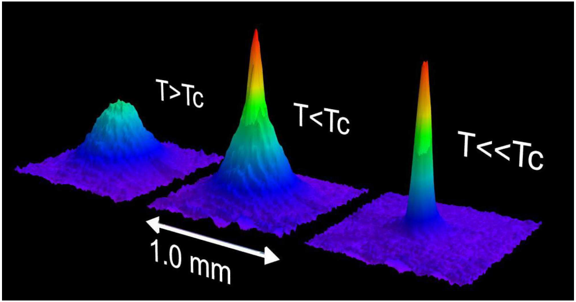
Usually density can be written as

$$n = \int_{\vec{p}} n(\vec{p})$$

with Occupation number $n(\vec{p})$. Example: Homogeneous Bose gas

$$n(\vec{p}) = n_c \, \delta^{(d)}(\vec{p}) + n_T(\vec{p}).$$

Occupation numbers are measured in time-of-flight experiments.



Picture from W. Ketterle, MIT.

Flow equations for occupation numbers

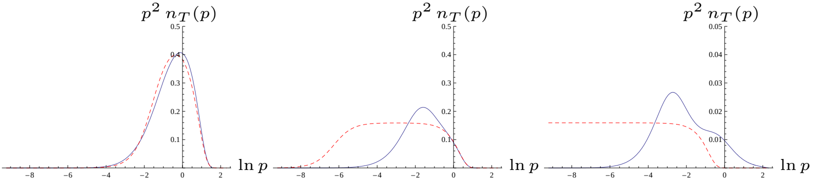
- Use momentum-dependent chemical potential $\mu = \mu(\vec{p})$

$$S = \int_p \varphi^*(p) \left[i p_0 + \vec{p}^2 - \mu(\vec{p}) \right] \varphi(p) + \ldots$$

- Obtain occupation numbers from

$$n(\vec{p}) = -\frac{\delta}{\delta \mu(\vec{p})} U.$$

- Flow equations for $n(\vec{p})$ can be derived (Wetterich 2008).
- Example: Bose gas in $d = 2$ with finite size.



$T > T_c,\ n_c = 0$ $\quad$ $T < T_c,\ n_c/n = 0.4$ $\quad$ $T \ll T_c,\ n_c/n = 0.9$

Fermi gases with different physics

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    (Efimov, Phys. Lett. **33B**, 563 (1970),
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  - On the lattice: Trion formation

    (Rapp, Zarand, Honerkamp, and Hofstetter, PRL **98**, 160405 (2007),
    Rapp, Hofstetter and Zarand, PRB **77**, 144520 (2008).)

Three component Fermi gas

- For equal masses, densities etc. global SU(3) symmetry

$$\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} \to u \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix}, \quad u \in \text{SU(3)}.$$

  Similar to flavor symmetry in the Standard model!
- For small scattering length $|a| \to 0$
  - BCS ($a < 0$) or BEC ($a > 0$) superfluidity at small T.
  - order parameter is conjugate triplet $\bar{\mathbf{3}}$ under SU(3)

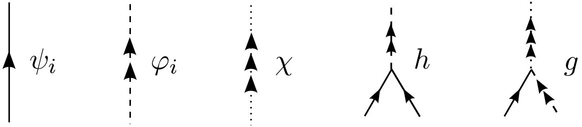
$$\varphi = \begin{pmatrix}\varphi_1\\ \varphi_2\\ \varphi_3\end{pmatrix} \sim \begin{pmatrix}\psi_2\psi_3\\ \psi_3\psi_1\\ \psi_1\psi_2\end{pmatrix}.$$

  - SU(3) symmetry is broken spontaneously for $\varphi \neq 0$.
- What happens for large $|a|$?

Simple truncation for fermions with three components

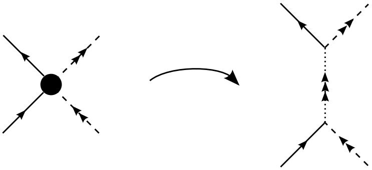
$$
\begin{aligned}
\Gamma_k \;=\; & \int_x \psi^\dagger(\partial_\tau - \vec{\nabla}^2 - \mu)\psi + \varphi^\dagger(\partial_\tau - \frac{1}{2}\vec{\nabla}^2 + m_\varphi^2)\varphi \\
& + \chi^*(\partial_\tau - \frac{1}{3}\vec{\nabla}^2 + m_\chi^2)\chi \\
& + h\epsilon_{ijk}(\varphi_i^*\psi_j\psi_k + h.c.) \quad + g(\varphi_i\psi_i^*\chi + h.c.).
\end{aligned}
$$

- Units are such that $\hbar = k_B = 2M = 1$
- Wavefunction renormalization for ψ, φ and χ is implicit.
- Γ_k contains terms for
  - fermion field $\psi = (\psi_1, \psi_2, \psi_3)$
  - bosonic field $\varphi = (\varphi_1, \varphi_2, \varphi_3) \sim (\psi_2\psi_3, \psi_3\psi_1, \psi_1\psi_2)$
  - trion field $\chi \sim \psi_1\psi_2\psi_3$

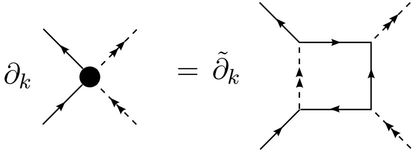


"Refermionization"

- Trion field is introduced via a generalized Hubbard-Stratonovich transformation



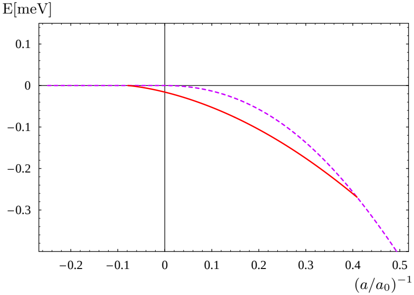
- Fermion-boson coupling is regenerated by the flow

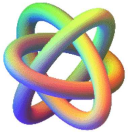


- Express this again by trion exchange
  (Gies and Wetterich, PRD **65**, 065001 (2002),
  Floerchinger and Wetterich, PLB **680**, 371 (2009).)

Binding energies

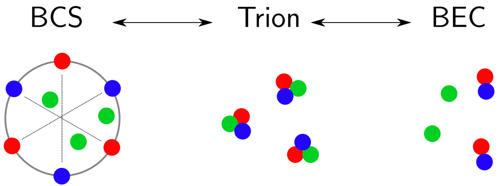
- Vacuum limit $T \to 0, \quad n \to 0.$





- Binding energy per atom for
  - molecule or dimer φ (dashed line)
  - trion or trimer χ (solid line)
- For large scattering length a trion is energetically favorable!
- Three-body bound state even for $a < 0$.
- There is actually a whole tower of bound states (Efimov effect, talk by S. Moroz).

Quantum phase diagram



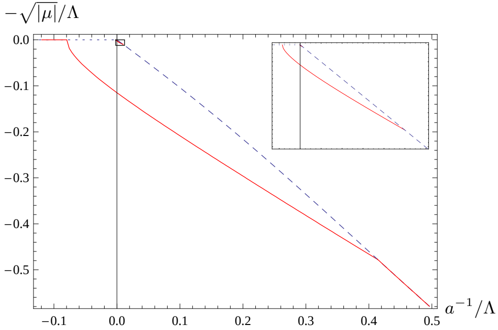
- BCS-Trion-BEC transition
  (Floerchinger, Schmidt, Moroz and Wetterich, PRA **79**, 013603 (2009)).
  - $a \to 0_-$: Cooper pairs, $SU(3) \times U(1) \to SU(2) \times U(1)$.
  - $a \to 0_+$: BEC of molecules, $SU(3) \times U(1) \to SU(2) \times U(1)$.
  - $a \to \pm\infty$: Trion phase, $SU(3)$ unbroken.
- Quantum phase transitions
  - from BCS to Trion phase
  - from Trion to BEC phase.

Conclusions

Conclusions

- Functional renormalization is a useful method to describe ultracold quantum gases.
- Quantitative precision seems reachable.
- Unified description of
  - Bosons and Fermions,
  - Weak and strong coupling,
  - Few-Body and Many-Body physics.

Efimov effect



- Self-similarity in energy spectrum.
- Efimov trimers become more and more shallow. At $a = \infty$

$$E_{n+1} = e^{-2\pi/s_0}\, E_n.$$

- Simple truncation: $s_0 \approx 0.82$.
- Advanced truncation: $s_0 \approx 1.006$ (exact result)

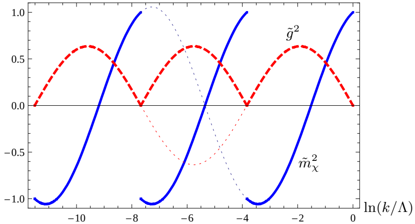
(Moroz, Floerchinger, Schmidt and Wetterich, PRA **79**, 042705 (2009).)

Renormalization group limit cycle

- For $\mu = 0$ and $a^{-1} = 0$ flow equations for rescaled couplings

$$k\frac{\partial}{\partial k}\begin{pmatrix}\tilde{g}^2\\ \tilde{m}_\chi^2\end{pmatrix} = \begin{pmatrix}7/25 & -13/25\\ 36/25 & 7/25\end{pmatrix}\begin{pmatrix}\tilde{g}^2\\ \tilde{m}_\chi^2\end{pmatrix}.$$

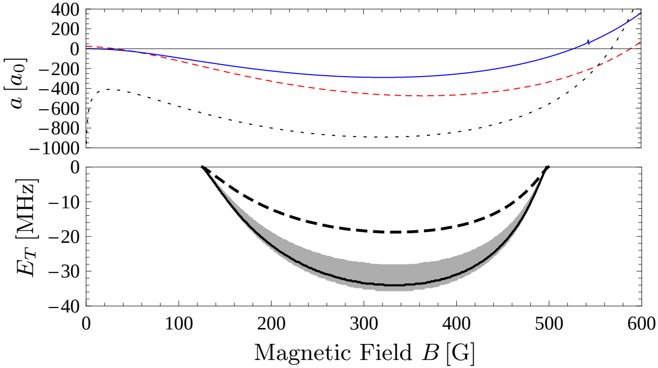
- Solution is log-periodic in scale.



- Every zero-crossing of $\tilde{m}_\chi^2$ corresponds to a new bound state.
- For $\mu \neq 0$ or $a^{-1} \neq 0$ limit cycle scaling stops at some scale k. Only finite number of Efimov trimers.

Contact to experiments

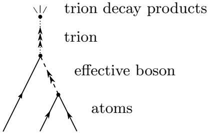
- Model can be generalized to case without SU(3) symmetry (Floerchinger, Schmidt and Wetterich, PRA A **79**, 053633 (2009)).
- Hyperfine states of ^{6}Li have large scattering lengths.

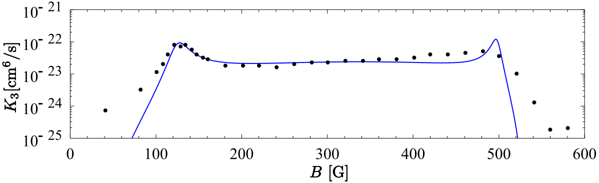


- Binding energies might be measured using RF-spectroscopy.
- Lifetime is quite short ~ 10ns.

Three-body loss rate

- Three-body loss rate measured experimentally (Ottenstein et al., PRL **101**, 203202 (2008); Huckans et al., PRL **102**, 165302 (2009))





- Trion may decay into deeper bound molecule states
- Calculate B-field dependence of loss process above.
- Left resonance (position and width) fixes model parameters.
- Form of curve for large B is prediction.
- Similar results obtained by other methods (Braaten, Hammer, Kang and Platter, PRL **103**, 073202 (2009); Naidon and Ueda, PRL **103**, 073203 (2009).)