Friedrich-Schiller-Universität Jena Winter term 2019/20
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Exercises to „QFT on the Lattice"

Sheet 1

Problem 1: Warm-up

1. Write a short `Hello World!` program in C/C++ or Fortran to get comfortable with your development environment (command line, editor, compiler).
2. Write a program that computes

$$\int_0^1 \mathrm{d}x\, e^x$$

   via the (i) rectangle rule and via (ii) Simpon's rule. Compare your result with the analytical one as a function of the interval length.

Problem 2: Monte-Carlo Integration

In everyday research, one has to solve n-dimensional integral numerically with n ranging from 1 or 2 to *very, very large*. Classical integration methods as used above are not suitable for this task. Instead, one uses so-called *Monte-Carlo* methods which yield an estimate of the integral by the use of randomly drawn samples. The precision of this estimate grows with the number of random numbers used.

As a simple example, we will calculate the area of a circle (with radius $R = 1.0$). To do so, we draw N uniformly distributed pairs of random numbers $x_i, y_i \in [0, 1)$ and count the number N_{in} of pairs that fall inside the circle ($x_i^2 + y_i^2 \leq R^2$). Then, we have

$$\frac{N_{\text{in}}}{N} \approx \frac{1}{4} \frac{A_{\text{circle}}}{A_{\square}}$$

where $A_{\square} = 1$ is the area of the first quadrant of the unit circle. One could even get an estimate of π via this method using $A_{\text{circle}} = \pi R^2$.

1. Write a program that returns the area of a circle ($n = 2$) and try how the estimate approaches the exact value for large N.
2. Generalize your program to arbitrary dimension and compare to the exact result

$$V = \begin{cases} \frac{\pi^k}{k!} & n = 2k, \\ \frac{2^{k+1}\pi^k}{(2k+1)!!} & n = 2k+1. \end{cases}$$