Problems in Supersymmetry

Sheet 6

Problem 19: Free Wess-Zumino model

Show that the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu A \partial^\mu A + \tfrac{1}{2}\partial_\mu B \partial^\mu B + \tfrac{i}{2}\bar{\psi}\slashed{\partial}\psi - \tfrac{1}{2}m^2(A^2 + B^2) - \tfrac{1}{2}m\bar{\psi}\psi = \mathcal{L}_0 + \mathcal{L}_m$$

with real scalar field A, real pseudo-scalar field B and Majorana spinor field ψ.

1. Show, that $\mathcal{L}$ is hermitean

2. Show that the supersymmetry transformations

$$
\begin{aligned}
\delta_\varepsilon A &= \bar{\varepsilon}\psi \\
\delta_\varepsilon B &= i\bar{\varepsilon}\gamma_5\psi \\
\delta_\varepsilon \psi &= -(i\slashed{\partial} + m)(A + i\gamma_5 B)\varepsilon
\end{aligned}
$$

   are compatible with the fact that A, B are real and ψ Majorana.

3. First calculate $\delta_\varepsilon \bar{\psi}$ and then calculate $\delta_\varepsilon \mathcal{L}$.

4. Calculate the commutator of two supersymmetry transformations (you may need the equations of motion!).

Problem 20: Noether current

Calculate the Noether current and charge of the susy-transformations in problem 20.

Problem 21: Interaction term

Add to the Lagrangian $\mathcal{L}_0$ of the non-interacting model the interaction term

$$\mathcal{L}_\mathrm{I} = -mgA(A^2 + B^2) - \tfrac{1}{2}g^2(A^2 + B^2)^2 - g\bar{\psi}(A - i\gamma_5 B)\psi.$$

1. What is the dimension of the coupling g?

2. What are the field equations of the interacting model with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\mathrm{I}$?

3. The susy-transformations of A and B are unchanged, but

$$\delta_\varepsilon \psi = -\left\{i\slashed{\partial} + m + g(A + i\gamma_5 B)\right\}\left\{A + i\gamma_5 B\right\}\varepsilon.$$