Friedrich-Schiller-University Jena Summer term 2016
Prof. Andreas Wipf
M.Sc. Tobias Hellwig Will be discussed: **22th week of year**

Problems in Supersymmetry

Sheet 8

Problem 22: Wess-Zumino model in 2 dimensions

Here we consider the Wess-Zumino model for a Majorana spinor and a scalar field in 1+1 dimensions. This is probably the simplest field theoretical realization of supersymmetry. We may choose a Majorana representation with $\gamma^0 = \sigma_2,\ \gamma^1 = i\sigma_1$ such that $\gamma_* = \sigma_3$.

- Restrict the general Fierz identitiy to 2 dimensions and show that

$$2\psi\bar{\chi} = -(\bar{\chi}\psi) - \gamma_\mu(\bar{\chi}\gamma^\mu\psi) - \gamma_*(\bar{\chi}\gamma_*\psi), \quad \gamma_* = \gamma^0\gamma^1.$$

- Our general results for the symmetry properties of fermionic bilinears reduce to

$$\bar{\psi}\chi = \bar{\chi}\psi, \quad \bar{\psi}\gamma^\mu\chi = -\bar{\chi}\gamma^\mu\psi \quad \text{and} \quad \bar{\psi}\gamma_*\chi = -\bar{\chi}\gamma_*\psi.$$

  Show this by explicit calculations.

- Do the same for the hermiticity properties:

$$\bar{\psi}\chi,\ \bar{\psi}\gamma_*\chi \quad \text{are hermitean and} \quad \bar{\psi}\gamma^\mu\chi \quad \text{is antihermitean.}$$

- Prove that for $2d$ Majorana spinors we have

$$(\bar{\alpha}\psi)(\bar{\alpha}\psi) = -\frac{1}{2}(\bar{\alpha}\alpha)(\bar{\psi}\psi), \quad (\bar{\psi}\gamma^\mu\alpha)\psi = -\frac{1}{2}(\bar{\psi}\psi)\gamma^\mu\alpha.$$

- The supersymmetry transformations are

$$\delta A = \bar{\varepsilon}\psi, \quad \delta\psi = (F + i\partial\!\!\!/ A)\varepsilon, \quad \delta F = i\bar{\varepsilon}\partial\!\!\!/\psi.$$

- Proof, that the action with Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu A\partial^\mu A - \frac{i}{4}\bar{\psi}\partial\!\!\!/\psi + \frac{i}{4}\partial_\mu\bar{\psi}\gamma^\mu\psi + \frac{1}{2}F^2 + FW'(A) - \frac{1}{2}W''(A)\bar{\psi}\psi$$

  is invariant under susy transformations.

- Determine the on-shell action.
- In order to have a unbroken supersymmetric theory the vacuum energy of the system has to be zero. Which implication follows for the expectation value of F in the physical minimum and which for the function $W'(A)$.
- Show that there is no one-loop correction to the vacuum energy. Is this also true in an on-shell calculation?

Problem 23: Superspace for $2d$ Wess-Zumino model

An elegant way to formulate supersymmetric theories is given by introducing superfields. A real superfield in 2 dimensions has the expansion

$$\Phi(x,\theta) = A(x) + \bar{\theta}\psi(x) + \frac{1}{2}\bar{\theta}\theta F(x)$$

with constant anticommuting Majorana parameter θ. The susy transformations are generated by the supercharge

$$Q = -i\frac{\partial}{\partial\bar{\theta}} - (\gamma^\mu\theta)\partial_\mu, \quad \delta_\varepsilon\Phi = i[\bar{\varepsilon}Q, \Phi].$$

- Prove that Q generates the susy transformations given in exercise 22.
- Argue, that $[\delta_1, \delta_2] = 2i(\bar{\varepsilon}_2\gamma^\mu\varepsilon_1)\partial_\mu$ implies

  $$\{Q_\alpha, \bar{Q}^\beta\} = 2(\gamma^\mu)_\alpha{}^\beta P_\mu.$$

- In order to have susy invariant derivatives of superfields one needs supercovariant derivatives which read in this case

  $$D = \frac{\partial}{\partial\bar{\theta}} + i(\gamma^\mu\theta)\partial_\mu \quad \text{and} \quad \bar{D} = -\frac{\partial}{\partial\theta} - i(\bar{\theta}\gamma^\mu)\partial_\mu.$$

  Which anticommutation rules do they satisfy? Show, that they anticommute with the supercharges.
- Show that the kinetic part of the Lagrangian is the $\bar{\theta}\theta$-term of

  $$\frac{1}{2}\bar{D}\Phi D\Phi.$$

  Thus an invariant action can be written as

  $$\frac{1}{2}\int d^2x d^2\theta \bar{D}\Phi D\Phi.$$

- Show that the interaction term of the Lagrangian is

  $$\int d^2x d^2\theta\, W(\Phi).$$

- Construct all independent and invariant higher order derivative terms that give rise to a maximum of four space time derivatives. To do so one may calculate $(\bar{D}D)^n$.
  What happened to the e.o.m. of the auxiliary field after adding above mentioned terms to the action?