Problems: Quantum Fields on the Lattice

Prof. Dr. Andreas Wipf   WiSe 2019/20
MSc. Julian Lenz

Sheet 4

---

9 Detailed Balance

A statistical system has two states $\omega = 1, 2$ with equilibrium probabilities P_ω. Construct the most general form of a stochastic matrix $W(\omega, \omega')$ such that detailed balance is fulfilled, i.e.

$$P(\omega)W(\omega, \omega') = P(\omega')W(\omega', \omega) \tag{1}$$

for $\omega, \omega' \in \{1, 2\}$. What is the optimal choice for W such that

$$W^{\mathrm{eq}} = \lim_{n \to \infty} W^n \tag{2}$$

is approached fastest?

10 Two-Dimensional Ising Model

Write a program for simulation of the two-dimensional Ising model via

1. the Metropolis algorithm

2. the Wolff algorithm.

Assume $J = 1$ such that the critical temperature is approximately $T_c \approx 2.269$. Implement the following observables:

1. magnetization

2. susceptibility

3. autocorrelation time of the magnetization.

Which differences do you find between the algorithms? Which differences do you find between cold and hot start?