Effective Models for Confining Gauge Theories: Analytical and Numerical Tests

Dissertation
zur Erlangung des akademischen Grades
doctor rerum naturalium (Dr. rer. nat.)

vorgelegt dem Rat der Physikalisch–Astronomischen Fakultät
der Friedrich–Schiller–Universität Jena

von Dipl.–Phys. Leander Dittmann
geboren am 3. November 1974 in Weida

Gutachter

1. Prof. Dr. Andreas Wipf, Jena
2. Prof. Dr. Michael Müller-Preussker, Berlin
3. Prof. Dr. Hugo Reinhardt, Tübingen

Tag der letzten Rigorosumsprüfung: 29. Januar 2004
Tag der öffentlichen Verteidigung: 5. Februar 2004

Contents

1. Introduction

Once upon a time ancient Greeks came up with the idea that all matter consists of some fundamental entities. While Anaximenes assumed the element Air to be the fundamental origin [1], soon afterwards Democrit and Leukipp supposed that matter consists of elementary particles which they called atoms according to the Greek word for indivisible. Since then, more than twothousand years passed by during which no considerable progress in particle physics was made. Things, however, changed dramatically within the last hundred years due to the invention of quantum field theory and the development of particle accelerators. In addition, theoretical and experimental physics have been crucially affected by the evolution of computer technology. Thanks to those efforts, a theoretical framework of particle physics has emerged which is known as the Standard Model. This is a gauge theory with symmetry group $SU(3) \times SU(2) \times U(1)$ where the $SU(2) \times U(1)$ symmetry is associated with the electro-weak theory [2, 3, 4], a unified description of two fundamental forces corresponding to electromagnetic and weak interactions. The $SU(3)$ gauge group is associated with strong interactions between hadrons[1] described by Quantum Chromodynamics (QCD). Gravitation as the fourth fundamental force, however, is not captured by the Standard Model. As shown in Tab. 1.1, on the subatomic level it is many orders of magnitudes weaker than all the other forces. Thus it is expected to play no role in this regime. Nevertheless, theoretical physicists and mathematicians keep working hard on finding an even more fundamental theory that treats all four forces in a unified way and thus might even describe phenomena near the very beginning of the universe.

The strong interactions provide the forces acting between quarks and gluons which yield the binding of protons and neutrons in nuclei. Many fundamental questions in particle physics are related to this force. On the other hand, however, the study

[1] Strongly interacting particles are called hadrons.

force	strong	electromagnetic	weak	gravitational
rel. strength	1	10^{-2}	10^{-5}	10^{-39}
range [m]	$\sim 10^{-15}$	∞	$\sim 10^{-18}$	∞
acts on	quarks, hadrons	electr. charges	leptons, quarks	all matter
mediator	gluons	photons	$W^{\pm}/Z^0$-bosons	gravitons

Table 1.1.: Properties of fundamental interactions. The relative strengths are measured between two up-quarks at distance $3 \cdot 10^{-17}$m.

of QCD is a very demanding task. It is this field where the present work aims to contribute to the scientific understanding of Nature.

Phenomenology of Strong Intercation

In the early twentieth century, Rutherford concluded from his scattering experiments that the atom is mostly empty space except for a small and dense core containing positively charged particles, the protons. But if so, one would expect this nucleus to burst apart due to the electromagnetic repulsion between the equally charged protons. Chadwick's discovery of the neutron as a second constituent of the nucleus still did not answer the question why it is stable. Obviously, there had to be another, yet unknown, mechanism responsible for that.

In the middle of the last century cosmic ray experiments and, even more important, the availability of newly developed particle accelerators led to a plethora of known particles, the 'particle zoo', which called for an explanation to bring order into this chaos.

The great variety of hadrons, subdivided into baryons and mesons, has been classified by Gell-Mann [5] and Zweig [6]. They invented the quark model according to which hadrons can be grouped into multiplets of $SU(3)_f$ associated with the quantum number 'flavor'. However, particles in the fundamental triplet carry fractional electric charge, a property which has never been observed in any experiment.

Later on, electron-nucleon scattering experiments confirmed Bjorken's prediction of the scaling of structure functions [7, 8] which could be explained by Feynman's parton model [9], stating that hadrons consist of point-like sub-particles, called partons. In

1971 scattering experiments with neutrinos and nucleons indicated that the data could indeed be accounted for if the partons had exactly the properties of the particles in the fundamental triplet of $SU(3)_f$. Therefore, Feynman's partons got identified with the quarks and antiquarks. These particles are fermions and come in six flavors as listed in Tab. 1.2. For example, the proton with charge 1 is built up from two up-quarks and a down-quark, uud, whereas the charge-zero neutron is a udd-state.

quark	u (up)	d (down)	s (strange)	c (charme)	b (bottom)	t (top)
mass [GeV]	0.003(2)	0.007(2)	0.117(38)	1.2(2)	4.25(25)	174(5)
charge [e]	2/3	−1/3	−1/3	2/3	−1/3	2/3

Table 1.2.: Masses and electromagnetic charges of the six quark flavors [10].

However, implementing the quark scheme ran into trouble because the properties of the Δ^{++}-resonance, originally discovered by Fermi, forced one to combine three identical fermions u into a completely symmetric ground state, $\Delta^{++} = uuu$. This is, of course, forbidden by the Pauli principle. Another unsatisfactory issue was that a number of possible combinations like quark-quark, antiquark-antiquark or single quarks had never been observed. Both problems were solved by introducing a new quantum number of quarks, called color, with corresponding symmetry group $SU(N_c)$. Quarks are supposed to come in three colors, red, green and blue, which implies $N_c = 3$. Thus, the quarks in the Δ-ground state are now distinguishable and hence not forbidden anymore. But in contrast with observations, there seem to be different kinds of protons or neutrons if one thinks of all color combinations of the quarks u and d. For this reason one assumes all particles observed in Nature to be colorless, or equivalently, to be unchanged under rotations in color space. In other words, observable particles are represented as color singlets, i.e. states combining all three colors or color-anticolor states. Quite recently, several groups have announced the observation of colorless combinations of five quarks [11, 12, 13, 14]. The fact that *there are no isolated particles in Nature with non-vanishing color charge* [15] is known as 'color confinement'[2]. Due to the different quark masses the flavor symmetry is not exactly realized in Nature in contrast to the color symmetry which is unbroken.

[2]See, however, [16] where weak confinement, i.e. the absence of free quarks and gluons, is distinguished from strong confinement which refers to an indefinitely rising potential. See Fig. 1.1.

QCD

This last fact prepared the ground to formulate a theory for the strong interaction, known as Quantum Chromodynamics [17, 18, 19]. Being a gauge theory with the non-Abelian symmetry group $SU(N_c)$ it describes the interaction of quarks with $\dim SU(N_c) = N_c^2 - 1$ color charged gauge bosons, the gluons. It is a local theory like all other theories in the Standard Model.

The QCD-Lagrangian,

$$\mathcal{L}_{QCD} = \mathcal{L}_f + \mathcal{L}_g \ , \tag{1.1}$$

decomposes into a fermionic part including the quarks,

$$\mathcal{L}_f = \sum_{\alpha,\beta=1}^{N_c} \sum_{f,f'=1}^{N_f} \bar{\psi}_{f\alpha}(\mathrm{i}\gamma^\mu D_\mu^{\alpha\beta} - m_{ff'}\delta^{\alpha\beta})\psi_{f'\beta} \ , \tag{1.2}$$

and a purely gluonic part

$$\mathcal{L}_g = -\frac{1}{4g^2} \sum_{a=1}^{N_c^2-1} F_{\mu\nu}^a F^{\mu\nu a} \ , \tag{1.3}$$

which describes the kinematic of the gluons and by itself is a non-trivial Yang-Mills theory. The field strength tensor belongs to the adjoint representation, and in terms of the Lie algebra valued gauge field $A_\mu = A_\mu^a T^a$ it is given by

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f_{abc} A_\mu^b A_\nu^c \ , \tag{1.4}$$

where the f_{abc} are the structure constants. The T^a denote a complete set of generators of the gauge group normalized according to

$$\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab} \ , \quad [T^a, T^b] = \mathrm{i} f_{abc} T^c. \tag{1.5}$$

The quark fields $\psi_{f\alpha}$ with masses m_f are labelled by their flavor and color quantum number, f and α respectively. g is the strong coupling constant, and the covariant derivative $D_\mu = \partial_\mu - \mathrm{i}A_\mu$ ensures local gauge invariance.

Let us note that it is possible to add a 'θ-term',

$$\mathcal{L}_\theta \propto \frac{\theta}{16\pi^2} \mathrm{tr}(\epsilon_{\alpha\beta\mu\nu} F^{\alpha\beta} F^{\mu\nu}) \ , \tag{1.6}$$

to the Lagrangian as a source for CP violation. Usually, such a term is discarded because it can be expressed as a divergence of a current and thus appears as a surface term in the action. However, a term of this form survives in non-Abelian gauge field theories because there are non-trivial instanton configurations. We nevertheless discard this term in this thesis because of the very small experimental upper bound for the QCD θ parameter, $\theta < 10^{-9}$ [20].

Apart from the realistic $N_c = 3$ for QCD, of great theoretical interest is also the study of Yang-Mills theory with arbitrary $N_c \geq 2$. There are crucial effects like asymptotic freedom and confinement of color charges that are generally believed to be a consequence of the non-Abelian nature of the gauge group and thus should occur for all $SU(N_c)$. Of particular interest, on the one hand, is the large N_c limit, where typical leading corrections scale with the small parameter $1/N_c$. On the other hand, there is the $SU(2)$-case which is the simplest for analytical and numerical investigations but nevertheless leads to reasonable results in understanding confinement.

Confinement

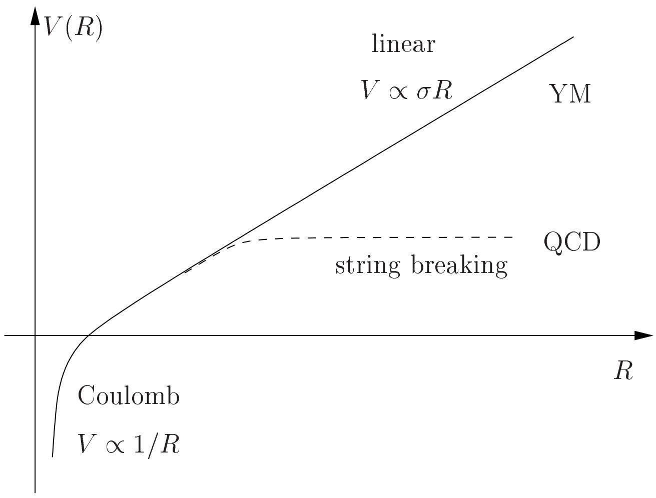
The most essential property of QCD is confinement [21] and its understanding is one of the most exciting challenges of modern physics [22].

Confinement and asymptotic freedom are closely related to the running coupling of QCD which has its origin in the non-Abelian nature of the theory. The gluons, also carrying color, not only interact with quarks but also among themselves. By turning into pairs of gluons they spread out the effective color charge of the quark and the closer one approaches the quark color charge, the more the measured charge decreases due to this anti-screening. This behavior shows up in the running coupling which decreases for high energy (short distance) and increases for low energy (large distance). The first fact leads to a vanishing interaction in the ultraviolet region at energy scale $Q^2 \to \infty$. Quarks become essentially free in this limit, and perturbation theory can be applied. The second fact signals the occurence of nonperturbative effects in the infrared region of the theory. However, this is the regime where one has to investigate issues like the hadron spectrum, the topological structure of the vacuum, the $U_A(1)$-problem, chiral symmetry breaking and also confinement of color.

The latter denotes the fact that color charges have never been observed in free space, i.e. at distances of about 1 fm, though visible in deep-inelastic scattering experiments at scales $\ll 1$ fm. To get some intuition one may imagine to separate a quark q and an antiquark $\bar{q}$ from each other. Inspired by superconductors where the electromagnetic field lines are expelled from the interior, Nambu proposed the QCD vacuum to behave like a dual superconductor where the chromoelectric field lines between quark and antiquark are squeezed into a thin flux tube with constant energy density per unit length, the string tension σ. Thus, the total energy of such a configuration is proportional to the $q\bar{q}$-separation. If the increasing energy is sufficiently large, an energetically favorable new quark-antiquark pair is created from the vacuum and breaks the string into two short pieces. This procedure ends when their energy has degraded into clusters of quarks and gluons, each colorless, while the strong color coupling turns them into hadrons as the particles to be detected.

To some extend the effects are already present in the gluonic part of the theory and it is much easier to restrict to that and look for their remnants in pure Yang-Mills theory.

In pure Yang-Mills theory the phenomenon of confinement is described by introducing a static quark potential $V(R)$ for quarks in the fundamental representation which is of Coulomb type for small distances R and rises linearly for large distances [23]. Due to the emerging virtual quark-antiquark pairs in full QCD or color charges in higher representations the string breaks and the potential flattens off, see Fig. 1.1. In that case the linear behavior is valid only in an intermediate range. Comparing different representations of the sources one observes Casimir scaling of the string tension [24, 25], which means that potentials of sources in different representations are proportional to each other with ratios given by the corresponding ratios of eigenvalues of the quadratic Casimir operators. It should be mentioned that there is another approach. According to [26, 27], for $SU(N)$ with $N \geq 4$ typical sources may be thought of as k fundamental charges, and there are new stable confining strings, called k-strings. For $SU(N)$ there are non-trivial stable k-strings up to a k-value equal to the integer part of $N/2$ with string tensions σ_k depending on k and N. The different models (e.g. Casimir scaling, M-theory approach to QCD [28], bag model


$V(R)$
linear
$V \propto \sigma R$
YM
QCD
string breaking
R
Coulomb
$V \propto 1/R$