Instanton-Induced Defects in Gauge Theories

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Contents

‘... the forces might become large enough to **confine** the quarks. That is the foremost problem of QCD.’ [1]

‘The outstanding problem in QCD is to explain long distance phenomena, in particular why we do not see quarks and gluons as physical objects - the so called problem of ”quark **confinement**”’. [2]

‘It is unlikely that one will ever prove from first principles that permanent **confinement** takes place...’ [3]

‘Quark **confinement** is not yet completely solved.’ [4]

‘The most essential property of QCD is **confinement**.’ [5]

‘Die spektakulärste Aussage der QCD ist zweifellos das Verbot freier, nicht in Hadronen gebundener Quarks.’ [6]

‘A long standing and yet unsolved problem is to explain color **confinement** in QCD.’ [7]

‘Over the last two decades various attempts have been aiming at a qualitative understanding and modelling of two basic properties of QCD: quark **confinement** and chiral symmetry breaking.’ [8]

‘Therefore, *understanding* **confinement**, in my opinion, is one of the most exciting challenges of modern physics.’ [9]

‘**Confinement** is something of a mystery. It is certainly the most striking qualitative phenomenon in QCD. Still we do not even have a satisfactory definition of what exactly is meant by this word.’ [10]

1. Introduction

1.1. The Gauge Theory of Strong Interactions

At present-day energies the *Standard Model* is the fundamental theory of elementary particles and their interactions (for a recent review see [11]). Its matter content consists of fermionic fields carrying a representation of the gauge group $U(1) \times SU(2) \times SU(3)$. The interactions are provided by gauge fields, i.e. vector fields in the Lie algebra of this gauge symmetry[1]. The part containing the strong interactions is named *Quantum Chromodynamics* (QCD); the Lagrangian density reads,

$$L = \sum_{k=1}^{N_f} \bar{\psi}^k (i\gamma^\mu \mathrm{D}_\mu - m^k)\psi^k - \frac{1}{2}\,\mathrm{tr}\, F^{\mu\nu}F_{\mu\nu}\,, \tag{1.1}$$

where $\{\psi^k\}_{k=1,\dots,N_f}$ stands for the fields of quark flavours ($N_f = 6$: up, down, strange, charm, bottom, top), m^k for their masses, $A_\mu = \sum_{a=1}^{8} A_\mu^a T_a$ for the gluon fields, $\mathrm{D}_\mu = \partial_\mu - iA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ for the covariant derivative and the field strength, respectively. $\{T_a\}_{a=1,\dots,8}$ are the Gell-Mann matrices, i.e. the generators of (the fundamental representation of) the Lie algebra $su(3)$ and $\{\gamma^\mu\}_{\mu=0,\dots,3}$ are the Dirac matrices in Minkowski space. Furthermore, the coupling constant g is set to unity, so are $\hbar$ and c; we suppress spinor indices and use the Einstein summation convention.

This Lagrangian is invariant under local gauge transformations with $g(x) \in SU(3)$,

$$\psi \to g\psi\,, \quad \bar{\psi} \to \bar{\psi}g^\dagger\,, \quad A_\mu \to gA_\mu g^\dagger + ig\partial_\mu g^\dagger\,, \quad (\mathrm{D}_\mu \to g\mathrm{D}_\mu g^\dagger\,, F^{\mu\nu} \to gF_{\mu\nu}g^\dagger)\,, \tag{1.2}$$

which may be thought of as rotations in a colour space spanned by 'red', 'green' and 'blue'. Due to the character of the matrix group $SU(3)$, such a gauge theory is called non-Abelian. Its quantum version exhibits very interesting features already at the perturbative level: the gauge fields interact among themselves via 3- and 4-vertices, they carry colour themselves. As a consequence the perturbative β-function shows[2] that the running coupling constant is small at high energies/short distances and large at low energies/long distances, respectively.

[1] Although the gravitational interaction can be described as a gauge theory as well, a quantum version of it is not well-defined yet; in our considerations gravitational effects are negligible.

[2] if there are not too many flavours as is realised in nature

The first fact, the so-called *asymptotic freedom* in the ultraviolet region, is the basis for many confirmations of QCD in deep inelastic scattering: probing strongly interacting particles at high energies one finds the quark constituents ('partons') to be essentially free. The second fact signals the occurence of *non-perturbative effects* in the infrared region of the theory. Indeed, *the fundamental quarks in the Lagrangian do not appear as asymptotically free states in nature.* Instead they are bound together to mesons, $\psi\bar{\psi}$-states, and baryons, $\psi\psi\psi$-states. These hadrons[3] are all singlets under the colour group $SU(3)$. In other words, *free coloured states have never been observed.* This effect is called colour *confinement.* It is generally believed to be a consequence of the non-Abelian nature of the gauge group, i.e. it should occur for all $SU(N)$, $N \geq 2$. However, its derivation from the Lagrangian (1.1) remains an *open problem of the Standard Model.*

One expects a similar effect to happen in the pure glue sector of QCD: at low energies *glueballs*, bound states of gluons, should appear. These objects have not been observed in experiments yet. Nevertheless, QCD sum rules and lattice simulations predict their masses to be around 1.5 GeV (see [12] for a review). Such a *mass gap* would force any correlation function in this theory to decay exponentially thus explaining the absence of long-ranged fields in QCD. Being one of the 'Millenium Prize Problems' [13] it is also interesting from a purely mathematical point of view.

In the chiral limit where the quark masses are neglected, QCD shows another non-perturbative property, the *chiral symmetry breaking.* Left handed and right handed quarks decouple in the Lagrangian (1.1) when $m = 0$. This amounts to two commuting flavour symmetry groups which can be rewritten as a product of a vector and an axial symmetry[4]. The latter would predict all hadrons to come in pairs of opposite parity, which is not the case. Chiral symmetry is broken by the *chiral condensate* $\langle\bar{\psi}\psi\rangle$ which couples left handed to right handed quarks like a mass term. The would-be Goldstone bosons for $SU(N_{\mathrm{f}} = 2)$ are the pions.

Presumably, QCD undergoes a phase transition at sufficiently high temperatures and/or densities: hadrons start to overlap and quarks and gluons are free to travel. Beyond this *deconfinement phase transition* a new state of matter occurs, the *quark gluon plasma.* It is assumed to be realised in the early universe and in neutron stars. Lattice simulations [14] predict the critical temperature[5] to be 170 MeV, but the observation of

[3] To be precise: hadrons have the same quantum numbers as if they consist of the given *valence* quarks; in fact, they also contain *sea* quarks and gluons induced by quantum fluctuations.

[4] In fact, the vector symmetry is a subgroup of the flavour symmetry group, while the axial symmetry is only a coset.

[5] for two flavours in the chiral limit

the quark gluon plasma in heavy ion collisions has not been achieved yet.

All these non-perturbative phenomena – as well as others we have not mentioned like the $U_A(1)$ problem – should follow from QCD as the fundamental theory or a proper effective theory thereof. In many cases the effects are mainly due to the pure glue part and it is easier to look at their remnants in *pure Yang-Mills* (YM) *theories* which are defined by neglecting the quark term in the Lagrangian (1.1). This is tantamount to treating the quarks as very heavy non-dynamical objects. Therefore, this approximation is also called *quenched QCD*. We will mainly adopt this point of view in due course.

To make progress in a better understanding of colour confinement is the main motivation of this work. Therefore, this phenomenon is described in detail in the next section, followed by a discussion of lattice gauge theory and two effective theories modelling confinement and glueball formation.

1.2. Confinement

The intuitive picture of confinement is the following (Figure 1.1): In order to separate a quark q and an anti-quark $\bar{q}$ (or three quarks) one has to bring more and more energy into the system. This energy is used to create a new quark anti-quark pair $q'\bar{q}'$ from the vacuum and one ends up with two hadrons instead of free quarks. In a field theoretic description this means that the lines of the gluon field are concentrated in narrow tubes between the quarks. The latter are the sources of the *chromoelectric field*. In contrast to that, the lines of the electric field between two electric sources are spread, leading to the well-known Coulomb potential which does not confine[6]. The pair production described above is also called *string breaking*, because it breaks the flux tube into two pieces.

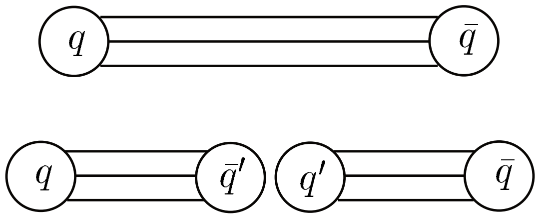
One can describe this phenomenon in *pure* YM theory by introducing a *heavy quark potential* $V_{q\bar{q}}(R)$ (Figure 1.1). For large separations R it rises linearly with R [16],

$$V_{q\bar{q}}(R) \to \sigma R \qquad \text{for large } R\,. \tag{1.3}$$

The quarks experience a *constant force*, as is very intuitive, since the densitiy of field lines is independent of R. We will refer to such a potential as confinement; the quarks are confined because an arbitrarily large amount of energy is needed to separate them.

The factor σ is called *string tension*. It can be estimated from the spectrum of charmonium J/Ψ (see e.g. [17]) since the charm quarks forming these hadrons are rather heavy. In the modern literature, the value of the string tension is $\sigma \simeq 1$ GeV/fm.

[6] One can show that in electrodynamics a tube-like configuration between electric sources is unstable and evolves in time to the Coulombic configuration [15].


q
$\bar{q}$
q
$\bar{q}'$
q'
$\bar{q}$