Preface

The theory of figures of equilibrium of rotating, self-gravitating fluids was developed in the context of questions concerning the shape of the Earth and celestial bodies. Many famous physicists and mathematicians such as Newton, Maclaurin, Jacobi, Liouville, Dirichlet, Dedekind, Riemann, Roche, Poincaré, H. Cartan, Lichtenstein and Chandrasekhar made important contributions. Within Newton's theory of gravitation, the shape of the body can be inferred from the requirement that the force arising from pressure, the gravitational force and the centrifugal force (in the corotating frame) be in equilibrium. Basic references are the books by Lichtenstein (1933) and Chandrasekhar (1969).

Our intention with the present book is to treat the *general relativistic* theory of equilibrium configurations of rotating fluids. This field of research is also motivated by astrophysics: neutron stars are so compact that Einstein's theory of gravitation must be used for calculating the shapes and other physical properties of these objects. However, as in the books mentioned above, which inspired this book to a large extent, we want to present the basic theoretical framework and will not go into astrophysical detail. We place emphasis on the rigorous treatment of simple models instead of trying to describe real objects with their many complex facets, which by necessity would lead to ephemeral and inaccurate models.

The basic equations and properties of equilibrium configurations of rotating fluids within general relativity are described in Chapter 1. We start with a discussion of the concept of an isolated body, which allows for the treatment of a single body without the need for dealing with the 'rest of the universe'. In fact, the assumption that the distant external world is *isotropic*, makes it possible to *justify* the condition of 'asymptotic flatness' in the body's far field region. Rotation 'with respect to infinity' then means nothing more than rotation with respect to the distant environment (the 'fixed stars') – very much in the spirit of Mach's principle. The main part of Chapter 1 provides a consistent mathematical formulation of the rotating fluid body problem within general relativity including its thermodynamic aspects. Conditions for parametric (quasi-stationary) transitions from rotating fluid bodies to black holes are also discussed.

Chapter 2 is devoted to the careful analytical treatment of limiting cases: (i) the Maclaurin spheroids, a well-known sequence of axisymmetric equilibrium configurations of homogeneous fluids in the Newtonian limit; (ii) the Schwarzschild spheres, representing non-rotating, relativistic configurations with constant mass-energy density; and (iii) the relativistic solution for a uniformly rotating disc of dust. The exact solution to the disc problem is rather involved and a detailed derivation of it will be provided here, which includes a discussion of aspects that have not been dealt with elsewhere. The solution is derived by applying the 'inverse method' – first used to solve the Korteweg–de Vries equation in the context of soliton theory – to Einstein's equations. The mathematical and physical properties of the disc solution including its black hole limit (extreme Kerr metric) are discussed in some detail. At the end of Chapter 2, we show that the inverse method also allows one to *derive* the general Kerr metric as the unique solution to the Einstein vacuum equations for well-defined boundary conditions on the horizon of the black hole.

In Chapter 3, we demonstrate how one can solve general fluid body problems by means of numerical methods. We apply them to give an overview of relativistic, rotating, equilibrium configurations of constant mass-energy density. Configurations with other selected equations of state as well as ring-like bodies with a central black hole are treated summarily. A *related website* provides the reader with, amongst other things, a computer code based on a highly accurate spectral method for calculating various equilibrium figures.

Finally, we discuss some aspects of stability of equilibrium configurations and their astrophysical relevance.

We hope that our book – with its presentation of analytical *and* numerical methods – will be of value to students and researchers in general relativity, mathematical physics and astrophysics.

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