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Winter term 2017/18

Problems in Advanced Quantum Mechanics

Problem Sheet 7

Problem 15: Resolvent 4+1 = 5 points

The operator

$$R_H(z) = \frac{1}{z - H}$$

is the resolvent of H. It enters the Lippmann-Schwinger equation discussed in the lecture.

1. Find the position representation of the resolvent of a free particle $\langle \boldsymbol{x}|R_{H^{(0)}}(z)|\boldsymbol{x}'\rangle$ for z with positive real part: $\Re(z) > 0$.
   Hint: the Hamiltonian of a free particle is simple in momentum space.
2. Find the asymptotic behaviour for $r = |\boldsymbol{x}| \gg r' = |\boldsymbol{x}'|$
   Hint: you probably have seen the result in electrodynamics.

Bonus: +1 *point if you evaluate the integral at point 1. explicitly, by some analytic method.*

Problem 16: Born approximation 4 points

Calculate the differential cross sections in the Born approximation for scattering at the following potentials:

$$V_1(r) = V_0\,\mathrm{e}^{-a^2r^2}$$
$$V_2(r) = V_0\,\mathrm{e}^{-ar}$$

Bonus: +2 *points if you evaluate the necessary integrals explicitly, by some analytic methods.*

Problem 17: Scattering phase 6 points

Determine the scattering phases for scattering at the potential $V = A/r^2$ and calculate the differential cross section for $0 \le \mu A/\hbar^2 \ll 1$, where μ is the reduced mass appearing in the Schrödinger equation.
Hint: In the radial Schrödinger equation for $u_{E\ell} = r f_{E\ell}$ set $u_{E\ell} = \sqrt{r} g_{E\ell}$. The differential equation for $g_{E\ell}$ should be familiar to you. Your will probably meet a sum over Legendre polynomials. This can be simplified with the identity

$$\sum_{\ell=0}^{\infty} P_\ell(\cos\theta) = \frac{1}{2\sin(\theta/2)}.$$

Submission date: Thursday, 7. December 2017, before the lecture begins.