

The phase diagram of a gauge theory with fermionic baryons

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The fermion-sign problem at finite density is a persisting challenge for Monte-Carlo simulations. Theories that do not have a sign problem can provide valuable guidance and insight for physically more relevant ones that do. Replacing the gauge group $SU(3)$ of QCD by the exceptional group G_2 , for example, leads to such a theory. It has mesons as well as bosonic and fermionic baryons, and shares many features with QCD. This makes the G_2 gauge theory ideally suited to study general properties of dense, strongly-interacting matter, including baryonic and nuclear Fermi pressure effects as relevant in compact stars and heavy-ion collisions. We present the first lattice simulations of the phase diagram of this theory at finite temperature and baryon chemical potential.

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Finite fermion density continues to be a serious challenge for Monte-Carlo simulations due to the fermion-sign problem [1, 2]. The sign problem appears in many areas of physics, but is of notorious importance to dense quark systems, especially in nuclei, heavy-ion collisions, and compact stellar objects. An alternative are models and continuum methods which do not have this type of problem [3–6]. However, these usually require approximations, and cross checks through lattice simulations remain desirable to improve systematic reliability.

To provide support from numerical simulations, two major strategies have been followed. One is to replace the baryon chemical potential by some quantity more amenable to simulations, e.g. an imaginary [7, 8] or isospin chemical potential [9, 10]. The other is to replace the theory with one accessible through numerical simulations at finite density. However, such theories usually differ from the original one in more or less important aspects.

One very well studied replacement of QCD for strongly interacting matter at finite density is two-color QCD [11–15]. In this case, the baryons are bosons instead of fermions, however. This leads to profound differences, such as Bose-Einstein condensation of a baryon superfluid with a BEC-BCS crossover at high densities instead of the usual liquid-gas transition of nuclear matter. While two-color QCD has many interesting aspects that deserve to be studied in their own right, the quantum effects due to the fermionic nature of baryons are expected to play a very significant role for nuclear matter and especially in the physics of compact stellar objects [16].

Therefore, a more realistic replacement theory in this regard should contain fermionic baryons. We employ such a theory without sign problem for Monte-Carlo sim-

ulations at finite baryon density here. It is obtained by replacing the $SU(3)$ gauge group of QCD with the gauge group G_2 [17]. All color representations of this theory are equivalent to real ones. As a consequence the Dirac operator has an anti-unitary symplectic symmetry which leads to an extended Pauli-Gürsey $SU(2N_f) \times Z_2$ flavor symmetry [11, 17], and it thus has a non-anomalous component even for a single flavor. In this letter we study the phase diagram for a single Dirac flavor of Wilson fermions, corresponding to a continuum $SU(2) \times Z_2$ extended symmetry in the chiral limit. Spontaneous or explicit breaking reduces this to $SO(2) \times Z_2 \sim U(1) \times Z_2$ [18]. The unbroken $U(1)$ relates to the baryon number to which the baryon chemical potential is coupled. The reality of the representation implies that the fermion determinant remains positive at finite baryon chemical potential even for a single flavor [12].

The physical bound states of this theory, besides the usual quark-antiquark and three-quark states, also contain hybrids of one quark with three gluons, as well as diquarks and further bound states with more than three quarks [17]. Thus, the hadronic spectrum contains both fermionic as well as bosonic baryons and mesons. These bound states are created by very similar interactions as in QCD, i.e. by a potential which rises linearly with the separation of the quarks before string-breaking sets in [17, 19–22]. Another appealing aspect is that it might be possible to continuously deform it to ordinary QCD by breaking the G_2 gauge group down to $SU(3)$ via a Higgs mechanism [17, 23], although this will likely require several Higgs fields and various Yukawa couplings and CKM-type explicit flavor violations. The surplus bound states would then become infinitely heavy and hence disappear from the spectrum. This implies that the sign problem would then reappear. Its gradual emergence controlled by the strength of the breaking might well lead to new insights and pathways for simulations and therefore this possibility deserves further study.

For our lattice simulations we employ an extension of the available HMC algorithm for scalars [23]. The techniques are generalizations from QCD simulations, for de-

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tails see [24, 25]. The introduction of temperature and chemical potential proceeds as in QCD. Our aim here is a first exploration of the phase diagram as proof-of-principle study. One obstacle is the presence of a bulk transition [21, 23, 26], which turns out to persist with dynamical fermions [18, 25]. To avoid this we chose a lattice with at least $N_t = 6$ time slices in finite temperature simulations. Simulations of this theory incur considerable computational costs so that we restricted our lattices to $N_s = 16$ points in spatial directions. Altogether, we investigated three different sets of lattice parameters:

(a) At zero density we varied the lattice coupling β between 0.9 and 1.0 in order to control temperature on our 6×16^3 lattice.

(b) At finite chemical potential μ we used $\beta = 0.9$ on both, 6×16^3 and 8×16^3 lattices, and $\beta = 1.0$ in zero-temperature simulations on a 16^4 lattice. The hopping parameter in the fermion determinant was fixed at $\kappa = 0.15625$ in all these thermodynamic simulations.

(c) For comparison we also varied κ from 0.15385 to 0.15625 on the symmetric 16^4 lattice with $\mu = 0$ and β in 0.85 to 1.1. Results from smaller lattices to assess systematic effects will be reported elsewhere [18, 25].

We measured three observables to study the phase diagram. One is the Polyakov loop. Unlike QCD, due to the trivial center, it is not an order parameter of the quenched theory [17], but it nevertheless reflects the corresponding first-order phase transition very well [21, 26]. In fact, it remains so small in the low-temperature phase that it is only possible to determine upper bounds. We found this to be true also with dynamical quarks. The second observable is the chiral condensate. The quenched G_2 theory has only one first-order transition at finite temperature which manifests itself also in the chiral condensate [27], a feature that it shares with QCD and two-color QCD. This is in contrast to QCD with adjoint quarks, where there is no sign problem either [11], but where separate chiral and deconfinement transitions occur at largely different temperatures [28, 29]. We normalize the chiral condensate to its (β -dependent) vacuum value, to avoid explicit renormalization. The third observable is the baryon number density, the derivative of the partition function with respect to the chemical potential. At large chemical potentials the density saturates to a temperature-independent value. This is observed also in two-color QCD, where this happens when each lattice site is occupied by the maximum number of fermions [30]. Here, the saturation is smaller, about 4.8/lattice site. The reason is yet unclear, but it may be that putting the simplest baryonic bound state at each lattice site is not saturating the lattice. To eliminate the scale, we normalize the baryon density to its saturation value.

In order to assess the phase diagram it is necessary to fix at least a relative scale. Using bound state masses for this purpose is rather challenging. We have determined in the vacuum the mass of the lowest states in the pseudoscalar meson and diquark channels, keeping only the

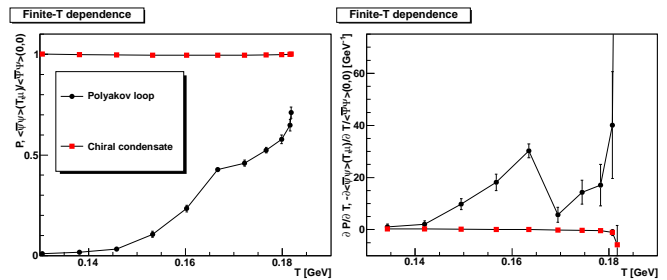


FIG. 1: The Polyakov loop and the chiral condensate at zero chemical potential (left panel) and the temperature derivatives (right panel).

connected contributions. The results [18] show strong systematic effects. The signal with the least lattice artefacts is the first excited state in the diquark channel, which varies for $\beta = 0.9 \dots 1.0$ at $\kappa = 0.15625$ from 2.3_{-4}^{+5} to 1.65_{-5}^{+1} . Setting its mass such that the zero density critical temperature is 160 MeV leads to lattice spacings between 0.25_{-4}^{+5} fm to 0.181_{-20}^{+1} fm, which is an acceptable systematic error for this investigation.

The results at zero chemical potential are shown in Figure 1. A (weak) transition is seen in the Polyakov loop at about $T = 160$ MeV, while the chiral condensate shows no response. This implies that either the chiral transition is displaced outside the investigated temperature region, the chiral condensate is not really capturing the transition, or the quarks are effectively very heavy. Further investigations will be needed to clarify this.

The situation at finite density is shown in Figure 2. At zero temperature a peak in the Polyakov loop is visible at around 1.4 GeV, and the derivative indicates two broad changes. The chiral condensate and the baryon density both show a transition roughly at the first peak of the Polyakov loop derivative, but this transition is rather broad. At large chemical potential the density saturates, as discussed above. If this is interpreted as effectively quenching the theory [30], this would explain the decrease of the Polyakov loop, similar as in two-color QCD [13]. This implies that severe lattice artifacts can be expected from the turning point of the Polyakov loop onwards.

On the present lattices our estimate of the lowest mass in the baryonic channel is too imprecise to explicitly verify that the point where the thermodynamic quantities start to respond to the chemical potential is half the diquark mass [31]. But investigations at larger masses on smaller lattices suggest that this so-called silver-blaze point could be expected around $\mu \approx 150 - 300$ MeV [18].

Finally, at both non-zero temperature and density not much change is observed, and only a gradual shift of the transition towards smaller chemical potential along with a washing-out of the transition is seen. The phase diagram, shown in interpolated form in Figure 3, therefore exhibits a rather rectangular shape, although this may be influenced by systematic effects in the scale setting.

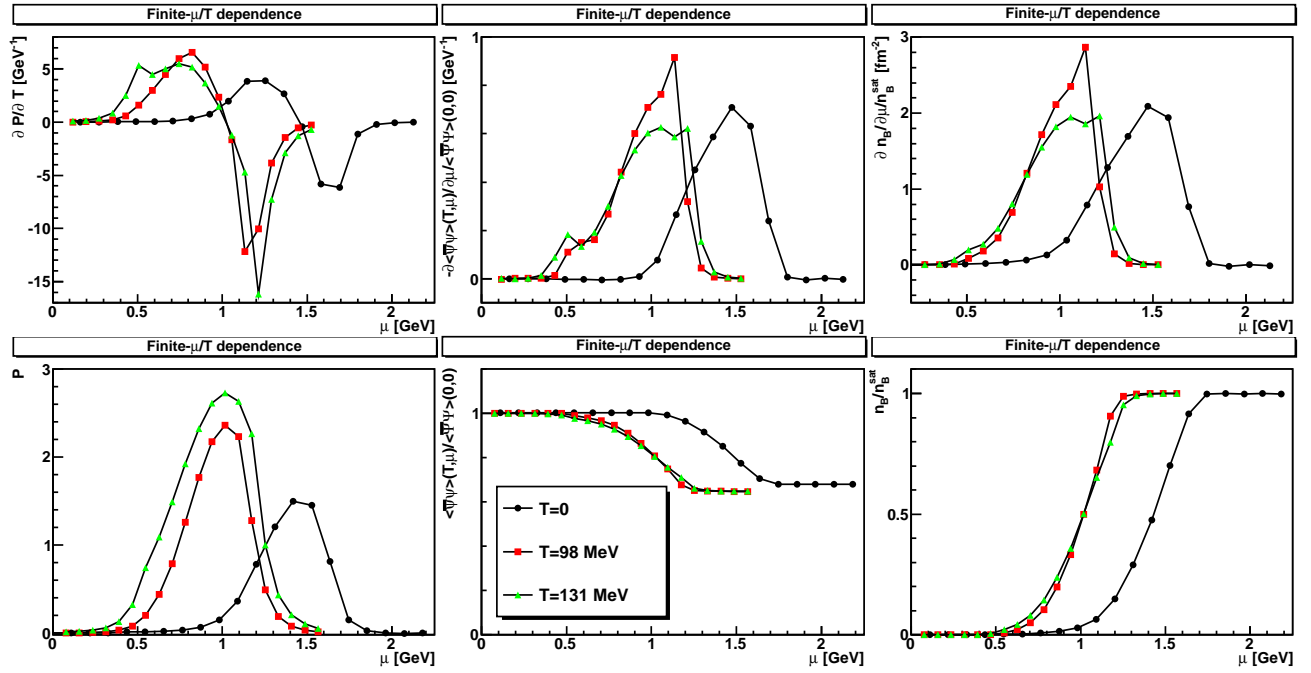


FIG. 2: The bottom row shows the raw data of the Polyakov loop (left panel), the chiral condensate (middle panel), and the normalized baryon density (right panel) at finite chemical potential and temperatures $T = 0, 98$ MeV and 131 MeV. The top row shows the numerical derivatives for the same quantities to identify regions of rapid changes.

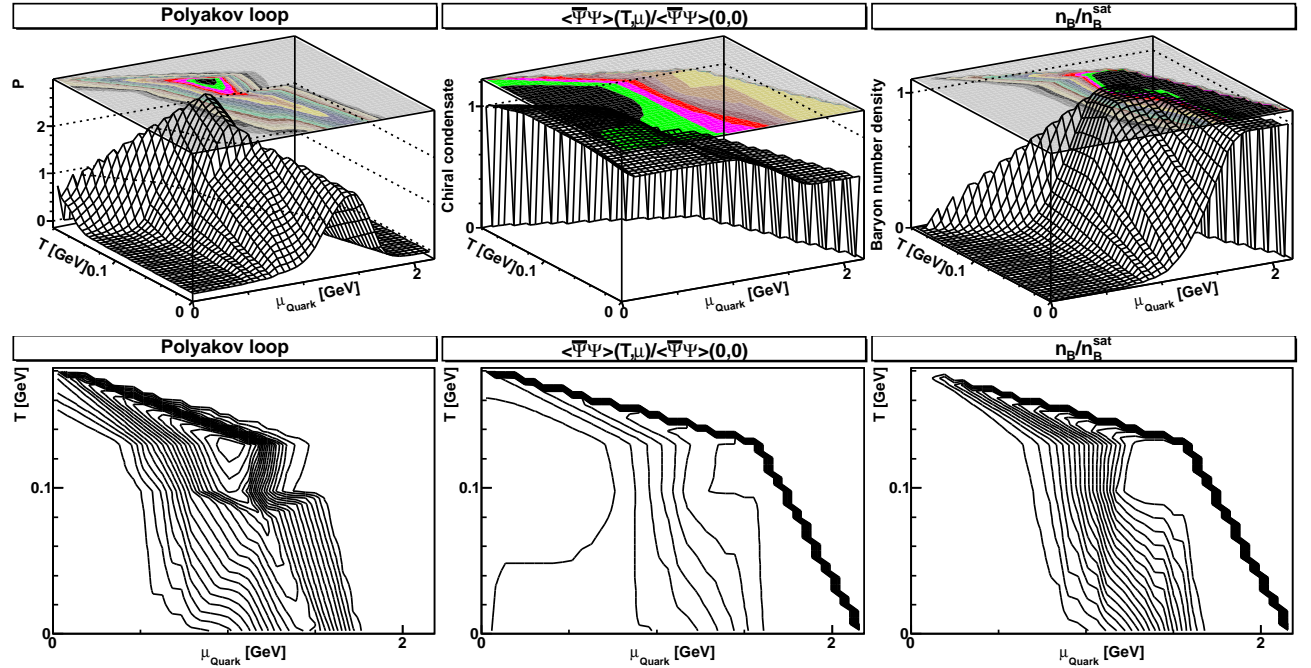


FIG. 3: The interpolated phase diagram for the Polyakov loop (left panels), chiral condensate (middle panels), and baryon density (right panels).

Also, the limited set of results and statistics so far does not permit to make any statement on the order of the transitions. Results on smaller lattices just reproduce the present results, albeit with larger systematic errors [18]. Nonetheless, the phase diagram so obtained is not

far from the one expected, and similar to the one of two-color QCD [12–15].

In total, the phase diagram at finite density shows a number of interesting features, but requires much more systematic investigations to see whether the effects ob-

served are genuinely physical, or whether some of them are lattice artefacts. In particular, the question of whether there is a phase transition or merely a crossover at finite density is a very interesting and important one.

Summarizing, we have determined the first full lattice phase diagram of G_2 QCD, a non-Abelian gauge theory with fermionic baryons. This opens up completely new horizons for high density studies of the strong nuclear force, as it is now possible to use first-principles calculations to assess the importance of the Fermi statistics of the baryons. Given the role this statistics plays for compact stellar objects, this is of utmost importance. In particular, it will be possible to investigate at which relative densities quark or hadron equations of state are more favorable, and thus contribute to an understanding of the question, whether just neutron stars or also quark stars could exist. Given the inflow of new astrophysical observational data on compact stellar objects, including major new discoveries, this is very timely and important.

This theory offers also the possibility to investigate the presence of a critical point in such theories, a question of prime importance for the ongoing heavy-ion collision experiments. Given sufficient computational resources, the complete phase diagram can be mapped with any desired precision. That is a novel perspective for a theory with properties so close to the ones of QCD. This may yield qualitatively new insights into low-energy heavy-ion

collisions and compact stellar objects.

Furthermore, these ab initio results open new avenues to approach the QCD phase diagram by effective models and functional methods. Firstly, it is possible to provide systematic checks of model assumptions and approximations in functional calculations. Secondly, if the G_2 -theory is continuously deformable to QCD [17, 18, 23], then proceeding along the lines developed for Yang-Mills theory [5, 6, 32], the following approach is feasible: Lattice calculations can provide guidance and cross-checks for the non-lattice approaches to a certain extent along the deformation to QCD, while the non-lattice methods can then provide the final step to QCD, providing its full phase diagram.

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